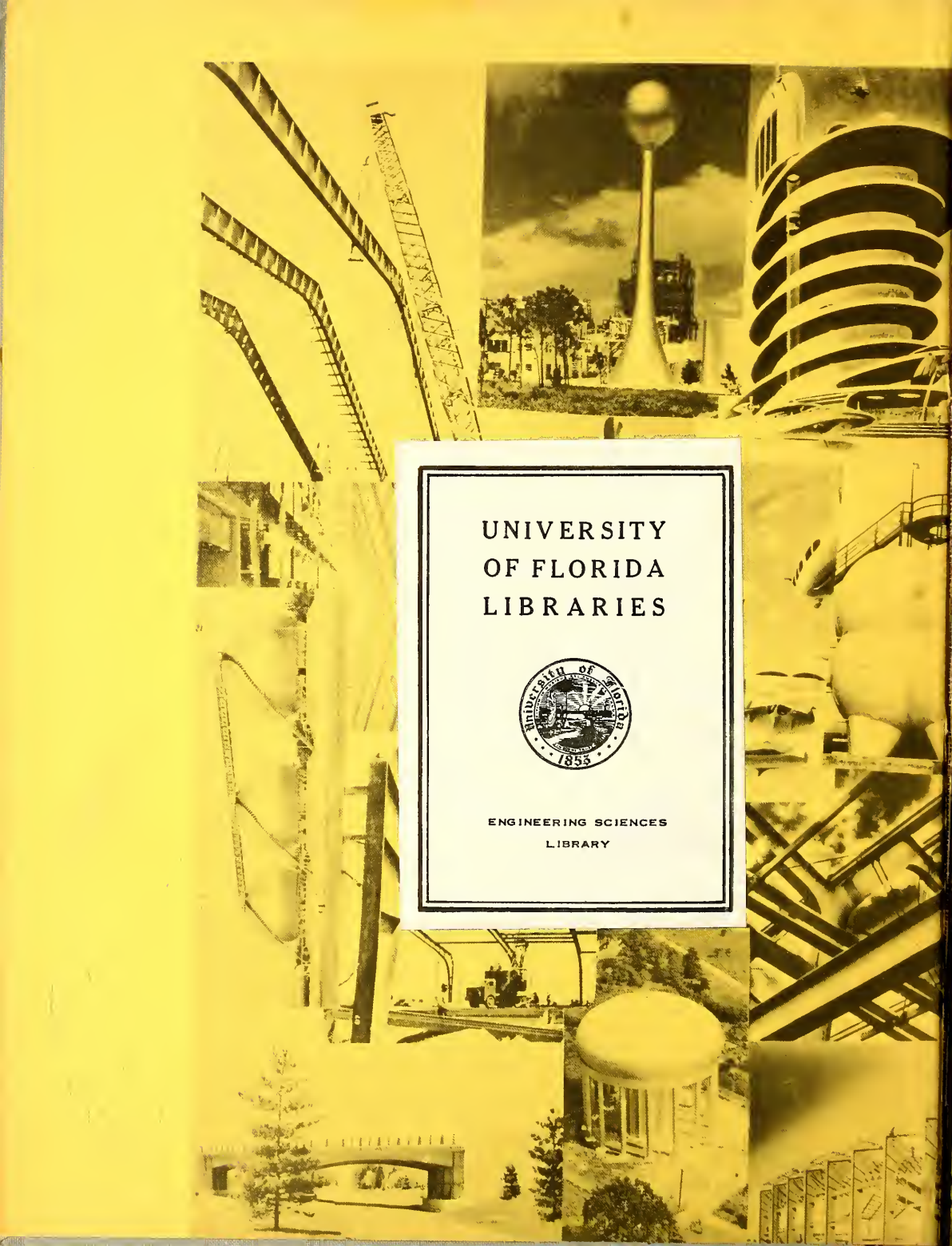
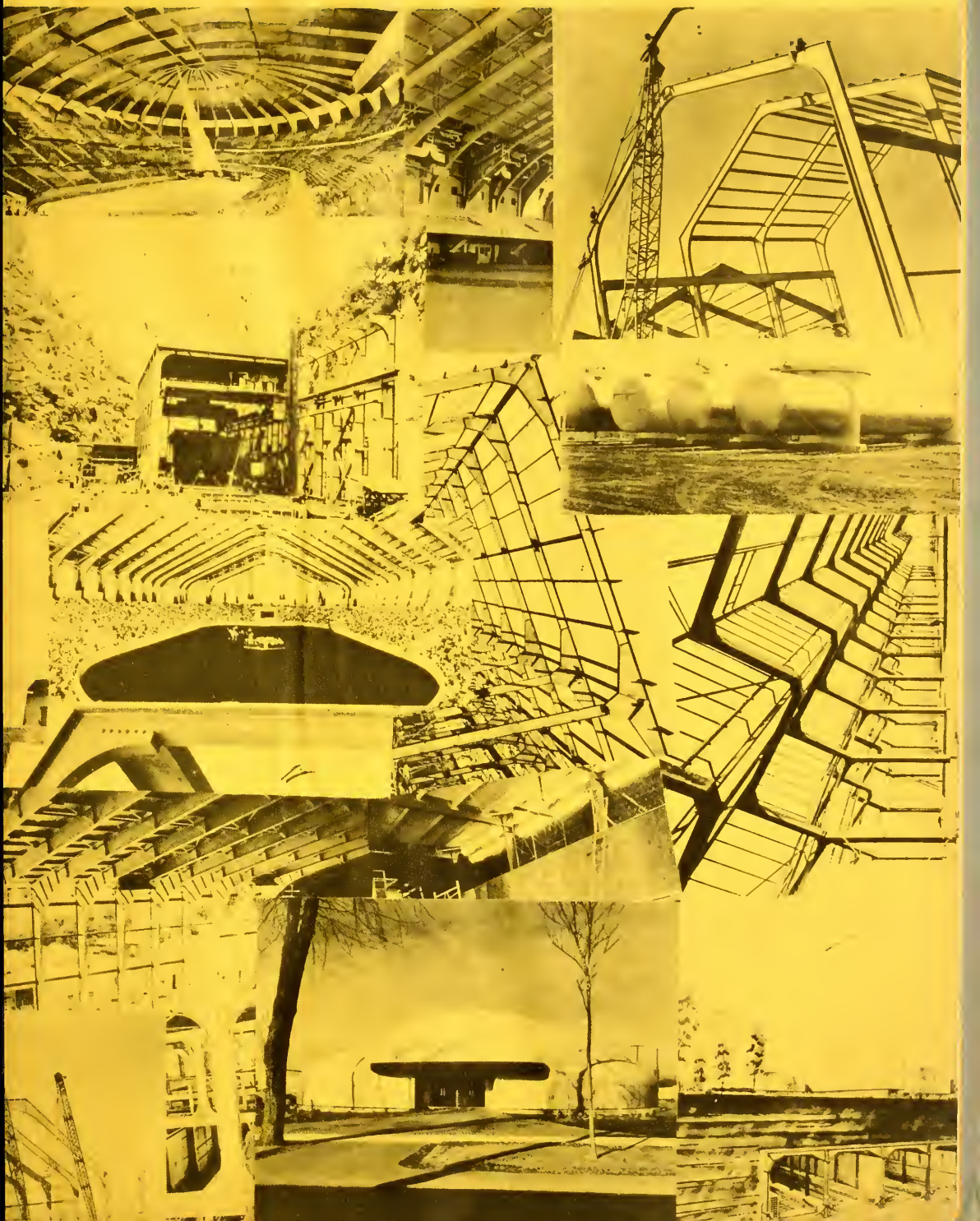
steel rigid frames
MANUAL
design & construction
By
MARTIN P. KORN



UNIVERSITY
OF FLORIDA
LIBRARIES

ENGINEERING SCIENCES
LIBRARY



UNIVERSITY
OF FLORIDA

The James F. Lincoln
Arc Welding Foundation
WELDING
LIBRARY

steel rigid frames MANUAL design & construction

MARTIN P. KORN, M.ASCE
CONSULTING ENGINEER

Member, Committee on Building Codes
American Welding Society

Copyright, 1953
MARTIN P. KORN

Published by J. W. EDWARDS, INC.
Ann Arbor, Michigan 1953

"Where there is no vision, the people perish"
Proverbs 29:18

Digitized by the Internet Archive
in 2011 with funding from
LYRASIS Members and Sloan Foundation

http://www.archive.org/details/steelrigidframes00korn

FOREWORD

This book may well be called a labor of love because it represents the author's contribution to the engineering profession—a contribution that is made with no thought other than that of service. In the preparation of this volume, the author has drawn from his own extensive experience of more than forty years as a Consultant and structural engineer and as a pioneer in the field of rigid frames, as well as from other "fontes et origines" not generally available to the designing engineer.

Within recent years the rigid frame has occupied an expanding area in the field of structures. Bulky riveted or bolted connections and details have been superseded largely by trim designs compatible with improved welding and fabrication technique.

Although this text will not be found to be highly theoretical in approach, sufficient fundamental theory is presented to give an adequate concept of the proper analysis and design of this bold, versatile and economical type of framing. The assemblage of basic information, design data, and an extensive compilation of current practice adapted for use by the student or the experienced practitioner makes this volume an excellent source book.

Unique in its field, this manual should fill a genuine professional need. The nicely balanced presentation of theory and practice illustrated by an unusual collection of actual designs is commendable. The design tables and other features will aid greatly in developing and stabilizing the judgment of the student and enable the experienced practitioner to design more or less routine structures without resort to laborious calculations.

Congratulations are due the author for this lucid, concise and original presentation.

C. L. Eckel, Dean
College of engineering
University of Colorado

Boulder
May, 1953

SAGE OF THE ROCKIES

Dean C. L. Eckel

A rigid frame pioneer with vision and courage
A teacher inspiring from majestic heights.

INTRODUCTION

This book is an outgrowth of lectures on steel rigid frames given by the author over a number of years before various professional groups and at universities. It follows requests that the lectures be published. Slides were shown of living structures in a moving parade of dramatic shapes as tableaux of adventures in creative design—the smallest beside the largest, the earliest and the latest.

Riveted structures passed in review in striking shafts of compacted strength, heavily armored in their splices and joints with fortifications of plates and regiments of rivets.

Welded rigid frames appeared naked by comparison, stripped for action, simple and streamlined, younger in years and lighter in the joints.

A few with stress misbehavior limped sadly along.

The rigid frames appeared as slender steel ribs—each a structural tour-de-force in monolithic form hurtling over space between outside walls, spectacular in flight, dynamic in action.

No steel truss could match their appearance, their graceful sweep, their show of so much strength in so little space. Lithe and adaptable to fancy or function, the steel rigid frame was everywhere acclaimed.

The lectures revealed living evidence of creativeness and daring and superior judgment by pioneering engineers. For steel rigid frames were practically unknown not too long ago. Structures were built before tests had been made. Even theory of design came along later.

Assembled here in a book without precedent, those pioneering structures are available for study, accompanied by rigid frames of contemporary design and pertinent subjects which were not in the lectures. To the practicing engineer, the designer, the architect, the builder, the book is a companion with practical experience. To the campus, it presents a pageant of living rigid frames for studies from life of theory in action. It is thus not a textbook, but a manual and guide encompassing the steel rigid frame from its conception to erection. The need for such a guide is evident from the complexity of the structures built to date and the limitations of textbooks which publish only theory.

Rigid frames come forth as structural expressions of designers' fancies, no two alike, with or without benefit of stress behavior knowledge. Theories vary from mathematical jamborees down to home-cooked recipes. The haunches follow individual tastes and knowledge,

swinging out towards center with individual thrusts. Some are loaded down with stiffeners, excessive welding or rivets. Others with similar spans come light by comparison—stark naked without any stiffeners or web or flange reinforcing. Each is distinctly individual to the designer. And each without fail always develops into a masterpiece. For no matter what technical concoction some designers make of a steel rigid frame, it always looks striking, even beautiful when completed—it is just that kind of structural phenomenon, especially the welded type.

But it will ever be thus because of its unlimited potential for creative design. Continuity is exploited into daring shapes of individual members and built-up assemblies. Already multi-dimensional frames are beginning to appear, foretelling space structures of the future. And with design possibilities so unlimited come license and possible threats to safety, examples of which were shown in my lectures. Boundaries for safety and deficiencies in design will be emphasized, therefore, throughout the book by comments based on tests, research and experience.

Requests from teachers for actual design data of the structures shown in the lectures came to me with other requests from engineers and architects for quick design data. Tables for preliminary design of single span frames up to 150 feet in span have, therefore, been prepared and included in the book.

However, in the cavalcade of living rigid frames which follows the design tables, the reader may well pause in meditation and study of spans larger than 150 feet. For there he will acquire wisdom from rigid frame pioneers, creative engineers of courage, vision and initiative.

Were it not for those great men and their original works, designers would have nothing to stand on! Because of them and their great structures which are among the largest rigid frames built, the designer will have something to stand on and work toward progress and better design.

To those great engineers and outstanding teachers of engineering who have here bequeathed their legacy of knowledge and experience, and to all others who have contributed to creative design of steel rigid frames, this book is dedicated as a public educational service.

Martin P. Korn

ACKNOWLEDGEMENTS

It is a pleasure to acknowledge the assistance rendered me by so many persons and organizations, and I wish especially to thank those whose names are listed below:

The management and associates of the nationally known firm of Giffels & Vallet, Inc., L. Rosetti, Associated Engineers and Architects of Detroit, Michigan, for their technical and financial assistance in research and development of the data assembled here and for their sponsorship of the series of lectures by the author which led to publication of this text.

Lloyd T. Cheney, Associate Professor of Civil Engineering, Wayne University, for his assistance and constructive criticism in producing the design tables and for directing the computation staff doing that work.

Frank K. McDanel, President, American Bridge Division, U. S. Steel Company, leading fabricator and pioneer of the split-beam arch, for unusual courtesies extended by all hands, enabling the author to collate data on the latest shop practice and erection methods of steel rigid frames.

Mr. E. R. Hoffman, Superintendent of Lighting, City of Seattle, and Mr. C. W. Cutler, Skagit Project Engineer, outstanding hydroelectric experts in the Northwest, for releasing to this book, design data, drawings and photographs of the Ross Powerhouse, a most unusual structure.

Dr. L. C. Maugh, Professor of Civil Engineering, University of Michigan, leading authority on theory of rigid frame design, for his review and constructive criticism of the author's text pertaining to knees.

M.P.K.

CONTRIBUTORS

AMERICAN BRIDGE Division, U. S. STEEL CO.
Pittsburgh 30, Pa.

A. AMIRIKIAN
Head Designing Engineer, Bureau of Yards & Docks, Navy Department, Washington 25, D.C.

DR. H. C. BOARDMAN
Director of Research, Chicago Bridge & Iron Company

J. R. BOUTEN
Director & Architect, Philips Electrical Works, Eindhoven, Holland

RALPH E. COBLENTZ
Chief Engineer, A. M. Strauss, Inc., Fort Wayne 2, Ind.

C. W. CUTLER
Skagit Project Engineer, City of Seattle, Wash.

C. L. ECKEL
Dean, College of Engineering, University of Colorado, Boulder, Colo.

PROF. WM. J. ENEY
Head, Dept. of Civil Engineering & Mechanics and Director, Fritz Engineering Laboratory, Lehigh University

PAUL J. FOEHL
Chief Engineer, Midwest Steel & Iron Works Co., Denver 17, Colo.

GIFFELS & VALLET, INC., L. ROSETTI
Associated Engineers & Architects, Detroit 26, Mich.

ARTHUR G. HAYDEN
Consulting Civil Engineer, St. Michaels, Md.

E. R. HOFFMAN
Superintendent of Lighting, City of Seattle, Wash.

PROF. GLENN W. HOLCOMB
Oregon State College, Corvallis, Ore.

J. F. JELLEY, REAR ADMIRAL, CEC, USN
Chief, Bureau of Yards & Docks, Washington 25, D.C.

DR. J. STERLING KINNEY
Head, Division of Structural Engineering, Rensselaer Polytechnic Institute, Troy, N.Y.

LINCOLN ARC WELDING FOUNDATION
Cleveland 17, Ohio

PROF. GEORGE M. McCONKEY
College of Architecture, University of Michigan, Ann Arbor, Mich.

FRED L. PLUMMER
Director of Engineering, Hammond Iron Works, Warren, Pa.

MILES K. COOPER & ROWLAND S. ROSE
Structural Engineers, Portland 4, Ore.

DR. JOHN L. SAVAGE
Consulting Engineer, Denver, Colo.

R. R. SHOEMAKER
Chief Harbor Engineer, Long Beach 2, Cal.

PROF. P. E. SONESON
Purdue University, Lafayette, Ind.

JOHN F. WILLIS
Engineer of Bridges & Structures, State of Connecticut, Highway Dept., Hartford 15, Conn.

CONTENTS

PART I

Page

1 What Is a Rigid Frame?
1 Difference Between Arch and Rigid Frame
1 Organizing Design of A Steel Rigid Frame Design
4 Selection of Type of Frame
7 Comparison Between Riveted and Welded Rigid Frames
7 A Buck Private in The Wacs
8 Shop Fabrication
8 Knees
15 Bases for Rigid Frames
16 Ties Between Column Bases
16 Struts and Bracing
17 Are the Big Fabricators Against Welding?
18 A Private Chat With Architects
22 Creative Design of Welded Steel Plate Structures
Dr. H. C. Boardman, Chicago Bridge & Iron Company
27 Derivation of Basic Equations for Single Span Frames
29 Tables of Design for Single Span Frames, Span 50 ft.-150 ft.
50 Analysis of Multi-Span Rigid Frames
Dr. J. Sterling Kinney, Rensselaer Polytechnic Institute
67 A Large Displacement Deformeter Apparatus for Stress Analysis with Elastic Models
Prof. Wm. J. Eney, Lehigh University
77 Use of Simple Cardboard Models To Obtain Design Data
Prof. Wm. J. Eney, Lehigh University
80 Measured Stresses in a Two-Hinged Beam Arch
Prof. P. E. Soneson, Purdue University

PART II

89 Contents Part II
90 City Auditorium, Denver, Colo., 215 ft. span
Paul J. Foehl
94 Rigid Frame Bridge, Mount Pleasant, N.Y.
Arthur G. Hayden, Consulting Civil Engineer
96 Two-Span Welded Rigid Frame Bridge on Wilbur Cross Parkway, Connecticut
J. F. Willis, Engineer of Bridges & Structures
105 Ross Powerhouse, City of Seattle, Wash., Stepped Column Rigid Frames
Dr. John L. Savage, Consulting Engineer
115 Allen County War Memorial Coliseum, Fort Wayne, Ind., 224 ft. span
Ralph E. Coblentz, Chief Engineer
123 Main Ave. Bridge, Cleveland, O.
Fred L. Plummer
126 Transit Shed, Long Beach, Cal., 200 ft. span
R. R. Shoemaker, Chief Harbor Engineer
131 Philips Electrical Works, Eindhoven, Holland, 3 hinged frames, 132 ft. span
J. R. Bouten, Director and Architect
135 Boston Navy Yard, Extension to Structural Shop, 102 ft. span
J. F. Jelley, Rear Admiral, Chief, Bureau of Yards & Docks
A. Amirikian, Head Designing Engineer, Bureau of Yards & Docks
141 Oregon State College Pavilion, Corvallis, Ore., 222 ft. span
Prof. Glenn W. Holcomb, Oregon State College
Miles K. Cooper & Rowland S. Rose, Structural Engineers
151 An Architect Speaks for Welded Rigid Frames
Prof. George M. McConkey, University of Michigan
154 N. Y. Subways, 4 span welded Rigid Frames
Lincoln Arc Welding Foundation
156 Photos of Varied Rigid Frame Structures
American Bridge Division, U. S. Steel Co.
164 Miscellaneous Photos of Rigid Frame Structures

PART I

What Is a Rigid Frame?

From National Bureau of Standards, Research Paper RP 1130, September, 1938, "A rigid frame is a portion of a structure in which ends of component members are connected in such a way as to prevent relative rotation of the ends at the connection."

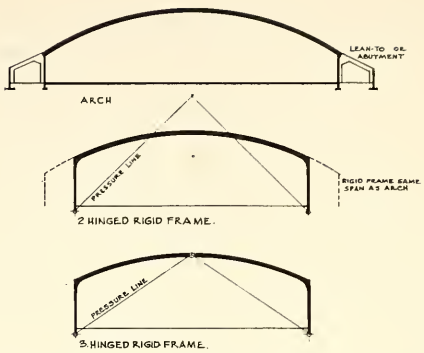
Try this: Take your pencil, sketch a simple straight beam, bend it, strengthen the bend for continuity and there you will have the elementary rigid frame. Two operations make it a rigid frame—the change in direction of its neutral axis and the addition of sufficient material at the bend or knee to obtain full continuity. (Knees can be deepened without reinforcement to obtain the same effect.) Add two more bends with legs turned downward, deepen or strengthen those bends for continuity and you will have a complete form applicable to a building.

In a simple beam, the straight line theory of stress analysis applies. With the addition of a bend or knee as above, the straight line theory does not apply at the knee. Thus, to the stress analyst, a rigid frame is any type of stress bearing member with one or more continuous bent joints.

Difference Between Arch and Rigid Frame

An arch is the most economical type of structure for long spans. It is ideal when you have room to stretch your structure out in space, as it requires abutments or lean-tos.

If you are cramped in width between property lines and still want a similar beautiful arch line and the same height as above for seats, etc., bend arch line, add two vertical legs and you will have a rigid frame. Use ties and you do away with abutments.



The sketches shown illustrate the above.

Thus you see that the rigid frame is nothing more than an arch with legs and knees. That makes it different. Advantage of the rigid frame is that it exploits to the full the width of identical arch spans within property lines.

The three-hinged rigid frame is more expensive than the two-hinged frame.

Organizing a Steel Rigid Frame Design

The manner in which you design a steel rigid frame is reflected right in your structure. Fear, boldness, caution, ignorance, all show up—even too many decimal points. You can't fool a structure, it lives to tell tales. You can't fool a fabricator, an erector, only your client.

Consultant Z, who asked me for push-button data, gets only one or two rigid frame jobs a year. Too little money, no time for long computations. So he loads down his structure and the finished auditorium looks just fine.

Did you ever see a steel rigid frame that did not look just fine?

Dr. A, fresh out of school, tees off for a round of column analogy—though it was only a small single span frame.

How he loves his decimal points!

The design took time and its costs were high. But that was not all. Just look at his score. Unnecessary hinges which are quite expensive, knee details poor and costly, the knee compression flanges not braced, only the tension flanges, bracing like a circus tent moorings. He missed badly on other putts. Now his rigid frame is up and does it talk back! But the client does not know for it looks spectacular.

Thousands of rigid frames dot the country, every day a new one and all with verve. They vary so much that if set to music their art would fit an extravaganza like a new version of "Showboat". Their forms spring of art, their structure of science. How about all those new Joes who are rigid framing?

There is a chasm between Consultant Z and Dr. A!

Too much theory, no experience, too little theory, lots of experience, they are the focal points in all design. Let us talk things over about that chasm.

Can there be too much theory in the design of a steel rigid frame?

No, if it is tempered by a philosophy of design.

Yes, if it lacks it. Dr. A lacked it; everyone does who hasn't cut his milk teeth on a living steel job.

Just what is a philosophy of design?
It is theory dunked in logic, realities and human equations.

Think of this before you begin design of a steel rigid frame.

You see, theory in action is mathematics.

Mathematical solutions are exact and soulless, not dependent on realities and human equations. Yet, they take root in dreamy metaphysics by accepting as ideal the theory of elasticity.

They accept the assumptions which are never realized. They then turn frigid—stiff as a corpse,—mathematics in action can do nothing else. Thus, your theory begins in dreamy metaphysics!

Why stop there?
Why not follow through?

Because you can't stop there. And if you wonder why, your thinking changes, you philosophize. You begin to consider the realities in your rigid frame, in the design of the bases, the hinges, the foundations, your knees. You wonder about your unit stresses. They were 20% higher on a neighboring war job. Yet, its steel is performing well without any arthritis. And you showing stresses to the nearest pound!

How about shop fabrication, those areas around the rivet holes stressed beyond the yield points, the locked-up stresses in the welded areas, stress concentrations relieved by plastic flow? They are not so bad as to be unsafe; nevertheless, they are secondary stresses not in your precise computations.

Plastic flow! Does that raise a question in your mind?

Plastic design would have been more economical than the theory of elasticity. How come, with all the study you did for your doctorate?

Oh! The boss!

The boss's look when you whispered "limit design"!

He raised his eyebrows. . . .

Then you dropped yours.

All right, so let's go on.

Your computations made no allowances for erection stresses.

Have you ever heard a drift pin talk to holes in field connections? Its language is fierce, its power terrific, unmatched holes yield and surrender quickly. Lots of secondary stresses not in your exact computations.

Your thinking is now accelerated, your fractions drop. Your computing has paused for realities. You study other structures in this book. You like this, you don't like that. You no longer use pounds for stresses. Your design gets better. It will cost the client less. You could get out and talk to a group of students and professionals on reasons for everything in your design, with or without its math. A structural behavior pattern is unfolded, a story of feasibility and practicability, your talk is based on a philosophy of design, your audience approves your design. Now a seasoned veteran, you confidently declare that all theory of design is based on assumptions! And on that concluding note, you feel like a junior Hardy Cross, steeped in his philosophy of design, dunked in his well of wisdom.

Can we now breach the chasm between Consultant Z and Dr. A?

Yes, we will cut corners after just a few words on methods available for the solution of rigid frame problems.

In an investigation done by the author many years ago for design procedures pertaining to continuity, a vast number were counted. All seemed to be developments in one form or other of the classical methods dating back in origin to Clapeyron, Castigliano, Muller-Breslau. Taking into account virtual work, moment areas, slope deflection, moment distribution and other accepted methods used commonly, there remained very many which from a practical standpoint had just as well be pickled as 57 varieties. They should soak and age first before use.

Stress analysis procedures used for jobs by contributors in this book include column analogy, slope deflection, moment distribution, exact methods. Would you have done it their way?

Whatever procedure you like best, that one in which you feel safe and comfortable, that is the one to use in the design of a steel rigid frame.

Now for straight drives down the fairway.

My advice to all who design steel rigid frames is to use a score card for processing your thoughts. You are then bound to play the first green first before the nineteenth. By the time you pause for refreshments, a philosophy on hooks and slices will have evolved. If people can generate high philosophy for mere little divots in life, designers should just as easily become mellowed into a philosophy for rigid frame design. Especially after the nineteenth.

The score card that follows is purely individual and one developed from a lifetime of experience in design and construction. Design of a large reinforced concrete rigid frame in 1910 and some steel rigid frame equipment before then made what has since become life everlasting with rigid frames. However, lay out your own course or no course at all, but do not fail to use a score card.

SCORE CARD

For processing steel rigid frame design

(Cross out unused terms)

1. Sketch desire shape
2. Indicate all dimensions
3. Welded Riveted
4. Indicate all loadings
5. Foundations
   (a) piles (b) soil bearing (c)
6. Ties Yes No
7. Base
   (a) hinged (b) fixed (c) partially fixed
8. When must steel be ready for bids?
9. Decide on design procedure you will use
   (Keep eye on 8)
   (Time elapsed 60 minutes)
10. Place red flags on knees, haunches, all bends or outline in red
11. Locate sway frames in roof

12. Rough in bracing and braced bays
13. Carry through on sides and in end bays
14. Expansion joints Yes No
15. Proceed with stress analysis of rigid frame
(Allow minimum 60 minutes)
16. (a) Have you done anything that departs from sound theory and practice?
(b) Have you done anything that departs from common sense?
17. Tackle your knees, don't remove red flag until you feel that
(a) Design is sound and safe
(b) Can be fabricated
(c) Can be erected
(d) You have braced bottom flange
Have you thought of the Osgood theory?
(Allow minimum 60 minutes)
18. Design col. base
19. Pause for refreshments
20. Is your steel design ready for draftsman? Give it the eagle eye
21. Design foundation
22. Prepare specification or have it prepared under your guidance
(Allow minimum 60 minutes)
23. Prayer followed by meditation
(minimum 30 minutes)

Selection of Type of Frame

As knees vary in form, so vary the paths for their stresses, their shears, their concentrations of stresses. In most cases also the center of gravity of knee section does not coincide with its neutral axis. That makes necessary variable theorems for the calculation of their stresses. Design of type shown in Fig. 2 is, for example, different from that of Fig. 3. In some cases no stress results in outer corner, particularly where its angle as in Fig. 2 is less than 90 degrees. That can be almost visualized from observation. Racing around the sharp inner corner, the compression stresses "gang up" there in a small concentrated area through added pressure from knee (double compression). The tension stresses having to cover a greater distance on the outside in the same time naturally tend to cut the corners and pass through the web. The unlimited variation possible in knee patterns consequently makes for unlimited theories for their design. From a practical standpoint such patterns can be restricted to a few types adaptable to various requirements and to the slide rule. Investigations conducted by the author theoretically and experimentally will be limited here to discussion of selected types.

A fundamental general principle of welded design insuring the greatest economy in costs is to limit all welding as much as possible to the knees and bent joints of frames—using rolled beams for the remaining members. This precept should be given primary consideration in the selection of a type of frame. Comparison of several different types should make for its understanding.

Figure 4 represents a rugged frame made up of separate plates and stiffeners welded continuously. Considerable steel in web plates is cut out and probably scrapped. Four lines of continuous fillet welds are required for full height and width of frame. Unnecessary costs result here due to fitting, steel scrap, cost of welding and weld material. This type does not develop welding's fullest economy.

Figure 5 illustrates one way to eliminate practically all waste steel and almost half of the welding material and labor if you particularly desire that type of frame. Two pieces are cut from one beam as shown in Fig. 6 and the shape of the frame is modified to permit the efficacious use of such pieces. The reduction of direct labor costs, as in this case, always means an additional savings in indirect costs—shop overhead.

The most economical type would employ rolled beam sections throughout, except for built-up knees.

Before any design is adopted that requires continuous welds throughout the frame, a study should be made to eliminate such welds by the substitution of rolled sections or pieces cut from them. A little ingenuity here will frequently make for master savings.

Figures 7a and 7b illustrate simple types of welded rigid frames suitable for assembly halls, schools, etc., and adaptable with slight modification to almost any purpose. Even the knees can utilize pieces cut from rolled sections in the manner previously explained. These types employ plates at all joints.

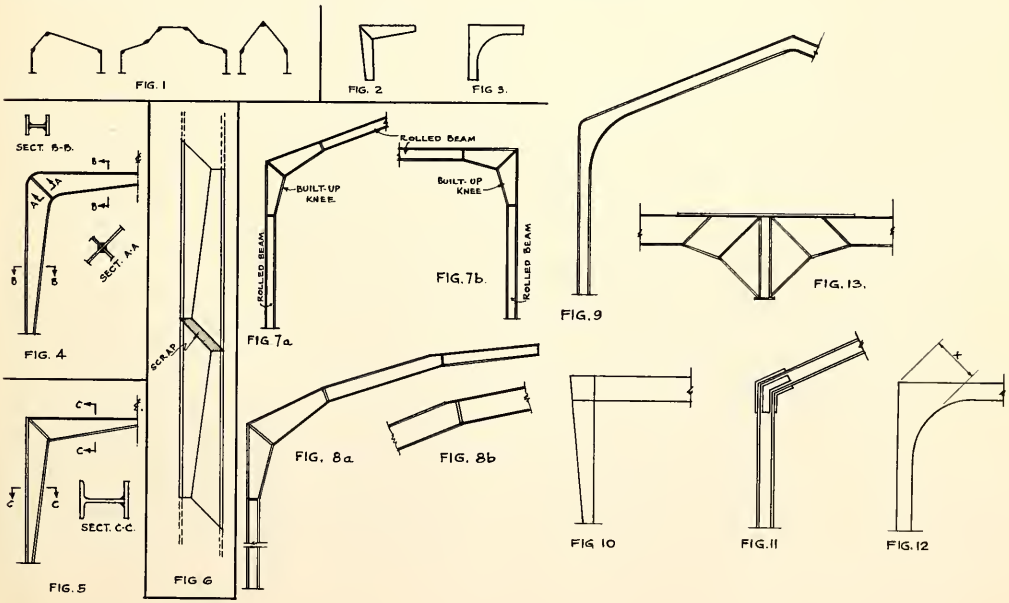
Figure 8a identical to 7a is suitable for very wide spans. Smaller beams can be used toward center for smaller moments. Connection between sections of varying depth is accomplished as in Fig. 8b. Triangular pieces of web are cut out at top or bottom and then flange is bent in and welded.

Figure 9 is modified type of Fig. 7a employing butt welded joints throughout.

Note employment of rolled beams in Figs. 7, 8 and 9 except for knees which are built up.

Type in Fig. 9 makes for somewhat better appearance than types 7 and 8. The butt-welded joints require shop preparation of edges. The cutting of knee web plates is done individually for small jobs, putting them in the "tailor-made" class with resultant higher costs of labor and overhead. Curved outline of knee in single web plate makes for waste steel. For a design of magnitude such plates can be cut in quantity in a single operation by automatic cutting apparatus at very low costs. Shop fitting in jigs must be accurate for butt-welded joints to avoid misfit of field joints. Flange plates of knees must be bent.

Above type is usually fabricated by cutting inner flange of column, bending it to desired form, inserting and welding additional piece of web plate.



Comparing types 7, 8 and 9, types 7 and 8 employing plates at all joints require preparation only of flanges for joints. More welding material and labor are required. Stiffeners make for small additional weight of steel. As against this, the bending of plates and the reentrant cuts are eliminated. Waste steel common to Fig. 9 is eliminated, as webs of knees are formed of separate plates welded to stiffeners. The stiffeners and milled edges of beams automatically make for correct shop joints without undue fitting and for better field joints. Their fabrication is economical for the smallest as for the largest jobs. The smallest shops can fabricate it. The joints are in many cases stronger against buckling, heavy loadings and impacts.

In general, the employment of angular design in a welded rigid frame as in Figs. 7 and 8 makes for greater rigidity, simpler fabrication and slightly lower costs.

Figure 10 represents a type of frame that does not develop welding's fullest economies, although more economical than simple beam design. It is most adaptable to bridges. The very shape of knee induces heavy concentrations of stresses at inner corner with practically no stress at outer corner. Obtuse angles both inside and outside as in Figs. 7, 8 and 9 make for lesser concentrations and more direct routing of stresses at knees, decreased moments in frame with commensurate economy.

Figure 11 represents a type of rigid frame employing beams without change in sections at knees—only the webs and flanges reinforced. It is not economical, except for short spans. Greater depth at knee than in contiguous members is fundamental for economical design of wide spans.

Consider now a case representing the other extreme of "too much knee" as in Fig. 12. You might consider it too much when dimension "x" is larger than 2½ times width of largest contiguous member. The outside corner carries no stress as previously mentioned. Flange plates are consequently not working where needed.

Figure 13 represents an economical assembly over columns for multiple spans employing all rolled sections throughout.

It seems fitting to pause here to make note of other fundamentals in design. Emphasis was placed previously on the importance of limiting all welding as much as possible to the knees and bent joints for economy. To that must be added the fact that the shapes and sizes of knees are vital factors affecting definitely the economical weights of the members in the frame proper, because of their effect on the moments. Particularly is that true for short spans with heavy loadings.

The selection of type of frame therefore involves not only limiting your welding to those knees, but also shaping those knees to such form and to such dimensions that the remaining members be of the lightest sections.

What makes for the lightest sections? Naturally, a reduction of moments in the frame proper. But a reduction of moments in the frame proper makes for heavy duty by the knees. Wherefore, we can truly conclude that the greater the amount of work assigned to the knees, the less steel will the remainder of the frame require. (This should not be construed as meaning the largest size.) If, now, this theorum be accepted as true, the question might be asked, "Why not first design the knees and then design the frame around them for the greatest economy?" The answer to that is that it is not at all impossible or unsound. In fact, it might be stated that that procedure alone does make for the most economical design for some types of frames where large quantities are involved. When it is realized that the knees

act as "traffic centers" directing the stresses in the entire frame and affecting their behavior, their importance will be properly considered in the selection of type of frame.

The types presented so far have been limited mostly to frames of simple spans. In general, the principles for their economical design apply equally to frames of multiple spans except that the calculation of stresses in knees between adjacent spans is different than for those at corners. We will confine ourselves first to understanding the stress behavior in single spans.

Comparison Between Welded and Riveted Rigid Frames

The riveted rigid frame is a structural unit pieced together from sundry rolled shapes — contiguous members meeting at joints that can never be eliminated, those joints requiring massed formations of splice plates and regiments of rivets to obtain continuity. The outline of a riveted rigid frame must follow closely the limitations of rolled shapes, thereby limiting creative form and design.

The welded rigid frame is a structural unit that can do anything a riveted rigid frame can do but without open joints, splice plates and excess metals. It can also do everything else not possible with riveted construction. Welding is synonymous with continuity. Its very process of fusion makes for natural continuity. Any form desired can be built. New designs and shapes can be created at will. Esthetic line can be exploited for fanciful or functional form without limit.

A Buck Private in the Wacs

Welded joints well thought out by a designer mentally aware of his structure's action can be as near the attainment of ideal joints as practical and economic considerations will allow. The question of their strength no longer forms a party line between engineers who weld and those who do not weld. It is now a matter of fact and living record. Examine for yourself some welds that are doing heavy duty; or, observe a test to destruction of a welded assembly, even a single joint. The welded joints alive and unbroken beside the ruptured parent material would definitely eliminate all doubts and erroneous impressions some have about them.

Fig. 14 of a knee in a riveted rigid frame for an existing field house poses a question for academic discussion and study. Observe the massed formations of field rivets required to make that joint in the knee. Observe also the fortifications of splice and reinforcing plates. Why all this excess metal? The answer is axiomatic. The joint is riveted, there must be continuity in that joint and with rivets there is nothing else you can do to obtain continuity but fortify to the edges.

Now hold that sight and—steady on the target. Do you notice that thin vertical line unarmed, unfortified and almost unseen? That too, is a joint—but it is a welded one!

Believe it or not, that single line of continuous weld, unarmed, unfortified and as meek-looking as a buck private in the Wacs is putting up as much resistance to maintain its joint as that whole regiment of rivets with its tank-like armor! And does she have power! On the flank, almost unseen, another single file of continuous weld stands guard with even greater power.

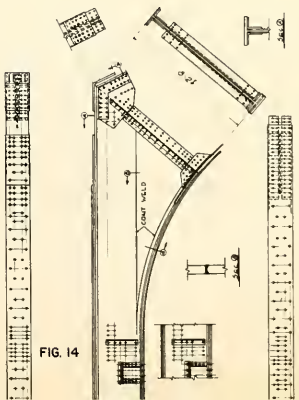


FIG. 14

Does this actual condition of a living rigid frame teach you anything about the difference between riveted and welded joints? Pass it along to others.

Let us detour for just a moment on a related subject.

Welding materials have become so highly developed and improved that you can know in advance what to expect from any recognized brand. Manufacturers' enthusiasm does not sell their products—only the joints that function by their products. Equally proficient have become the leading fabricators and welding contractors who follow the established codes. They—your tools and workers at welding—assure safety through qualified tests. A welded joint will sidle up and become all yours for life, if only you would give it a chance and understanding.

Shop Fabrication

The designer who thinks in terms of rigid frames must be a visionary type. Having, therefore, the talent for projected views, he should visualize fabrication and erection of his structure as his preliminary designs take form in varying outlines.

The fabricator's estimates are based upon your details. For they alone definitely affect and determine the unit costs on your job. The more exact your details, the more exact your price. The simpler the design and details, the lower the price.

Duplication of pieces, particularly those cut to special shapes, should be planned even if in some cases it means the use of additional steel. Details should be standardized or made alike. Think in terms of quantity of pieces and make their joints consistent with best practice. Reduce variety of joints to a minimum. Do not use every possible size of fillet weld. Straight cuts should be given preference to curved or re-entrant cuts, except for small quantities. Field welding should be reduced to a minimum. Overweld welding should be eliminated except where absolutely necessary. Specifications should comply with latest established codes. Design procedures wherever indicated on drawings should have a provision to permit fabricator or contractor to substitute his own procedure subject to your approval.

Comb final design for small corners and other details that look innocent on the drawing board, but cost plenty. For it is on the details that the contractor estimates his unit costs, not on the mere rolled sections or cut lengths whose prices he knows to the third decimal point.

We have now covered a number of points describing in general, operations that make for economical fabrication. A good welding handbook* plus a little experience will do more than a ton of words to reduce to habit the matter of designing for economy.

*RECOMMENDED:
Manual of Design for Arc Welded Steel Structures, Air Reduction, New York 17, N.Y.
Procedure Handbook of Arc Welding Design and Practice by The Lincoln Electric Co., Cleveland 17, Ohio
Welding Handbook, American Welding Society, New York 18, N.Y.

KNEES

Functioning of Knees in Rigid Frames

Stresses within rigid frames must turn the corners at the knees. In order that a force change its direction, it must be acted upon by another force or forces with a transverse direction. These transverse forces occur within knees in the form of radial stresses. The radial stresses combine with the circumferential stresses and curve the paths of stress around the knee.

The tension area of resistance sprawls out over most of the knee. The compression area of resistance has so much smaller space in which to perform even more work. In consequence, the compression stresses become much greater than the tension stresses. Two different behavior patterns thus exist unlike each other and both different from the normal.

An analysis of the stresses in a knee involves, therefore, the analysis of two individual parts. And if you consider them as separate bodies, according each individual treatment, they will not be difficult to understand.

If the knees be weak, it might mean failure to your structure. For the roof or deck beams can be overstressed even beyond the elastic limit and the structure will stand up, if they can resist the shear and thrust, so long as the knees and the columns are adequate.

If the rigid frame be overloaded or its legs give ground to the earth, the knees will feel the strains first. Designed to dominate, they will at once make the roof beams and the columns extend themselves to cover an emergency. And in many cases, the roof beams and columns can take up to 50% more stress than ordered in the design computations and return to normal after an emergency. We see then how unusually safe your rigid frame can be if only you give the knee which constitutes so small a part of the rigid frame the upper hand.

A good design for a rigid frame should, therefore, set up a life line around the knees. This is not to imply loading down the knee with excess steel and excess welding, which might do more harm than good.

We shall now proceed with a study of a knee as in Fig. 15. We shall study separately its tension area of resistance and its compression area of resistance by cutting the knee along its neutral axis as in Figs. 15A, 15B and 15C. The diagrams shown have been drawn out of proportion solely for illustrative purposes.

Behavior Characteristics of Tension Resistance Area

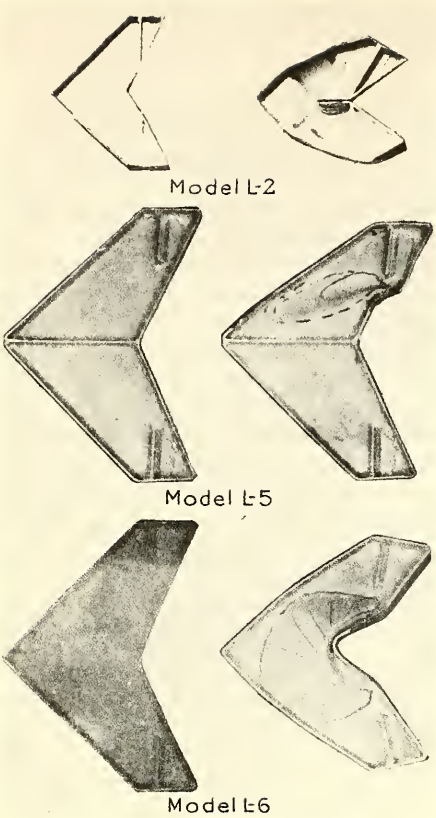
Let us concentrate on Fig. 15A as though it were an individual member.

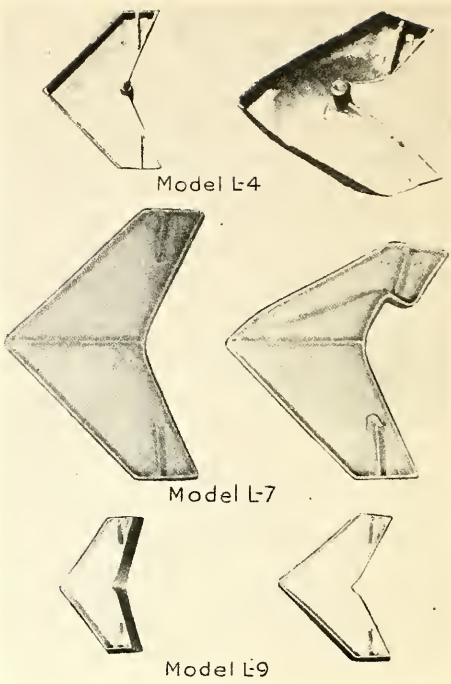
If certain forces were applied to this member so that its stresses became exactly like its prototype in the knee proper, those forces would appear as in Fig. 15A except for shearing stresses which are not shown. The external forces "x" and "y" at ends of member would make for tension in the member. The external forces "z" along the cut edge (neutral axis of knee) would induce radial compression stresses in the member. The tension strains would be maximum at the flanges, diminishing to zero at the cut edge.

On Fig. 15A have been shown the paths of stress. We know that the strains will cut the outer corner and pass through the web. It is obvious even to inspection that somewhere along plane "b—b", there would occur the maximum tension. It is also abvious that along the path of stress, the compression normal to the path will be diminishing from its maximum value which occurs somewhere near the cut edge (neutral axis of knee).

If you were to design a structural member acted on by forces indicated in Fig. 15A, you would probably provide one or more web stiffeners or make web thick enough to resist buckling without stiffeners. The simple picture you see of the stress behavior in this member is that exactly of its prototype in the full knee except for shearing stresses which are not shown.

Let us now examine Fig. 15B, another view of member 15A. Imagine, if you can, that each fiber within this member stood out to the





naked eye in the manner in which it is stressed. Such a picture cannot be illustrated, so that for the purpose only of illustration, we shall assume that the member had only eight fibers as shown.

Fiber 1 would obviously get little or no tension, but it would get a high radial compression. Fiber 3 would have far more tension, but less radial compression. You will observe that fiber 3 continues down into the web, above it fiber 4 into the flanges. Fibers 5 to 7 similarly run into the flanges. The maximum flange stress consequently occurs between fibers 4 and 7. (Actually it occurs only at one of those fibers).

It must be remembered that these fibers do not represent comparative unit stresses within the member or its prototype in knee. They are illustrated as shown to assist the reader to visualize just how the material must work to carry the stresses around curve.

Let us see if we can pick up similar points of maximum flange stress in some of the tested models. Note the bend points in the flanges in models L-2 and L-4. In the fibers between the bend points on these models occurs the maximum tension corresponding to that in fibers 4 to 7 in Fig. 15B.

The compression at any point along the cut edge (or neutral axis) from the tension are equals the summation of the radial compres-

sion stresses in all fibers in radial plane through that point.

If for Fig. 15B you visualized myriads of thin stranded wires replacing the eight steel fibers, those wires stayed laterally—you would see the wires with the maximum tension strains bearing down and tending to roll over one another. They would of course have "anchorage" in the flanges. Actually it is the other way around. The tension in the flanges pulling at those fibers tend to make them roll over one another. Does this action portray anything to you? Here occurs what we might call a transformation—the tension becomes shear in the web! Let us repeat for sure understanding.

A force in top flange pulling to right represented by arrow develops its maximum flange stress at "c", veers down into the web, then into the vertical outside flange and down into the column. Those tension stresses in the flanges and in the web both horizontal and vertical become both horizontal and vertical web shear forces. It is thus plain that the maximum shear in the web of the knee occurs above the neutral axis within approximately a square area, with flanges forming two sides of that square. Understanding this action, you would naturally place one or more diagonal stiffeners in the web.

With the maximum tension in the stranded wires obtaining between points "b" and "c", and with stress in each wire decreasing down toward the cut edge, can you now further imagine a condition where around center of cut edge, you might possibly have "slack wires"? By slack wires I mean only loose wires without anchorage subject only to compression. Such a condition actually occurs creating areas possessing no tension strains above the neutral axis in a knee. This point will become clearer as we look into the compression area.

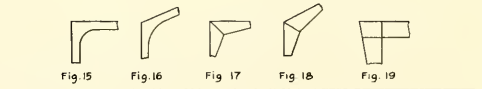
An effort has been made to focus attention in simple language to the critical points of stress through diagrammatic illustration of the stress behavior action. Knowing where the critical points of stress occur in any member is what makes for good and safe design. If your computed stresses deviated 10% or even 20% from the true stresses based on the critical points of stress, they would produce a better and safer structure than mere mathematical analyses of an inanimate body varying 3% from some hypothetical assumption.

If you understand the above illustrations, you will understand the knee's behavior above the neutral axis.

Behavior Characteristics of Compression Resistance Area

Fig. 15C represents only the compression area of resistance cut from the knee Fig. 15 along its neutral axis. Let us now concentrate on Fig. 15C as though it were an individual member.

If certain forces were applied to this member so that its stresses became exactly like its prototype in the knee proper, those forces would appear in magnitude and direction as illustrated in Fig. 15C except that the shearing stresses are not shown. The entire member



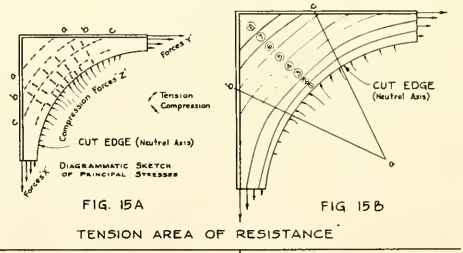


FIG. 15A FIG. 15B

TENSION AREA OF RESISTANCE

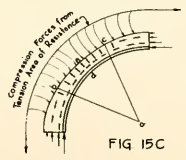


FIG 15C

COMPRESSION AREA OF RESISTANCE

Fig 20

would be in compression. The maximum stress would occur along the flange and it would decrease towards the cut edge or neutral axis. The maximum compressive stress would occur in the flange at a point between 12 and 15 degrees from the point of tangency if the flange area is kept constant around the curve.

Acted upon by both bending and radial com pression, the bottom flange would shorten along the web center line between the points of tangency. It can do nothing for relief, pressed as it is at both ends and at top. But that restraint fades away towards the outstanding portions of its flange plate. And, as a result, the edges of the flange plate remain almost in their original position, retain almost their original length and in effect turn upward. Or, to say it differently, the compression flange will bend inward due to the radial component of the compressive force. Obviously, then, the edges of the flange plate do not get the high compression stress of the flange along the center line. Those edges, consequently, are critical points for design of bottom flange. Depending upon the flange width, thickness and the curve radius ($\frac{b^2}{tr}$ governs the amount) the outstanding portions of the flange plate might be only 50% effective if ignored in your design. A natural step then, is to provide stiffeners or reinforce the flanges.

Radial stiffeners should be used to prevent inward bending of the compression flange if $\frac{b^2}{tr} > 1.2$ where b equals width of flange in inches, t equals thickness, r radius in inches of the bottom flange. However, the stability of the web may control the arrangement of the stiffeners.

If the shearing stress in the web reaches the ultimate strength of the material in shear, then the ultimate strength of the knee is reached regardless of the arrangement of the stiffeners. (This was shown by tests on corners of aircraft-carriers)

Where flange angles were used instead of a plate, the outstanding legs, just like flange plates, would bend up in relief through deformation and thus not develop their design capacity unless reinforced or held by stiffeners.

Test model L-6 without stiffener shows the failure effect in the heavy compression area. Test model L-7 with stiffener indicates the strengthening effect of the stiffener.

For the purpose of determining the value and effect of stiffeners, test model L-9 was designed with thick web and thinner flanges—the section having the same moment of inertia as section in Model L-5. It took more steel and had less capacity than L-5.

The action outlined above is exactly that of its prototype for the compression resistance area in a knee.

In the light of these facts would you omit bracing the bottom flange of knees?

The Neutral Axis in Knees

According to our present accepted theory of strains and stresses in elastic materials, the strains from bending are proportional to their distance out from the neutral axis in beams with straight axes.

In beams with curved axes, such is not the case. The fiber lengths between sections vary in length and if plane sections remain plane after bending, the unit strains are not proportional to their distance out from the axis. Any standard textbook on strength of materials will explain that point.

In both cases of straight and curved axes, bending will produce a rotation of all cross-sectional planes normal to the axis. This rotation is caused by the tension and compression stresses acting on the fibers between two adjacent cross-sectional planes normal to the axis.

If we are to have flexure in the knees, and they are composed of the same elastic material and subjected to bending moment as the rest of the structure, then the neutral axis must assume a position such that cross-sectional planes normal to the elastic axis will not overlap and will be at least an infinitesimal distance apart at any point.

This is a physical or boundary condition which causes the elastic axis to deviate from the center of gravity of the section within the range of influence of this boundary condition. In order to have cross-sectional planes normal to the axis which do not intersect within the section, the neutral axis must be curved and of a radius greater than the distance from the neutral axis to the boundary of the material.

The location of the neutral axis then is dependent on the radius which it assumes and the laws of equilibrium between the internal stresses and the external forces together with the principle of conservation of energy. Since the elastic axis becomes curved to a relatively small radius within the knees, radial stresses necessarily increase in importance.

Summary on Knees

The above condensed notes on knee behavior focused attention to the critical points of stress with which you are mainly concerned for practical design. A knee as in Fig. 15 was cut along its neutral axis after the knee had been loaded and stressed. Forces were substituted on the individual members that would result in the same strains in their prototypes in the knee proper. Observations were then taken on those separated bodies illustrated in Fig. 15A, 15B and 15C. Combining these observations should give you a picture of the stress behavior in a complete knee. Let us summarize them in terms of fundamentals essential to good knee design.

The maximum compression strains are much higher than the maximum tension strains. And they have much less living space than the tension strains in which to perform even more work. The compression area of resistance must, therefore, be more heavily armored.

The web around the center of the compression flange is stressed in all directions. That invites both buckling and crushing. Thickening the web here is one solution, but stiffeners in this highly compressed area make for greater strength and economy. Model L-7 with stiffener proved 60% stronger than model L-6, within the elastic limit.

This highly compressed area likewise makes for buckling in the compression flange. Web stiffeners will consequently serve both the web and the flange of the compression area for strength and economy. They should be welded to the compression flange.

In the tension area of resistance, the web can buckle from shear forces both horizontal and vertical. Web stiffeners will offset that. They need not be welded to the tension flange unless flanges are wide.

In brief, a good design would "heavy up" on the compression area with greatest resistance in all directions around center of compression flange. The tension area of resistance would be lighter. Your knee would then possess both strength and flexibility—all potentialities to dominate the frame. Dominance implies the ability of the knee to take over and carry stresses caused by extraneous forces not in design computations. Such stresses might result from uneven settlement, from sidesway not assumed in design, from other causes. Effective toward insuring such dominance are web stiffeners. They make for an increase of more than 50% in strength of knee throughout the critical area by tapping the concentrations and redistributing the web stresses. They also induce

stress concentrations. Where knees are large and more than one pair of stiffeners are used diagonally, short stiffeners above the compression flange would in many cases suffice to strengthen the heavily stressed area.

At the beginning of this discussion, Figs. 15 to 19 were mentioned as being typical of most knees. The behavior pattern in Fig. 15 varies in the other types only to extent that you either relieve or intensify the concentrations.

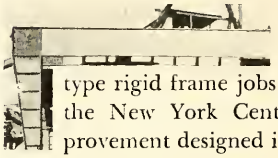
Fig. 16 develops lesser concentration of compression stress than Fig. 15.

A knee as in Fig. 17 with sharp inner corners should have stiffeners. (See models L-2 and L-5).

A knee as in Fig. 18 develops great strength and economy.

In the square type knee shown in Fig. 19, the highest stresses occur at the inner corner. Beyond this critical point the flange and web stresses diminish rapidly. And quite naturally, the compression flanges, unrestrained, just beyond that corner and free to move tend to buckle under that crushing compression. Reinforcing the critical areas of the flanges and the web might prove least costly. The strength of the knee panel definitely limits the carrying capacity of this type of frame. The web has a tendency to buckle as in Fig. 20. That is why some designers put a diagonal stiffener in knee panel in addition to vertical and horizontal ones. Rounding off the outside corner will not decrease strength of the knee but flange should be continuous.

The pioneering rigid frame designers shied clear of the rectangular type of frame because of the unfathomable knees. Faced with such a design in 1908 for the support of a crane, I loaded the inner corner with steel, stacked the rivets shoulder to shoulder so that the boss could not get a cuss word between them. He took one piercing look, scowled his approval. Neither of us was certain of what we were doing. But we rode through with horse sense, as designers of that era were frequently doing in uncharted design alleys.



Among the earliest of the rectangular type rigid frame jobs was the subway for the New York Central West Side Improvement designed in 1935, by Madigan & Hyland., shown here.

Rounding off this summary would probably suit some of my lecture fans if I would pull back the curtain on four pages of charts with curves for push-button design of knees. Then the client, interested in a heavenly auditorium, could be asked to take a seat and before he made third base on the sports page, Mr. Engineer would flash a home run and begin wrapping up his rigid frame design. But after striving so hard to make you talk to your structure, think its design through and give it life without parental misbehavior, I am for making you roll your own knee analysis and synthesis on behavior characteristics just outlined. Then it won't be wrong. It might perhaps be a bit fat in the knees but it will be safe, and above all it will stick mentally for life. I have tried that psychology on several and it worked gloriously. Bibliography will be appended for real study. But I will also throw in some notes off the cuff discussed with Consultant Z and others privately following lectures.

Cuff notes: If you are pushed, make haste slowly, get your "H" (Horizontal reaction), polish off a knee with a fair radius, if round, so that it could yield and your design will be well under way. Knee should be strong but ductile, braced properly. Min. rad. 2½ times width of largest contiguous member to knee except in grade crossings where clearance conditions might limit radius. No need for splitting hairs about moment at crown.

For dispatch of knee, sketch a symmetrically shaped tapering beam, bend it at center, plot the neutral axis. Divide knee radially into sections 12" or longer along neutral axis, design as assymmetrical units subject to bending and compression. You may use center of gravity axis for tension area if you want extra safety. But use neutral axis for design of compression area.

In round knees where roof pitch is between 30 and 45 degrees, set the neutral axis (N.A.) 0.35 of length of diagonal through center line. Where roof pitch is 15 to 29 degrees set N.A. 0.25 of diagonal length. For shallower frames and rectangular types, place N.A. at distance of 0.18 of length of diagonal through center line. These locations might require adjustments up or down in accordance with ratios of height of column (along gravity axis) to span.

The horizontal force "H" which you have determined instantly along the center of gravity axis of the frame should be kicked up just as quickly. You know that "H" increases as knee grows bigger and stiffer. You know also that it increases rapidly as ratio of column height to span decreases. The manner or amount in which you increase "H" may not be theoretically correct but it is surely safer than complete neglect. Many designers do neglect it completely.

Approximate increases in "H" for the purpose of determining knee moment only are in round figures 5% where height is 0.20 or more of span, 10% where it is 0.15 of span, 15% where it is 0.12 of span, 20% where it is 0.10 of span, and in the latter case check by Osgood or Bleich method.

I have now done everything but design your frame. You are asked to do something in return for efforts made here to save your time. Take off one night to read Physical Properties That Affect the Behavior of Structural Members by Wilbur M. Wilson in the ASCE Proceedings of December, 1942, or any other similar material. Reading time 15 min., should be followed by meditation, 15 min. It will sink in and make you think of another world in which you live and of which you don't know or have forgotten. Then read Plasticity of Metals—Mathematical Theory and Structural Application by D. C. Drucker published in the ASCE Proceedings for August, 1950. Don't miss their discussions in Separate No. D-27-D, May, 1951, for fun and mental pabulum. These brief papers will merely initiate you into fields every designer of rigid frames should traverse occasionally.

Recommended Bibliography:

National Bureau of Standards, Research Paper RP1130, Strength of a Riveted Steel Rigid Frame Having Straight Flanges, by A. H. Stang, Martin Greenspan and W. R. Osgood.

National Bureau of Standards, Research Paper RP1161, Strength of a Riveted Steel Rigid Frame Having a Curved Inner Flange by A. H. Stang, M. Greenspan and W. R. Osgood.

Proceedings, American Society of Civil Engineers, November, 1940, an Investigation of Steel Rigid Frames by I. Lyse and W. E. Black with discussions in Proceedings for February, March, April, Sept. & Oct., 1941.

Progress Reports 1 to 10 on Stress Distribution in Steel Rigid Frames by A.I.S.C., 1936.

Design of Rigid Frame Knees by Friedrich Bleich, A.I.S.C. Publication 1943.

Welded Continuous Portal Frames by Lynn S. Beedle, A. A. Topractsoglou and Bruce Johnson, published in the Welding Journal, July, 1951, August, 1951, November, 1952.

University of Illinois Bulletin No. 23, An Investigation of Rigid Frame Bridges, Part I by F. E. Richart, T. S. Dolan and T. A. Olson.

Bases for Rigid Frames

The question of whether to use fixed or hinged bases depends entirely upon the specific problem and the designer. We can evaluate the effect of all types of bases on the stresses in frames. But it is important that we first agree

on what we mean by fixed or partially fixed bases. For it is a fact that some designers do not fully realize that their fixed bases remain fixed only in their design notebooks.

Strictly speaking, no base of a structure can be considered as fixed except one in rock or hardpan. Steel piles frequently settle under loading, likewise all types of spread footings.

Because settlement may occur, it is to be expected that some rotation of the bases will follow, since neither maximum or minimum loading conditions will produce equal distribution of soil-bearing pressures on the base. If, therefore, you design rigid frames with fixed bases not bearing in rock, you have no assurance that your structure will be fixed. You can more safely assume that it will not be fixed. The restraining moment acting on the base will be something else than the moment necessary to completely fix the base.

The only result you can be sure of when you design for fixed bases is that your steel rigid frame and bases will act together toward absorption of the stresses due to loadings. In other words, you change the behavior of the steel frame when you unite it with the footings. You change, likewise, the behavior of the footings. Briefly, you make your foundation an integral part of the steel frame with the stresses in the structure depending entirely upon the extent and manner of its union with your foundation.

In a steel rigid frame having hinged bases, the footings serve only as "soil-bearing plates" just as wall plates for wall-bearing beams.

Comparing briefly the results of a design with fixed bases to one having hinged bases, the stresses in the columns vary the greatest, the stresses in the knees in a lesser degree, the stresses in the deck or roof beams in a still smaller amount, not enough to change your design should you change the type of base. This applies only to symmetrical loadings.

Of special interest are the bases for the Ross Powerhouse, Seattle, and the Boston Navy Yard, shown in this book.

Ties Between Column Bases

Rigid frames can be designed with safety without ties just like arches. But with arches the leantos or abutments reach up above ground. Rigid frame foundations are almost always underground. Thus, for large spans, they would be massive and uneconomical without ties.

In general, the use of ties serves for economy, for safety and for the alignment of columns during erection.

When ratio of height to span is very small, ties are definitely necessary. For an average building of 70 ft. span or less, ties are unnecessary especially if there is a floor slab. This statement must be qualified with the warning that the foundations must be designed, not just poured, as the author discovered in investigation of a failure.

Study the structure in this book and you will acquire years of experience on ties in the briefest time.

Struts and Bracing

If you would examine closely the numerous photographs in this book, you would observe considerable variety in the manner of bracing rigid frames. There are several structures with no sway frames (portals) between the knees, others without even a strut between the bottom flanges of the knees.

Those structures look good, moreover they are standing. On the basis of that living evidence, you might be tempted towards a "strip tease" in your design and omit bracing the knees despite my serious sermon on that subject.

Don't be tempted.

Lateral instability can cause premature failure without developing the working stresses of your steel rigid frame in bending about the major axis. Such a failure was shown in some of my lectures. As to the odd number of rigid frames with their knees bare and not braced which have not failed, discuss among yourselves why they stand. That will prime your minds towards decisive thinking.

For safe and sensible design, laced struts, portals or sway frames are suggested for use as follows:

In spans up to 40 ft., none are necessary where you have purlins or deep metal deck. Rod bracing would not be out of order for one or more bays.

In spans from 41 ft. to 60 ft., on diagonal center line of haunch or knee and at ridge (Intermediate members might be necessary for the 60 ft. spans)

In spans over 60 ft., at haunches and along ridges with additional intermediate sway frames spaced at a distance equal to 90 times the least radius of gyration and not to exceed 100 times that dimension.

Depth of portals or laced struts should not be less than one-half depth at knees and at crown and they should slope at ends down to bottom flanges. Braced bays in spans over 60 ft. should have bracing in sidewalls if for no other reason than for erection purposes, irrespective of portals at the knees. Wall bracing may be omitted for small spans.

Where deep metal panels are used without purlins, they can in most cases be considered as bracing of the top flanges throughout as shown in photograph here (Courtesy of Detroit Steel Products Co.) The compression flanges of the knees should be stayed laterally.

A quick way to route your bracing thoughts is to consider your building as a bridge with only two rigid frames, and then things would start humming. A portal would drop into place automatically. Heavy sway frames and bracing would follow. You would need no prompting on spacing of struts. After a few minutes your bridge would look similar to the problem Professor Quickee gave you in your finals. Then start easing up for you know that you were only fooling. Remember now, that you have a building with many bents, not a bridge. Lighten up on your assumed powerful members but hold that bracing layout. Let it serve as the braced bay of your building. Carry on with lines of struts at least in alternate bays. And that will be it!

Think straight, be safe and you can't help but be right.

Are the Big Fabricators Against Welding?

Visit Oscar Seidel, Plant Manager, American Bridge at Ambridge, who has been welding since the early twenties, and he will show you stacks of welded jobs they have turned out from the world's largest to those requiring the extreme in precision and accuracy. Or call on genial and debonair J. Hagood Zorn, Assistant to President, Pittsburgh, mention my name and you will get a free lunch—that is, provided you have a job in your pocket. He will string out jobs from coast to coast done by American Bridge Division, U. S. Steel Company.

Deep Metal Panels Brace Top Flanges →
(Courtesy of Detroit Steel Products Co.)

The Bethlehem Steel Company has turned out a 4,000 ton forming press of 75% welded construction of which they are proud and justly so. To quote from their own advertisements, Bethlehem weldments are "economical . . . . versatile . . . .they eliminate excess weight without eliminating rigidity . . . . they offer freedom in design." Moreover, Bethlehem is producing new steels which can be "bent, pressed or shaped prior to welding without harm to its physical structure."

Stop off at R. C. Mahon Co. in Detroit; see their new welding shop with X-ray and every type of welded work. But first ask for J. R. Stitt, welding wizard and engineering consultant, whose magic technique can make pretzels out of steel or straighten steel which had been accidentally pretzeled.

Out on the West Coast, see D. B. Bertone, Manager of the progressive Pacific Iron & Steel Co. in Los Angeles, inspect their shop and learn of the quantity and quality of welded work they are doing. Photos of the Long Beach Transit Shed shown in this book were taken in that shop, where the work was fabricated.

Let us stop here because this is a lecture series, not a directory. But there are hundreds of fabricators around the country who weld with pleasure and profit. Welded contract bids may sometimes be so high as to discourage your interest in planning welded rigid frames. One reason may be an overload and backlog of orders for the welding shop with a resulting courtesy bid which is not even courtesy. Another reason—your lousy welded design.

If you design from a score card and follow the suggestions on shop practice given here, your design should be good. The fabricators will spot you as a choice customer with extra calendars for the New Year and sharp bidding for your work. There are such jobs in this book.

Shop Fabrication

Undulating Shafts of Extended Rhythm →

Once in a while, you will run across a fabricator's district sales manager who for various reasons, good or bad, would suggest that you change your welded design to a riveted one or to a truss job. Ask him to state all of his reasons. Don't say a word for fifteen minutes. Such silence frequently brings forth much more than a mere spectrum in your visitor's face.

There are rigid frame jobs in some isolated locations where shipping facilities and limitations govern the design. Do not fail to give consideration to the use of high strength bolts, or equal, for joint splices in such cases and elsewhere.

A Private Chat with Architects Space Structures, Dreams and Fantasy

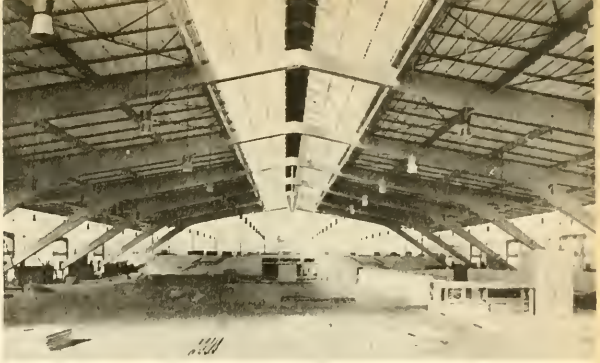
In September, 1939, the Association of Collegiate Schools of Architecture assembled at Washington, D. C., at their national meeting. Before that distinguished group there passed in review Design of Today and Tomorrow. The influence of the great pioneer architect, Louis Sullivan, of the imaginative Hugh Ferris and of the creative Eliel Saarinen had already left their imprint on contemporary design in vertical architecture, as exemplified in our towering skyscrapers. But by this time there was already becoming manifest new functional horizontal architecture, low, long and horizontal structures of singular beauty under the influence of creative designers like Gropius, Aalto and Van der Rohe.

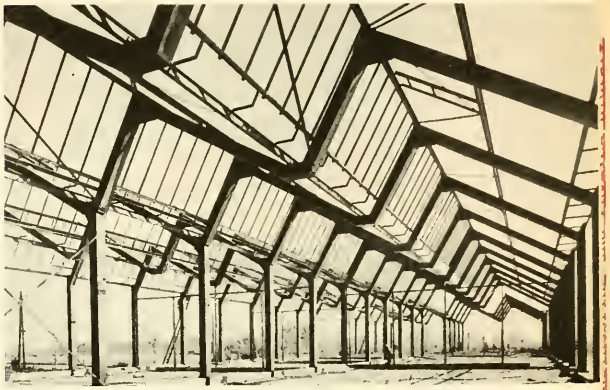
Most challenging, however, in horizontal form was a comparatively unknown school of design of welded steel rigid frames that began to appear here and there, stretching out their power in undulating shafts of extended rhythm.

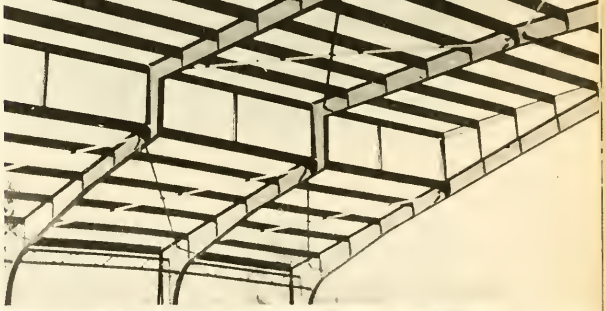
The all-welded rigid frame plant of the Lincoln Electric Co., Cleveland, was such an example, designed and built by the Austin Co. in 1937. What did that mode of functional steel ribs portend, holding sway in single leaps where we were accustomed to see trusses zigzagging aloft?

With a view to learning the effect welding might have on the future of creative architectural design, the late E. W. P. Smith, Consulting Engineer, Lincoln Electric Co., was invited to address the above convention. (The author advised him and prepared his material for that talk, a custom we had as close friends of helping each other.) Emphasized was the fact that the welded rigid frame was an acrobatic wonder that could swing up hill, take hurdles, wind around corners in buildings, bridges, boats, in everything. And look what happened since 1939—only yesterday to me!

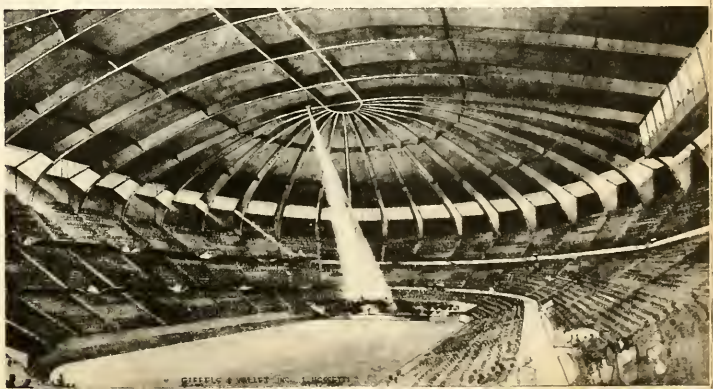
Behold the Europa Hall, a welded steel rigid frame at Hanover, Germany, its appearance suggestive of a surrealist symbol by Dali. Observe also the industrial plant in Holland; the clerestory frames at the Ft. Collins High School, in Colorado; the proposed Convention Hall 300 x 400 ft. for the Detroit Civic Center—steel woven in space as figures conceived in the "mind's eternal heaven where dreams are begot."







(Photos top to bottom)
Europa Hall—Hanover, Germany.
Chemical Plant, Holland.
(Courtesy of DeVries Robbe & Co., Holland)
Fort Collins High School, Colo.
(Courtesy of Milo S. Ketchum, Cons. Engr., Denver)
Proposed Convention Hall, Detroit Civic Center
Giffels & Vallet, Inc., L. Rosetti
Associated Engineers & Architects
Detroit, Mich.



Multiplanar steel structures are now on drawing boards planning yesterday's dreams for construction tomorrow. Their skeletons hide moments, shears and thrusts not palatable to architects whose tastes revolve about eye appeal—the skin, the face, the curves, the lines. How, now, can architects plan their future soulful designs which are so dependent on slide-ruled soulless engineers? They must do unto them what painting had done to architecture. They must take a lesson from their own development, their own association with art. Let us look back for a moment.

Throughout the ages there has been a close relationship between painting and architecture. Both arts had their baroque periods. Present day interest concentrates on good clean design and the most effective use of materials. To modern painters, "architectonic" is an adjective in great favor. Conversely, painting has influenced architecture. The compositions of the painter, Mondrian, have their exact counterpart in the Mies Van der Rohe's Metallurgical Research Building at the Armour Research Foundation, Chicago.

Welding has liberated steel from its old limitations to such a great extent that it is now possible to make almost any kind of design. We could reproduce the angles of Picasso; we could experiment with steel as the painters did with paint. But whereas the painters can be escapist or bizarre, the architect must consider function, moments, shears and thrusts. The future for creative design, therefore, rests on complete integration of architecture and engineering. Such unification of mind, soul and sliderule is now taking place as Engineer-Architect entities. An interesting example which I shall outline is the proposed Convention Hall for the Detroit Civic Center, designed by the Engineer-Architect firm of Giffels & Vallet, Inc., L. Rosetti. It will be a dome-shaped rigid frame structure 300 ft. x 400 ft., part of the proposed Convention Hall & Exhibits Building with a total length of 1200 feet.

Development of the structural design proceeded from the start by the integration of architecture and engineering into single planning center. Limitation in width of property dictated an oval shaped structure for the large capacity planned. Concrete was adopted for the immense seating bowl for fireproofing purposes. Two-hinged welded rigid frames were selected for the roof support because of economy, savings in depth of steel and esthetic appearance.

Beginning with an arch form, legs were added on each side to exploit to the limit the available width of the building and the natural rise of the arch. Otherwise lean-tos or abutments would have been required. Thus the rigid form took form and evolved from the arch.

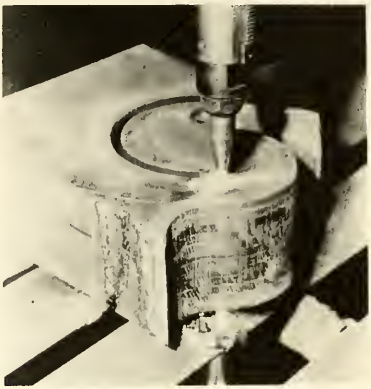
Since the concrete bowl had an upper tier of seats, concrete piers were run up for their supports. Then as a very natural step in progress of the design, the exterior half of the concrete piers were run up as cantilevers, taking the wind pressure from the walls and away from the rigid frame legs; the other half of the pier served at once as a support of the rigid frame.

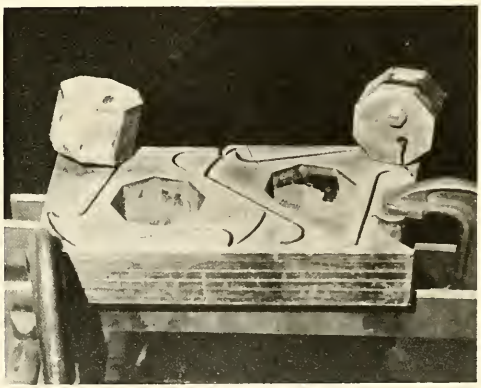
At this stage in the development of the structural design, steel rigid frames shaped as segments from the sky, were covering the bowl. Their legs rested on piers supporting that bowl. Free of those legs to permit the steel to breathe, the piers ran upward to the roof. The piers were now ready to function for a triple purpose. Needed to complete the structural design was anchorage for the base of the rigid frame legs. But already waiting for good anchorage were the mass weight and rigid framing of the concrete bowl. Thus was completed this preliminary design, natural in its develop-

ment, natural in its form, integrated without division between architecture and engineering. The architectural rendering shown here depicts their unification. Of course, like any municipal project of magnitude, this preliminary design may be changed. But the Engineer-Architect integration that produced it remains as its symbol of successful unification.

I will now close this private chat in order to present the next speaker who will take you into the field of creative plate design. It is my strong conviction that welded plate construction will provide the means for future creative structural design. Welded box bridges of light plate construction have already been built offering great savings, simple and clean design, most natural form. Europa Hall shown here is of box construction. You may begin now to plan your dream shapes in space structures made from plates because the means and the men are here to help you, the engineer for stress analysis, the apparatus for strain measurements of model of your dream structure, steel plates unlimited, equipment with magic-like potentials for cutting your steel to tailor-made designs, singly or in dozens at a time. Three photos here, courtesy of the Linde Air Products Co., shows magic-like cutting operations done with their equipment. Not shown here is the well-known Airco No. 50 Travograph Gas Cutting Machine which operates on the pantograph principle and can cut an unlimited variety of shapes up to 12 feet in diameter, sold by Air Reduction Sales Company.

Steel, the symbol of strength, now beckons all architecture towards new horizons with creative plate design.





Cutting Intricate Shapes in Quantity →
(Courtesy of Linde Air Products Co.)

Creative Design of Welded Steel Plate Structures

by
DR. H. C. BOARDMAN*
Director of Research
Chicago Bridge & Iron Company

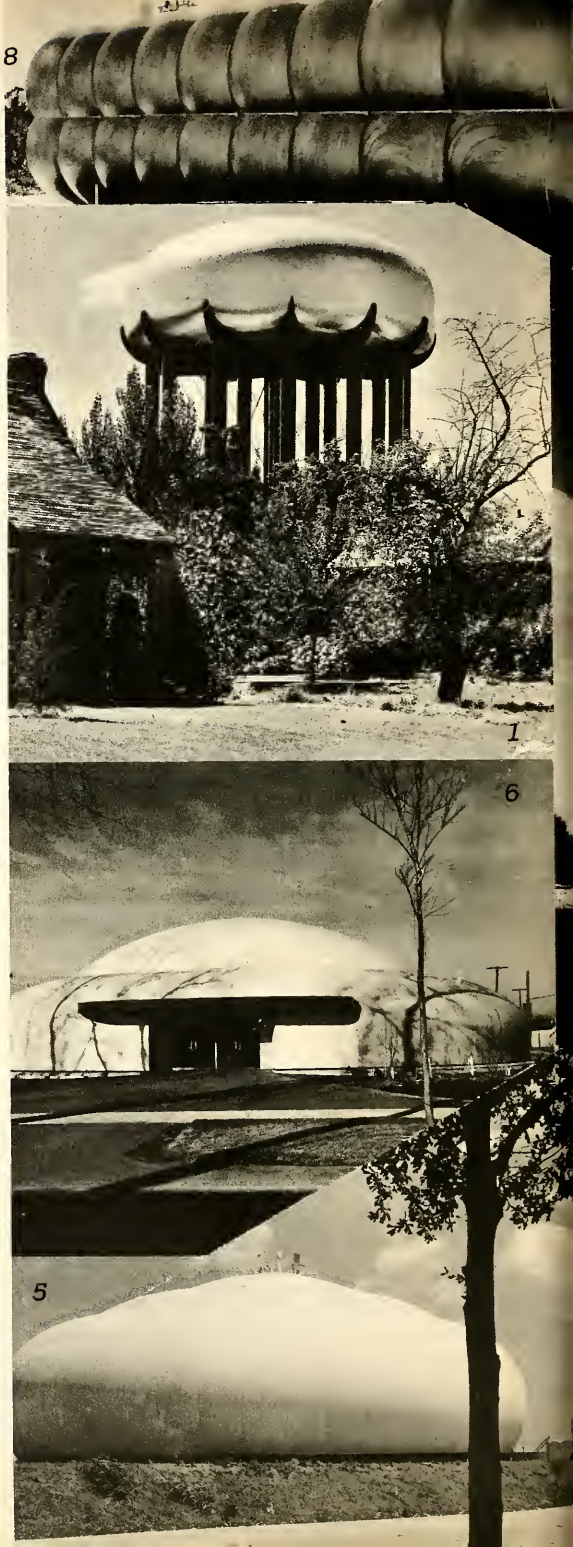
The possibilities for creative design of welded steel plate structures are practically unlimited and largely unrealized although the photographs herewith attest to their utility and beauty.

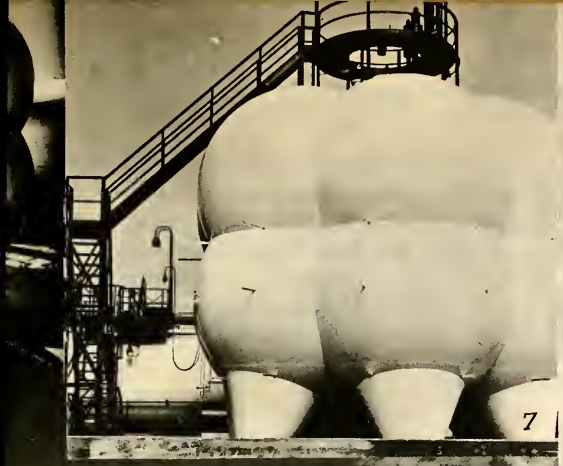
Much has been thought, said, written and done about thin domes and barrels (cylinders or partial cylinders) of reinforced concrete, but relatively little about much thinner domes and barrels of steel — doubtless because the very thinness of the steel suggests buckling and discourages investigation. Nevertheless, steel plates in many forms can do surprising things in spanning wide spaces.

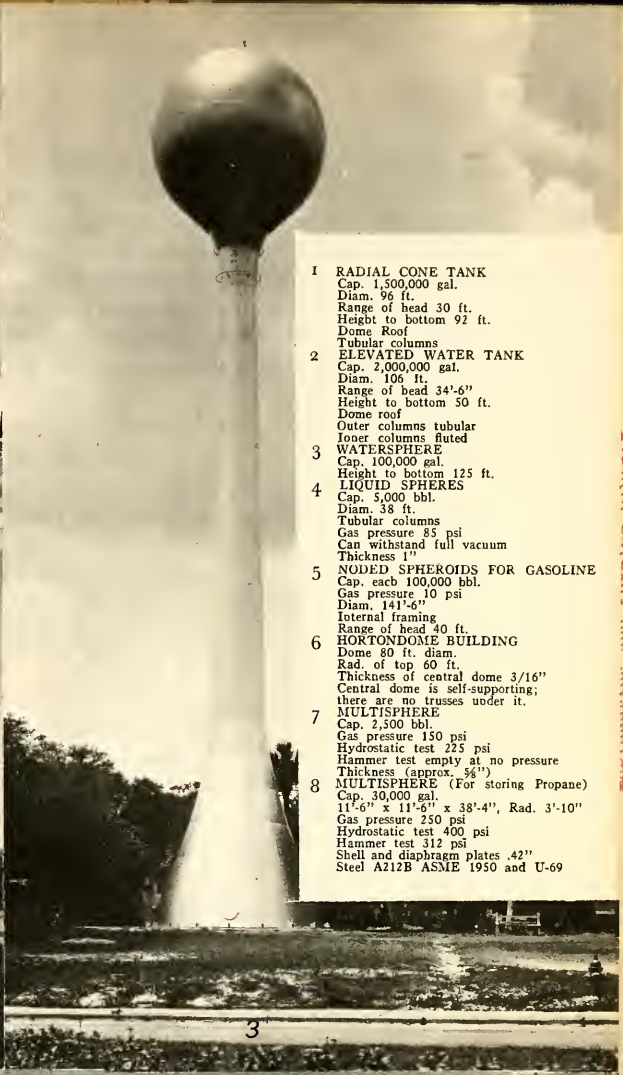
On the other hand, the inability of concrete per se, and the ability of steel per se, to sustain loads by tensile stresses, are established and generally known.

Pressure vessels and storage tanks demand materials able to resist gas and liquid pressure by tension and to carry roof and column loads by compression. Welded steel plates meet these needs. Hence, in the pressure vessel and storage tank industry, as perhaps in no other, the potentialities of thin welded steel plate structures to act efficiently, whether they pull or push, *are* appreciated and utilized. What is done in this field points the way to definite exploitation in other fields—particularly in those of buildings and bridges.

*Author's Note: Few men anywhere have had Dr. Harry Boardman's long and rich experience in creating, developing and designing of bent plate structures. Associated with one of the world's leading pioneers of that type of work, he exhibits here what appears like a "fairyland of dream structures" beckoning creative engineers and architects towards new horizons with plate design. No limit exists on shape. Design can follow functional or fanciful form, figures, curves or angles. Welding's magic now stands revealed before you in striking performance. For buildings and bridges it augurs near miracles in economy and appearance through hitherto unexplored plate design. M.P.K.







1 RADIAL CONE TANK
Cap. 1,500,000 gal.
Diam. 96 ft.
Range of head 30 ft.
Height to bottom 92 ft.
Dome Roof
Tubular columns

2 ELEVATED WATER TANK
Cap. 2,000,000 gal.
Diam. 106 ft.
Range of head 34'-6"
Height to bottom 50 ft.
Dome roof
Outer columns tubular
Inner columns fluted

3 WATERSPHERE
Cap. 100,000 gal.
Height to bottom 125 ft.

4 LIQUID SPHERES
Cap. 5,000 bbl.
Diam. 38 ft.
Tubular columns
Gas pressure 85 psi
Can withstand full vacuum
Thickness 1"

5 NODED SPHEROIDS FOR GASOLINE
Cap. eacb 100,000 bbl.
Gas pressure 10 psi
Diam. 141'-6"
Ioternal framing
Range of head 40 ft.

6 HORTONDOME BUILDING
Dome 80 ft. diam.
Rad. of top 60 ft.
Thickness of central dome 3/16"
Central dome is self-supporting;
there are no trusses under it.

7 MULTISPHERE
Cap. 2,500 bbl.
Gas pressure 150 psi
Hydrostatic test 225 psi
Hammer test empty at no pressure
Thickness (approx. 5/8")

8 MULTISPHERE (For storing Propane)
Cap. 30,000 gal.
11'-6" x 11'-6" x 38'-4", Rad. 3'-10"
Gas pressure 250 psi
Hydrostatic test 400 psi
Hammer test 312 psi
Shell and diaphragm plates .42"
Steel A212B ASME 1950 and U-69

The API (American Petroleum Institute) Std. 12C Specification[1] for Welded Oil Storage Tanks provides for plain welded spherical steel dome roofs (each a spherical zone of one base) which are supported only at their peripheries and which otherwise sustain their own weights, wind forces, snow loads, and such concentrated loads as are imposed by valves, other fittings, and personnel.

Suitable formulas for the design of such plain spherical dome roofs are:

$$(1)\quad L = \frac{330t}{\sqrt{1+5t}} \qquad \text{and}$$

$$(2)\quad t = \left(\frac{L^2}{43600}\right)\left(1+\sqrt{1+\frac{17400}{L^2}}\right)$$

wherein:

L = radius of curvature of dome, ft.

t = thickness of dome, in.

Formulas (1) and (2) may be applied safely to hemispherical, and to less than hemispherical, butt-welded steel spherical domes.[2] According to them, a plain steel hemispherical dome 200 ft. in diameter could be safely made of 5/8 in. plate. Obviously, a hemispherical dome of this diameter could be of much thinner plate if reinforced by meridian ribs. Furthermore, it is easy to see that, instead of conforming to a spherical shape, the dome plates could be bulged or barreled outwardly or inwardly between the ribs; or they could be flat from rib to rib so that each meridian section would define a circular arc and each horizontal section would define a regular polygon of as many sides as ribs. The variations possible are many. Only tantalizing glimpses can now be seen of what might be done with very thin double-walled steel domes with ribs between the walls as in airplane wings. Certain it is that careful study and experimentation along these lines would amply reward investigators in the field of building construction.

Corresponding formulas for the design of plain butt-welded ellipsoidal dome roofs—each with the hight (1/2 of the minor axis) equal to 1/4 of the diameter (major axis) are:

$$(3)\quad D = \frac{370t}{\sqrt{1+5t}} \qquad \text{and}$$

$$(4)\quad t = \left(\frac{D^2}{54800}\right)\left(1+\sqrt{1+\frac{21900}{D^2}}\right)$$

wherein:

D = diameter (major axis), ft.

t = thickness of dome, in.

According to formulas (3) and (4), a plain butt-welded ellipsoidal roof 150 ft. in diameter with a diameter to height ratio of 4 could be safely made of 1 in. plate, and a similar roof 100 ft. in diameter could be safely made of 1/2 in. plate. Otherwise expressed, a plain ellipsoidal roof 150 ft. in diameter and 37.5 ft. high requires 1 in. plating, and a similar roof 100 ft. in diameter and 25 ft. high requires 1/2 in. plating.

These assurances of the structural adequacy of plain and ribbed steel domes are a challenge in the building field where problems of insulation, condensation, heating, ventilation, and decoration, tease and intrigue the designer.

For many years tubular columns[3] to support elevated tanks full of water have been designed by the formula:

$$(5)\quad \frac{P}{A} = XY, \text{ wherein;}$$

P = total axial load, lb.

A = cross-sectional area, sq. in.

$$X = \frac{18000}{1+\dfrac{L^2}{18000r^2}} \quad \text{or 15,000 psi,}$$

whichever is the smaller

(X = direct compression alone, or combined direct and bending compression)

$$Y = 2/3\left(100\frac{t}{R}\right)\left[2 - 2/3\left(100\frac{t}{R}\right)\right] \text{ for values of } \frac{t}{R} \text{ less than } 0.015 \text{ and } Y = 1.00 \text{ for values of } \frac{t}{R} \text{ equal to or more than } 0.015.$$

L = effective length of column in.
(The effective length depends on the nature of the lateral and end support).
R = outside radius of the column, in.
t = thickness of column in.
(minimum allowable t= ¼ in.)
r = radius of gyration of column
= .707 R in.

When $\frac{L}{r} = 60$, X = 15,000 psi. Therefore, when (L ÷ .707R) is equal to or less than 60, or, what is the same thing, when L is equal to or less than 42.42 R, the full cross-sectional area of a tubular column can be assigned the maximum design stress of 15,000 psi in marked contrast to rolled-section columns of the same length but much greater slenderness $\frac{L}{r}$ ratio. It is for this reason that a single tubular column can support a 250,000 gal. elevated tank full of water and 100 ft. or more above the foundation whereas an adequate support made of rolled sections would be a complex space frame.

Surely the bridge engineer should look into the use of tubular compression members and of multiple flat plate or bar tension members for thus all welding could be done on plates thin enough to require little or no preheating; yet without limiting the total composite area. Without doubt, also, the building engineer would, if he really looked, find unexpected and rewarding uses for tubular, but not necessarily circular, elements.

For at least two decades makers and users of butt-welded horizontal cylindrical tanks and pipe lines[4] have taken advantage of their remarkable ability to act as beams, provided they are held round, at or near the supports, by encircling rings, or by heads, or by diaphragms. Thus long horizontal cylindrical pressure vessels utilize the 50% of their longitudinal strength which is not required to resist internal pressure. The ordinary beam formulas may be applied keeping in mind the facts that the beam has two curved webs and that the circumferential shearing stresses are tangential instead of vertical. The stress system is such that neither the metal weight nor the contained liquid weight causes out-of-roundness. Only near the supports, where the shell is constrained against normal expansion under pressure, are there longitudinal bending stresses in the shell in addition to the longitudinal direct stresses due to beam action.

Obviously, a horizontal cylindrical shell could be split lengthwise in the plane of a horizontal diameter and the cylinder axis, and each half-cylinder[5] could then be made to serve as a beam provided:

(a) A suitable bar or flange were installed along each cut edge to take from the half-cylindrical shell the stresses which, in a full cylinder, would act between the upper and lower half cylinders;
(b) The ends, or two sections set equally in from the ends, were held round, by suitable devices, and supported on or near to such devices.

Such complete and half cylindrical shells are able to support astonishingly large localized loads[6] by beam action.

References:

1. API Std 12C, Eleventh Edition, September, 1952, pars. 3.5.8, 3.5.9, and 3.5.10. Issued by *American Petroleum Institute,* Division of Production, Dallas, Texas.
2. Vacuum Test of Sphere to Failure—Leonard P. Zick and Carl E. Carlson. November, 1947, issue of the *Water Tower,* published by the Chicago Bridge & Iron Company, Chicago, Illinois.
3. AWWA D100-52, pars. 3.5.2, 3.5.3 and Tables 1, 2, and 3. Issued October, 1952.
4. Design of Large Pipe Lines.—Herman Schorer. Transaction ASCE, Vol. 98 (1933), p. 101.
5. Line Load Action on Thin Cylindrical Shells—Herman Schorer. Transactions ASCE, Vol. 101 (1936), p. 767.
6. University of Illinois Experiment Station Bulletin No. 331.

DERIVATIONS OF FORMULAS (1), (2), (3) and (4)

Assume a uniform dome thickness of t inches, and a snow load of $25\cos^2\phi$ lb. per sq. ft. of dome surface.
Metal wt. = 41t lb. per sq. ft.
Differential dome area

$$= Ld\phi\ (2\pi L \sin\phi)$$
$$= 2\pi L^2 \sin\phi d\phi$$

Total load supported by the dome

$$= \int_0^\theta (2L^2 \sin\phi\ d\phi)\ (41t + 25\cos^2\phi)$$

$$= \pi L^2[82t\ (1-\cos\theta) - 16\tfrac{2}{3}\ (\cos^3\theta - 1)]$$

By inspection of the sketch, the equation of equilibrium of vertical forces on the dome is

$$T_2 \sin\theta\ (2\pi L \sin\theta = -\pi L^2[82t\ (1-\cos\theta) - 16\tfrac{2}{3}\ (\cos^3\theta - 1)]$$

Hence,

$$T_2 = -\frac{L}{2}\left[\frac{82t}{1+\cos\theta} + 16\tfrac{2}{3}\ \left(1 + \frac{\cos^2\theta}{1+\cos\theta}\right)\right]$$

NOTE:— If T_2 is +, it indicates tension.
Let T_1 = lb. per ft. of meridian arc.

NOTE:— If T_1 is +, it indicates tension.

Then the equation of equilibrium of radial forces on a sq. ft. of dome area is the spherical soap bubble formula,

$$\frac{T_1 + T_2}{L} - (41t + 25\cos^2\theta)\cos\theta,$$

Hence, $T_1 = -[T_2 + L\ (41t + 25\cos^2\theta)\cos\theta]$ or

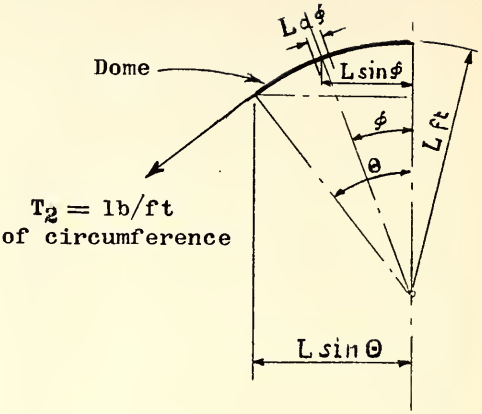
$$T_1 = +\frac{L}{2}\left[\frac{82t}{1+\cos\theta} + 16\tfrac{2}{3}\ \left(1 + \frac{\cos^2\theta}{1+\cos\theta}\right) - (82t + 50\cos^2\theta)\cos\theta\right]$$

When $\theta = 0$, $T_2 = T_1 = -\frac{L}{2}(41t + 25)$.

When $\theta = 90°$, $T_2 = -\frac{L}{2}(82t + 16\tfrac{2}{3})$ and

$$T_1 = +\frac{L}{2}(82t + 16\tfrac{2}{3}).$$

For all values of t greater than .2 in., the value of T_2 for $\theta = 90°$ is more than the value of T_2 for $\theta = 0°$. Since .2 in. is approximately $\frac{3}{16}$ in. and since no roofs thinner than $\frac{3}{16}$ in. are likely to be built, and since, obviously, T_1 is



always numerically equal to or less than T_2, it follows that all domes having θ values ranging from 0° to 90° may safely and practically be designed for the value of T_2 for $\theta = 90°$, namely, a compressive stress of $\frac{L}{2}(82t + 16\tfrac{2}{3})$ lb. per ft. or closely, $L\ (3.42 + \frac{.7}{t})$ psi.

The ASME Unfired Pressure Vessel Code allowable compressive stress in spherical domes is $\left(\frac{900{,}000\ t}{12\ L}\right)$psi.

Therefore, $\frac{900{,}000\ t}{12\ L} = L\ (3.42 + \frac{.7}{t})$: hence,

(1) $L = \frac{330t}{\sqrt{1 + 5t}}$ ft. and

(2) $t = \left(\frac{L^2}{43600}\right)\left(1 + \sqrt{1 + \frac{17400}{L^2}}\right)$ in.

For ellipsoidal dome roofs with the height equal to ¼ of the diameter, the crown radius is approximately .9 D, where D is the diameter in feet. Substituting .9 D for L in equations (1) and (2)),

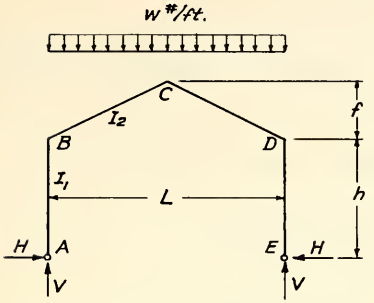
(3) $D = \frac{370t}{\sqrt{1 + 5t}}$ ft. and

(4) $t = \left(\frac{D^2}{54800}\right)\left(1 + \sqrt{1 + \frac{21900}{D^2}}\right)$ in.

SHEET No. 1 OF 2

DERIVATION OF BASIC EQUATIONS

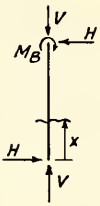
Consider Only Bending Deformation
(usual analysis)



By Least Work :-

$$\frac{\partial W}{\partial H} = \sum \int M \frac{\partial M}{\partial H} \frac{ds}{EI} = 0$$

Free Body of Column:-



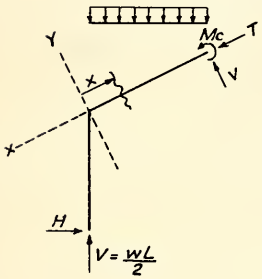
$$M = -Hx \Big]_0^h$$

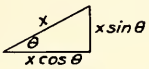
$$\frac{\partial M}{\partial H} = -x$$

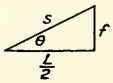
$$\int_0^h (-Hx)(-x)\frac{dx}{EI_1}$$

other column is the same.

Free Body of Half Frame:-







$$\sin\theta = \frac{f}{s} \qquad \cos\theta = \frac{L}{2S}$$

$$M = -H(h + x\sin\theta) + Vx\cos\theta - wx\cos\theta\,\frac{x\cos\theta}{2}$$

$$M = -H\left(h + \frac{xf}{s}\right) + \frac{wL}{2} \times \frac{L}{2s} - \frac{wx^2L^2}{8s^2}$$

$$M = -H\left(h + \frac{xf}{s}\right) + \frac{wL^2x}{4s} - \frac{wx^2L^2}{8s^2}$$

$$\frac{\partial M}{\partial H} = -1\left(h + \frac{fx}{s}\right)$$

$$\int_0^s \left[-H\left(h + \frac{fx}{s}\right) + \frac{wL^2x}{4s} - \frac{wx^2L^2}{8s^2}\right]\left[-\left(h + \frac{fx}{s}\right)\right]\frac{dx}{EI_2}$$

Other rafter is the same thus:-

$$\frac{\partial W}{\partial H} = 0 = 2\int_0^h (-Hx)(-x)\frac{dx}{EI_1} + 2\int_0^s \left[-H\left(h + \frac{fx}{s}\right) + \frac{wL^2x}{4s} - \frac{wL^2x^2}{8s^2}\right]\left[-\left(h + \frac{fx}{s}\right)\right]\frac{dx}{EI_2}$$

$$\frac{\partial W}{\partial H} = 0 = 2\int_0^h Hx^2\frac{dx}{EI_1} + 2\int_0^s \left[H\left(h + \frac{fx}{s}\right)^2 - \frac{wL^2x}{4s}\left(h + \frac{fx}{s}\right) + \frac{wL^2x^2}{8s^2}\left(h + \frac{fx}{s}\right)\right]\frac{dx}{EI_2}$$

$$\frac{\partial W}{\partial H} = 0 = 2\int_0^h Hx^2\frac{dx}{EI_1} + 2\int_0^s H\left(h + \frac{fx}{s}\right)^2\frac{dx}{EI_2} - 2\int_0^s \frac{wL^2xh}{4s}\frac{dx}{EI_2} - 2\int_0^s \frac{wL^2fx^2dx}{4s^2\,EI_2} + 2\int_0^s \frac{wL^2x^2h\,dx}{8s^2\,EI_2} + 2\int_0^s \frac{wL^2fx^3dx}{8s^3\,EI_2}$$

SHEET No. 2 OF 2

from last line on Sheet 1

$$\frac{\partial W}{\partial H} = 0 = 2\int_0^h \frac{Hx^2\,dx}{EI_1} + 2\int_0^s \frac{H\left(h+\frac{fx}{s}\right)^2 dx}{EI_2} - 2\int_0^s \frac{wL^2xh}{4s}\,\frac{dx}{EI_2} - 2\int_0^s \frac{wL^2fx^2\,dx}{4s^2\,EI_2} + 2\int_0^s \frac{wL^2x^2h}{8s^2}\,\frac{dx}{EI_2} + 2\int_0^s \frac{wL^2fx^3}{8s^3}\,\frac{dx}{EI_2}$$

$$\frac{\partial W}{\partial H} = 0 = \frac{2Hx^3}{3EI_1}\Big]_0^h + \frac{2H\left(h+\frac{fx}{s}\right)^3}{3\frac{f}{s}\,EI_2}\Big]_0^s - \frac{2wL^2hx^2}{8s\,EI_2}\Big]_0^s - \frac{2wL^2fx^3}{12s^2\,EI_2}\Big]_0^s + \frac{2wL^2hx^3}{24s^2\,EI_2}\Big]_0^s + \frac{2wL^2fx^4}{32s^3\,EI_2}\Big]_0^s$$

$$\frac{\partial W}{\partial h} = 0 = \frac{2Hh^3}{3EI_1} + \frac{2Hs\left(h+\frac{fx}{s}\right)^3}{3fEI_2}\Big]_0^s - \frac{wL^2hs}{4EI_2} - \frac{wL^2fs}{6EI_2} + \frac{wL^2hs}{12EI_2} + \frac{wL^2fs}{16EI_2}$$

$$\frac{\partial W}{\partial H} = 0 = \frac{2Hh^3}{3EI_1} + \frac{2Hs(h+f)^3}{3fEI_2} - \frac{2Hsh^3}{3fEI_2} - \frac{wL^2hs}{6EI_2} - \frac{10wL^2fs}{96EI_2}$$

$$\frac{\partial W}{\partial H} = 0 = \frac{2Hh^3}{3EI_1} + \frac{2Hs(h^3+3h^2f+3hf^2+f^3)}{3fEI_2} - \frac{2Hsh^3}{3fEI_2} - \frac{wL^2hs}{6EI_2} - \frac{5wL^2fs}{48EI_2}$$

$$\frac{\partial W}{\partial H} = 0 = \frac{2Hh^3}{3EI_1} + \cancel{\frac{2Hsh^3}{3fEI_2}} + \frac{2Hsh^2}{EI_2} + \frac{2Hshf}{EI_2} + \frac{2Hsf^2}{3EI_2} - \cancel{\frac{2Hsh^3}{3fEI_2}} - \frac{wL^2hs}{6EI_2} - \frac{5wL^2fs}{48EI_2}$$

$$\frac{\partial W}{\partial H} = 0 = \frac{2Hh^3}{3EI_1} + \frac{2Hsh^2}{EI_2} + \frac{2Hshf}{EI_2} + \frac{2Hsf^2}{3EI_2} - \frac{wL^2hs}{6EI_2} - \frac{5wL^2fs}{48EI_2}$$

$$\frac{wL^2hs}{6I_2} + \frac{5wL^2fs}{48I_2} = \frac{2Hh^3}{3I_1} + \frac{2Hsh^2}{I_2} + \frac{2Hshf}{I_2} + \frac{2Hsf^2}{3I_2}$$

$$wL^2\left(\frac{hs}{6} + \frac{5fs}{48}\right) = H\left(\frac{2h^3I_2}{3I_1} + 2sh^2 + 2shf + \frac{2sf^2}{3}\right)$$

$$\frac{wL^2s(8h+5f)}{48} = H\left(\frac{2}{3}h^2s\left[\frac{hI_2}{sI_1} + 3 + \frac{3f}{h} + \frac{f^2}{h^2}\right]\right)$$

$$\frac{wL^2\cancel{s}(8h+5f)}{32h^2\cancel{s}\left[\frac{hI_2}{sI_1} + 3 + \frac{3f}{h} + \frac{f^2}{h^2}\right]} = H$$

$$\boxed{\frac{wL^2(8h+5f)}{32h^2\left[\frac{hI_2}{sI_1} + 3 + \frac{3f}{h} + \frac{f^2}{h^2}\right]} = H}$$

$$M_B = -Hh = \boxed{\frac{wL^2(8h+5f)}{32h\left[\frac{hI_2}{sI_1} + 3 + \frac{3f}{h} + \frac{f^2}{h^2}\right]} = M_B = M_D}$$

$$M_C = -H(h+f) + \frac{wL^2}{4} - \frac{wL^2}{8}$$

$$M_C = -H(h+f) + \frac{wL^2}{8} \qquad \boxed{M_C = \frac{wL^2}{8} - H(h+f)}$$

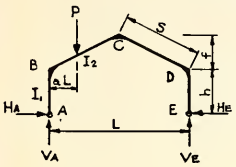
Single Span Rigid Frames

DESIGN TABLES—SPAN 50 FT.-150 FT.

CONSTANTS:

$$A = \frac{I_2}{I_1}\cdot\frac{h}{s} \qquad C = \frac{f}{h} \qquad K = 4(A+3+3C+C^2)$$

CASE I-VERTICAL CONCENTRATED ROOF LOAD AT ANY SECTION



$$V_A = P - P_a$$

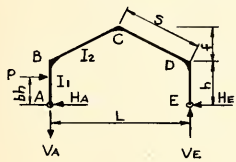
$$V_E = P_a$$

$$H_A = H_E = H = \frac{PLa(6-6a+3C-4Ka^2)}{hK}$$

$$M_B = M_D = -Hh$$

$$M_C = \frac{P_aL}{2} - H(h+f)$$

CASE II-HORIZONTAL CONCENTRATED COLUMN LOAD AT ANY SECTION



$$V_A = V_E = V = \frac{Pbh}{L}$$

$$H_E = \frac{Pb}{K}(3A-b^2A+G+3C)$$

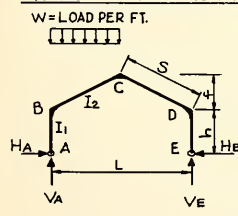
$$H_A = P - H_E$$

$$M_B = H_Ah - Ph(1-b)$$

$$M_C = \frac{Pbh}{2} - H_E(h+f)$$

$$M_D = -H_Eh$$

CASE III-UNIFORM VERTICAL ROOF LOAD ON HALF SPAN



$$V_A = \frac{3WL}{8}$$

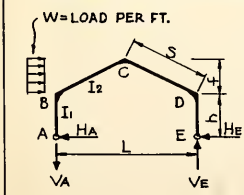
$$V_E = \frac{WL}{8}$$

$$H_A = H_E = H = \frac{WL^2}{16hK}(8+5C)$$

$$M_B = M_D = -Hh$$

$$M_C = \frac{WL^2}{16} - H(h+f)$$

CASE IV-UNIFORM HORIZONTAL ROOF LOAD



$$V_A = V_E = V = \frac{Wf(2h+f)}{2L}$$

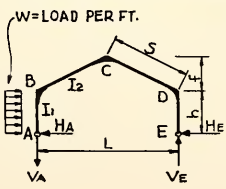
$$H_E = \frac{Wf}{4K}(8A+24+20C+5C^2)$$

$$H_A = Wf - H_E$$

$$M_B = H_Ah \qquad M_D = -H_Eh$$

$$M_C = \frac{VL}{2} - H_E(h+f)$$

CASE V-UNIFORM HORIZONTAL COLUMN LOAD



$$V_A = V_E = V = \frac{Wh^2}{2L}$$

$$H_E = \frac{Wh}{4K}(5A+12+6C)$$

$$H_A = Wh - H_E$$

$$M_B = H_Ah - \frac{Wh^2}{2} \qquad M_D = -H_Eh$$

$$M_C = \frac{Wh^2}{4} - H_E(h+f)$$

The tables that follow are essentially self-explanatory.

Values shown are based upon a uniformly distributed load of one (1) kip per foot. Other values are proportional.

Positive bending moments cause tension on the inside surface of the frame.

General equations for varying conditions encountered frequently, other than those given in the tables, are shown on this page.

Maximum positive moments represented by M_x are given for spans over 100 ft.

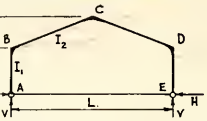
All computations for the tables are based on the center of gravity axis.

The design tables are for hinged bases only, as fixed bases are seldom used. (Read chapter on BASES)

Temperature stresses have not been given, as they are usually inconsequential for the spans listed in the tables. They are generally ignored in practice. Investigation should be made for spans greater than 200 ft.

Although the effect of the increased depth in curved knees is also generally ignored in practice while computing the horizontal reactions, the actual knee moments in many cases will be very much higher than those computed theoretically. See chapter on KNEES for adjustment values for the horizontal reactions, especially for the extreme cases of wide flat spans with short columns.

1 KIP PER FT.

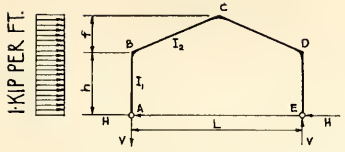


L=50

Dimensions are in feet
Loads are in kips.
Moments are in ft.-kips.

$\frac{I_2}{I_1} = 1$						$\frac{I_2}{I_1} = \frac{1}{2}$			
h	f	$-M_B$	$+M_C$	H_A	V_A	$-M_B$	$+M_C$	H_A	V_A
12	5.0	161.0	84.5	13.4	25	169.1	73.0	14.1	25
12	7.5	151.8	65.8	12.6	25	158.2	55.5	13.2	25
12	10.0	143.2	50.1	11.9	25	148.1	40.9	12.3	25
12	12.5	135.1	36.7	11.3	25	139.0	28.7	11.6	25
12	15.0	127.6	25.4	10.6	25	130.7	18.4	10.9	25
12	17.5	120.7	15.7	10.1	25	123.2	9.7	10.3	25
14	5.0	161.0	94.0	11.5	25	170.9	80.6	12.2	25
14	7.5	153.6	76.6	11.0	25	161.6	64.3	11.5	25
14	10.0	146.5	61.5	10.5	25	152.9	50.4	10.9	25
14	12.5	139.6	48.3	10.0	25	144.8	38.5	10.3	25
14	15.0	133.1	36.9	9.5	25	137.3	28.2	9.8	25
14	17.5	127.4	26.9	9.1	25	130.3	19.3	9.3	25
16	7.5	154.3	86.0	9.6	25	163.8	71.9	10.2	25
16	10.0	148.3	71.5	9.3	25	156.2	58.6	9.8	25
16	12.5	142.5	58.6	8.9	25	149.1	47.0	9.3	25
16	15.0	137.2	46.7	8.6	25	142.3	36.8	8.9	25
16	17.5	131.5	37.2	8.2	25	135.9	27.9	8.5	25
18	5.0	158.9	109.5	8.8	25	172.0	92.7	9.6	25
18	7.5	154.1	94.3	8.6	25	165.2	78.5	9.2	25
18	10.0	149.2	80.4	8.3	25	158.6	65.8	8.8	25
18	12.5	144.3	67.9	8.0	25	152.2	54.6	8.5	25
18	15.0	139.5	56.7	7.8	25	146.2	44.5	8.1	25
18	20.0	130.3	37.5	7.2	25	134.9	27.7	7.5	25

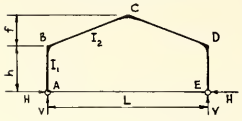
L=50



Dimensions are in feet
Loads are in kips.
Maments are in ft.-kips.

$\frac{I_2}{I_1} = 1.$								$\frac{I_2}{I_1} = \frac{1}{2}$					
h	f	$+M_B$	$-M_C$	$-M_D$	H_A	H_E	V_A	$+M_B$	$-M_C$	$-M_D$	H_A	H_E	V_A
12	5.0	72.7	11.8	59.3	12.1	4.9	2.9	73.5	10.7	58.5	12.1	4.9	2.9
12	7.5	92.2	18.3	69.8	13.7	5.8	3.8	93.1	16.9	68.9	13.8	5.7	3.8
12	10.0	112.1	25.5	80.0	15.3	6.7	4.8	112.9	24.0	79.1	15.4	6.6	4.8
12	12.5	132.1	33.5	89.9	17.0	7.5	6.0	133.0	31.8	89.1	17.1	7.4	6.0
12	15.0	152.6	42.2	99.7	18.7	8.3	7.3	153.1	40.3	98.9	18.8	8.2	7.3
12	17.5	172.5	51.5	109.5	20.4	9.1	8.7	173.4	49.5	108.6	20.4	9.1	8.7
14	5.0	91.1	14.1	76.9	13.5	5.5	3.6	92.3	12.5	75.7	13.6	5.4	3.6
14	7.5	113.8	21.5	88.2	15.1	6.4	4.6	115.0	19.5	88.0	15.2	6.3	4.6
14	10.0	136.7	29.6	101.3	16.8	7.2	5.8	138.1	27.3	99.9	16.9	7.1	5.8
14	12.5	160.0	38.4	113.0	18.4	8.1	7.0	161.3	35.9	111.7	18.5	8.0	7.0
14	15.0	183.4	47.8	124.6	20.1	8.9	8.4	184.7	45.1	123.3	20.2	8.8	8.4
14	17.5	207.0	58.0	136.0	21.2	9.7	9.9	208.3	55.1	134.8	21.9	9.6	9.9
16	7.5	137.0	24.9	111.0	16.6	6.9	5.5	138.8	22.3	109.2	16.7	6.8	5.5
16	10.0	163.2	33.9	124.9	18.2	7.8	6.8	165.0	30.8	123.0	18.3	7.7	6.8
16	12.5	189.6	43.5	138.4	19.8	8.7	8.1	191.5	40.1	136.5	20.0	8.5	8.1
16	15.0	216.2	53.8	151.8	21.5	9.5	9.6	218.1	50.2	149.9	21.6	9.4	9.6
16	17.5	243.1	64.8	164.9	23.2	10.3	11.2	244.9	60.9	163.1	23.3	10.2	11.2
18	5.0	133.2	19.5	118.8	16.4	6.6	5.3	135.5	16.6	116.5	16.5	6.5	5.3
18	7.5	162.0	28.6	135.0	18.0	7.5	6.5	164.5	25.2	132.5	18.1	7.4	6.5
18	10.0	191.3	38.5	150.7	19.6	8.4	7.8	193.8	34.5	148.2	19.8	8.2	7.8
18	12.5	220.8	49.0	166.2	21.3	9.2	9.3	223.5	44.6	163.6	21.4	9.1	9.3
18	15.0	250.7	60.1	181.3	22.9	10.1	10.9	253.3	55.3	178.7	23.1	9.9	10.9
18	20.0	311.1	84.3	211.0	26.3	11.7	14.4	313.6	79.0	208.4	26.4	11.6	14.4

1 KIP PER FT.

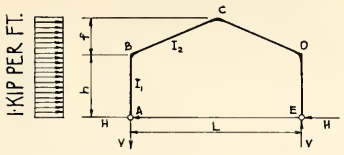


L = 55

Dimensions are in feet
Loads are in kips.
Moments are in ft.-kips.

$\frac{I_2}{I_1} = 1$						$\frac{I_2}{I_1} = \frac{1}{2}$			
h	f	$-M_B$	$+M_C$	H_A	V_A	$-M_B$	$+M_C$	H_A	V_A
12	7.5	184.9	77.6	15.4	27.5	192.0	66.1	16.0	27.5
12	10.0	174.1	58.9	14.5	27.5	179.7	48.6	15.0	27.5
12	12.5	164.1	43.0	13.7	27.5	168.5	34.0	14.0	27.5
12	15.0	154.9	29.6	12.9	27.5	158.4	21.7	13.2	27.5
12	20.0	138.7	8.2	11.6	27.5	141.0	2.3	11.7	27.5
14	7.5	187.5	90.3	13.4	27.5	196.4	76.5	14.0	27.5
14	10.0	178.4	72.3	12.7	27.5	185.6	59.9	13.3	27.5
14	12.5	169.4	56.7	12.1	27.5	175.2	46.5	12.5	27.5
14	15.0	161.7	43.2	11.5	27.5	166.5	33.3	11.9	27.5
14	17.5	154.1	31.4	11.0	27.5	158.0	22.7	11.3	27.5
14	20.0	147.0	21.1	10.5	27.5	150.2	13.4	10.7	27.5
16	7.5	188.5	101.3	11.8	27.5	199.3	85.4	12.5	27.5
16	10.0	180.9	84.1	11.3	27.5	189.9	69.6	11.9	27.5
16	12.5	173.5	69.0	10.8	27.5	180.9	55.8	11.3	27.5
18	7.5	188.6	111.0	10.5	27.5	201.1	93.2	11.2	27.5
18	10.0	182.3	94.6	10.1	27.5	192.9	78.1	10.7	27.5
18	12.5	176.0	79.9	9.8	27.5	185.0	64.7	10.3	27.5
20	7.5	187.9	119.7	9.4	27.5	202.2	100.1	10.1	27.5
20	12.5	177.4	89.8	8.9	27.5	187.9	72.8	9.4	27.5
22	7.5	186.8	127.6	8.5	27.5	202.7	106.3	9.2	27.5
22	12.5	178.1	98.9	8.1	27.5	190.1	80.0	8.6	27.5
					L = 60				
12	7.5	221.4	90.3	18.5	30	229.2	77.5	19.1	30
12	10.0	208.2	68.3	17.4	30	214.4	56.9	17.9	30
12	12.5	196.1	49.7	16.3	30	200.9	39.7	16.7	30
12	15.0	184.9	34.0	15.4	30	188.8	25.2	15.7	30
12	20.0	165.4	9.0	13.8	30	167.9	2.3	14.0	30
14	7.5	224.7	104.9	16.1	30	234.6	89.7	16.8	30
14	10.0	213.6	83.9	15.2	30	221.6	70.1	15.8	30
14	12.5	203.0	65.7	14.5	30	209.6	53.3	15.0	30
14	15.0	193.2	49.9	13.8	30	198.5	38.9	14.2	30
14	20.0	175.4	24.1	12.5	30	178.9	15.4	12.8	30
16	7.5	226.3	117.7	14.2	30	238.2	100.1	14.9	30
16	10.0	216.9	97.6	13.6	30	226.8	81.5	14.2	30
16	12.5	207.8	79.9	13.0	30	216.0	65.2	13.5	30
18	7.5	226.6	129.0	12.6	30	240.6	109.1	13.4	30
18	10.0	218.7	109.8	12.2	30	230.5	91.4	12.8	30
18	12.5	210.9	92.6	11.7	30	220.9	75.6	12.3	30
20	7.5	226.2	139.0	11.3	30	242.1	117.1	12.1	30
20	10.0	219.4	120.9	10.9	30	233.2	100.3	11.7	30
22	10.0	219.5	130.8	10.0	30	235.0	108.2	10.7	30
22	12.5	213.8	114.7	9.7	30	227.3	93.6	10.3	30

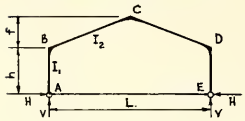
L=55



Dimensions are in feet
Loads are in kips
Moments are in ft.-kips

$\frac{I_2}{I_1} = 1.$								$\frac{I_2}{I_1} = \frac{1}{2}$					
h	f	$+M_B$	$-M_C$	$-M_D$	H_A	H_E	V_A	$+M_B$	$-M_C$	$-M_D$	H_A	H_E	V_A
12	7.5	92.4	18.1	69.6	13.7	5.8	3.5	93.2	16.8	68.8	13.8	5.7	3.5
12	10.0	112.2	25.3	79.8	15.3	6.7	4.4	113.0	23.8	79.0	15.4	6.6	4.4
12	12.5	132.2	33.3	89.8	17.0	7.5	5.5	133.0	31.6	89.0	17.1	7.4	5.5
12	15.0	152.4	41.9	99.6	18.7	8.3	6.6	153.2	40.1	98.8	18.8	8.2	6.6
12	20.0	193.0	61.3	119.0	22.1	9.9	9.3	193.7	59.4	118.3	22.1	9.9	9.3
14	7.5	114.0	21.2	89.0	15.1	6.4	4.2	115.2	19.4	87.9	15.2	6.3	4.2
14	10.0	136.9	29.2	101.1	16.8	7.2	5.2	138.2	27.1	99.8	16.9	7.1	5.2
14	12.5	160.2	38.0	112.8	18.4	8.1	6.4	161.4	35.7	111.6	18.5	8.0	6.4
14	15.0	183.6	47.5	124.4	20.1	8.9	7.6	184.8	44.9	123.2	20.2	8.8	7.6
14	17.5	207.1	57.6	135.9	21.8	9.7	9.0	208.3	54.9	134.7	21.9	9.6	9.0
14	20.0	230.8	68.5	147.2	23.5	10.5	10.5	232.0	65.6	146.0	23.6	10.4	10.5
16	7.5	137.3	24.5	110.7	16.6	6.9	5.0	139.0	22.0	109.0	16.7	6.8	5.0
16	10.0	163.4	33.4	124.6	18.2	7.8	6.1	165.2	30.6	122.8	18.3	7.7	6.1
16	12.5	189.8	43.0	138.2	19.9	8.6	7.4	191.6	39.9	136.4	20.0	8.5	7.4
18	7.5	162.4	28.1	134.6	18.0	7.5	5.9	164.7	24.9	132.3	18.1	7.4	5.9
18	10.0	191.7	37.9	150.4	19.6	8.4	7.1	194.0	34.2	148.0	19.8	8.2	7.1
18	12.5	221.2	48.3	165.8	21.3	9.2	8.5	223.7	44.2	163.4	21.4	9.1	8.5
20	7.5	189.3	32.0	160.8	19.5	8.0	6.9	192.2	27.9	157.8	19.6	7.9	6.9
20	12.5	254.3	54.0	195.7	22.7	9.8	9.6	257.5	48.7	192.5	22.9	9.6	9.6
22	7.5	217.8	36.1	189.2	20.9	8.6	7.9	221.6	31.0	185.4	21.1	8.4	7.9
22	12.5	289.1	59.9	228.0	24.1	10.4	10.8	293.2	53.5	223.8	24.3	10.2	10.8
							L=	**60**					
12	7.5	92.5	17.9	69.5	13.7	5.8	3.2	93.2	16.7	68.8	13.8	5.7	3.2
12	10.0	112.3	25.1	79.7	15.4	6.6	4.0	113.1	23.7	78.9	15.4	6.6	4.0
12	12.5	132.3	33.0	89.7	17.0	7.5	5.0	133.1	31.5	88.9	17.1	7.4	5.0
12	15.0	152.5	41.7	99.5	18.7	8.3	6.1	153.2	40.0	98.8	18.8	8.2	6.1
12	20.0	193.1	61.1	118.9	22.1	9.9	8.5	193.8	59.2	118.2	22.1	9.9	8.5
14	7.5	114.1	20.9	88.9	15.2	6.3	3.9	115.2	19.2	87.8	15.2	6.3	3.9
14	10.0	137.1	28.9	100.9	16.8	7.2	4.8	138.3	27.0	99.7	16.9	7.1	4.8
14	12.5	160.3	37.7	112.7	18.5	8.0	5.9	161.5	35.5	111.5	18.5	8.0	5.9
14	15.0	183.7	47.1	124.3	20.1	8.9	7.0	185.0	44.7	123.0	20.2	8.8	7.0
14	20.0	230.9	68.2	147.1	23.5	10.5	9.6	232.0	65.5	146.0	23.6	10.4	9.6
16	7.5	137.6	24.1	110.4	16.6	6.9	4.6	139.1	21.8	108.9	16.7	6.8	4.6
16	10.0	163.7	33.0	124.3	18.2	7.8	5.6	165.3	30.4	122.7	18.3	7.7	5.6
16	12.5	190.1	42.6	137.9	19.9	8.6	6.8	191.8	39.6	136.3	20.0	8.5	6.8
18	7.5	162.8	27.6	134.2	18.0	7.5	5.4	164.9	24.6	132.1	18.2	7.3	5.4
18	10.0	192.0	37.3	150.0	19.7	8.3	6.5	194.2	33.9	147.8	19.8	8.2	6.5
18	12.5	221.6	47.8	165.5	21.3	9.2	7.8	223.8	43.9	163.2	21.4	9.1	7.8
20	7.5	189.7	31.4	160.3	19.5	8.0	6.3	192.5	27.5	157.5	19.6	7.9	6.3
20	10.0	222.0	42.0	178.0	21.1	8.9	7.5	225.0	37.6	175.0	21.2	8.8	7.5
22	10.0	253.8	46.9	208.2	22.5	9.5	8.5	257.5	41.4	204.5	22.7	9.3	8.5
22	12.5	289.6	59.0	227.4	24.2	10.3	9.9	293.5	53.0	223.5	24.3	10.2	9.9

1 KIP PER FT.

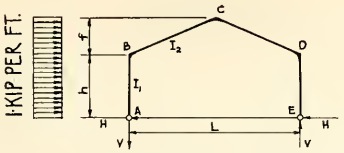


L = 65

Dimensions are in feet
Loads are in kips.
Moments are in ft.-kips.

$\frac{I_2}{I_1} = 1$						$\frac{I_2}{I_1} = \frac{1}{2}$			
h	f	$-M_B$	$+M_C$	H_A	V_A	$-M_B$	$+M_C$	H_A	V_A
12	7.5	261.1	103.8	21.8	32.5	269.7	89.8	22.5	32.5
12	10.0	245.4	78.3	20.4	32.5	252.1	65.9	21.0	32.5
12	12.5	231.0	56.5	19.2	32.5	236.3	45.7	19.7	32.5
12	17.5	205.3	23.4	17.1	32.5	208.9	14.6	17.4	32.5
12	22.5	184.4	−1.9	15.4	32.5	186.6	−8.4	15.5	32.5
14	7.5	265.3	120.7	18.9	32.5	276.2	103.9	19.7	32.5
14	12.5	239.5	74.8	17.1	32.5	246.6	61.3	17.6	32.5
14	17.5	216.5	41.1	15.5	32.5	221.3	30.2	15.8	32.5
14	22.5	196.8	15.1	14.1	32.5	200.1	6.5	14.3	32.5
16	7.5	267.5	135.1	16.7	32.5	280.6	115.9	17.5	32.5
16	10.0	256.0	112.1	16.0	32.5	267.0	94.3	16.7	32.5
16	12.5	245.0	92.1	15.3	32.5	254.1	75.6	15.9	32.5
16	20.0	215.5	43.3	13.5	32.5	220.7	31.4	13.8	32.5
20	7.5	268.0	159.7	13.4	32.5	285.5	135.5	14.3	32.5
20	10.0	259.7	138.6	13.0	32.5	274.9	115.9	13.7	32.5
20	12.5	251.5	119.5	12.6	32.5	264.5	98.4	13.2	32.5
20	17.5	235.2	87.2	11.8	32.5	245.0	68.8	12.2	32.5
24	7.5	265.5	179.7	11.1	32.5	287.2	151.2	12.0	32.5
24	10.0	259.0	161.1	10.8	32.5	278.7	133.3	11.6	32.5
24	12.5	254.0	142.1	10.6	32.5	270.5	116.9	11.3	32.5
						L = 70			
12	5.0	324.0	153.5	27.0	35	336.1	136.4	28.0	35
12	7.5	304.0	118.5	25.3	35	313.4	103.1	26.1	35
12	10.0	285.6	89.0	23.8	35	293.0	75.4	24.4	35
12	12.5	268.5	64.4	22.4	35	274.3	52.4	22.9	35
12	17.5	238.6	25.9	19.9	35	242.4	16.6	20.2	35
14	7.5	309.4	137.4	22.1	35	321.3	119.1	22.9	35
14	12.5	278.5	85.5	19.9	35	286.4	70.4	20.5	35
14	17.5	251.6	46.3	18.0	35	257.0	34.4	18.4	35
16	7.5	311.5	155.0	19.5	35	326.1	133.6	20.4	35
16	12.5	286.0	103.0	17.9	35	295.5	86.1	18.5	35
16	17.5	261.8	64.4	16.4	35	268.7	49.8	16.8	35
18	7.5	313.5	168.5	17.4	35	330.2	144.7	18.3	35
18	10.0	301.7	143.2	16.8	35	316.0	120.9	17.6	35
20	7.5	313.4	181.6	15.7	35	332.7	155.1	16.6	35
20	10.0	304.0	156.5	15.2	35	320.3	132.1	16.0	35
24	7.5	311.6	203.6	13.0	35	335.0	172.8	14.0	35
24	10.0	303.7	182.3	12.7	35	324.8	152.4	13.5	35

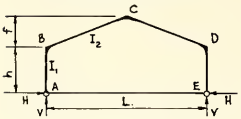
L=65



Dimensions are in feet
Loads are in kips
Moments are in ft.-kips.

$\frac{I_2}{I_1}=1$								$\frac{I_2}{I_1}=\frac{1}{2}$					
h	f	$+M_B$	$-M_C$	$-M_D$	H_A	H_E	V_A	$+M_B$	$-M_C$	$-M_D$	H_A	H_E	V_A
12	7.5	92.6	17.7	69.4	13.7	5.8	2.9	93.3	16.6	68.7	13.8	5.7	2.9
12	10.0	112.4	24.9	79.6	15.4	6.6	3.7	113.1	23.6	78.9	15.4	6.6	3.7
12	12.5	132.4	32.8	89.6	17.0	7.5	4.6	133.1	31.4	88.9	17.1	7.4	4.6
12	17.5	172.8	50.8	109.2	20.4	9.1	6.7	173.5	49.2	108.5	20.5	9.0	6.7
12	22.5	213.6	71.7	128.4	23.8	10.7	9.2	214.2	69.9	127.8	23.9	10.7	9.2
14	7.5	114.3	20.7	88.7	15.2	6.3	3.6	115.3	19.1	87.7	15.2	6.3	3.6
14	12.5	160.5	37.3	112.5	18.5	8.0	5.4	161.6	35.3	111.4	18.5	8.0	5.4
14	17.5	207.4	57.0	135.6	21.8	9.7	7.6	208.5	54.6	134.5	21.9	9.6	7.6
14	22.5	254.6	79.8	158.4	25.2	11.3	10.2	255.8	76.8	157.2	25.3	11.2	10.2
16	7.5	137.8	23.8	110.2	16.6	6.9	4.2	139.3	21.7	108.8	16.7	6.8	4.2
16	10.0	163.9	32.6	124.1	18.2	7.8	5.2	165.4	30.2	122.6	18.3	7.7	5.2
16	12.5	190.3	42.2	137.7	19.9	8.6	6.2	191.9	39.4	136.1	20.0	8.5	6.2
16	20.0	330.6	75.1	177.4	24.9	11.1	10.0	272.2	71.6	175.8	25.0	11.0	10.0
20	7.5	190.1	30.8	159.9	19.5	8.0	5.8	192.7	27.2	157.3	19.6	7.9	5.8
20	10.0	222.4	41.4	177.6	21.1	8.9	6.9	225.2	37.2	174.8	21.3	8.7	6.9
20	12.5	255.1	52.6	194.9	22.8	9.7	8.1	258.0	48.0	192.0	22.9	9.6	8.1
20	17.5	321.3	77.2	228.7	26.1	11.4	10.8	324.2	71.8	225.8	26.2	11.3	10.8
24	7.5	249.4	38.9	218.6	22.4	9.1	7.6	253.6	33.4	214.4	22.6	8.9	7.6
24	10.0	297.3	51.2	240.1	24.0	10.0	8.9	292.3	44.9	235.7	24.2	9.8	8.9
24	12.5	326.8	64.2	261.2	25.6	10.9	10.2	331.4	57.2	256.6	25.8	10.7	10.2
							L=	70					
12	5.0	73.1	11.2	58.9	12.1	4.9	2.1	73.7	10.3	58.3	12.1	4.9	2.1
12	7.5	92.7	17.6	69.3	13.7	5.8	2.7	93.3	16.5	68.7	13.8	5.7	2.7
12	10.0	112.5	24.7	79.5	15.4	6.6	3.5	113.2	23.5	78.8	15.4	6.6	3.5
12	12.5	132.5	32.7	89.5	17.0	7.5	4.3	133.2	31.3	88.8	17.1	7.4	4.3
12	17.5	172.9	50.6	109.1	20.4	9.1	6.2	173.5	49.1	108.5	20.5	9.0	6.2
14	7.5	114.4	20.5	88.6	15.2	6.3	3.3	115.5	19.0	87.6	15.2	6.3	3.3
14	12.5	160.6	37.2	112.4	18.5	8.0	5.0	159.7	35.2	111.4	18.5	8.0	5.0
14	17.5	207.5	56.7	135.5	21.8	9.7	7.1	208.6	54.5	134.5	21.9	9.6	7.1
16	7.5	138.0	23.5	110.0	16.6	6.9	3.9	139.4	21.5	108.6	16.7	6.8	3.9
16	12.5	190.5	41.9	137.5	19.9	8.6	5.8	192.0	39.2	136.0	20.0	8.5	5.8
16	17.5	243.9	63.1	164.1	23.2	10.3	8.0	245.4	60.0	162.7	23.3	10.2	8.0
18	7.5	163.3	26.8	133.7	18.1	7.4	4.6	165.2	24.2	131.8	18.2	7.3	4.6
18	10.0	192.6	36.5	149.4	19.7	8.3	5.6	194.5	33.4	147.5	19.8	8.2	5.6
20	7.5	192.1	30.4	159.6	19.6	7.9	5.4	192.9	27.0	157.1	19.6	7.9	5.4
20	10.0	222.8	40.9	177.2	21.1	8.9	6.4	225.4	36.9	174.6	21.3	8.7	6.4
24	7.5	249.9	38.2	218.1	22.4	9.1	7.1	253.9	33.0	214.1	22.6	8.9	7.1
24	10.0	288.4	50.4	239.6	24.0	10.0	8.3	292.6	44.5	235.4	24.2	9.8	8.3

1 KIP PER FT.

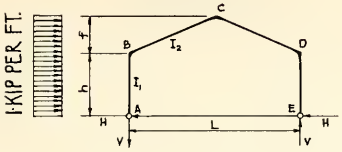


L=75

Dimensions are in feet
Loads are in kips.
Moments are in ft.-kips.

$\frac{I_2}{I_1}=1$						$\frac{I_2}{I_1}=\frac{1}{2}$			
h	f	$-M_B$	$+M_C$	H_A	V_A	$-M_B$	$+M_C$	H_A	V_A
12	12.5	309.0	72.3	25.8	37.5	315.3	59.3	26.3	37.5
12	15.0	291.0	48.1	24.3	37.5	295.8	37.6	24.7	37.5
14	7.5	356.8	155.1	25.5	37.5	369.7	135.3	26.4	37.5
14	10.0	338.2	123.3	24.2	37.5	348.7	105.4	24.9	37.5
14	12.5	320.8	95.9	22.9	37.5	329.3	79.8	23.5	37.5
16	7.5	361.0	173.1	22.6	37.5	376.2	150.6	23.5	37.5
16	10.0	344.4	143.6	21.5	37.5	357.3	122.5	22.3	37.5
16	12.5	329.0	117.1	20.6	37.5	339.9	97.7	21.2	37.5
18	7.5	362.0	190.1	20.1	37.5	379.6	165.4	21.1	37.5
18	10.0	348.3	161.3	19.4	37.5	363.8	137.1	20.2	37.5
20	7.5	363.0	204.1	18.2	37.5	383.0	176.5	19.2	37.5
20	10.0	350.6	177.3	17.6	37.5	368.6	150.3	18.4	37.5
24	7.5	361.0	229.1	15.1	37.5	386.5	195.9	16.1	37.5
24	10.0	352.0	204.6	14.7	37.5	373.6	173.8	15.6	37.5
					L=80				
12	12.5	352.4	80.6	29.4	40	359.2	66.6	29.9	40
12	15.0	331.5	53.5	27.6	40	337.1	41.5	28.1	40
14	12.5	366.1	107.1	26.2	40	375.3	89.7	26.8	40
14	15.0	347.5	80.5	24.8	40	355.1	64.5	25.4	40
16	12.5	376.0	130.0	23.5	40	387.3	110.1	24.2	40
16	15.0	359.1	104.2	22.5	40	368.9	85.3	23.1	40
20	12.5	387.0	171.0	19.4	40	404.2	143.2	20.2	40
22	10.0	402.7	218.3	17.9	40	425.6	185.3	18.9	40
25	10.0	402.8	236.1	16.1	40	428.9	199.5	17.2	40
25	12.5	393.0	210.5	15.7	40	415.9	176.1	16.6	40
					L=85				
12	15.0	375.0	60.1	31.3	42.5	380.9	46.0	31.7	42.5
14	12.5	414.4	118.8	29.6	42.5	424.2	100.2	30.3	42.5
14	15.0	393.0	89.1	28.1	42.5	400.8	71.8	28.6	42.5
16	12.5	425.4	145.6	26.6	42.5	438.0	123.0	27.4	42.5
20	12.5	439.3	189.3	22.0	42.5	457.4	159.9	22.9	42.5
22.5	12.5	444.0	212.1	19.8	42.5	465.4	179.2	20.7	42.5
25	12.5	446.0	234.1	17.9	42.5	471.1	196.4	18.8	42.5

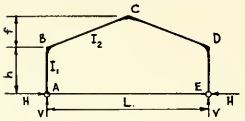
L = 75



Dimensions are in feet
Loads are in kips.
Moments are in ft.-kips.

$\frac{I_2}{I_1} = 1.$								$\frac{I_2}{I_1} = \frac{1}{2}$					
h	f	$+M_B$	$-M_C$	$-M_D$	H_A	H_E	V_A	$+M_B$	$-M_C$	$-M_D$	H_A	H_E	V_A
12.	12.5	132.6	32.5	89.4	17.0	7.5	4.0	131.2	31.2	88.8	17.1	7.4	4.0
12.	15.0	152.7	41.1	99.3	18.7	8.3	4.9	153.4	39.7	98.7	18.8	8.2	4.9
14.	7.5	114.6	20.3	88.4	15.2	6.3	3.1	115.5	18.9	87.5	15.2	6.3	3.1
14.	10.0	137.5	28.2	100.5	16.8	7.2	3.8	138.5	26.6	99.5	16.9	7.1	3.8
14.	12.	160.7	36.9	112.3	18.5	8.0	4.7	161.7	35.1	111.3	18.6	7.9	4.7
16.	7.5	138.2	23.3	109.9	16.6	6.9	3.7	139.5	21.4	108.6	16.7	6.8	3.7
16.	10.0	164.3	32.1	123.7	18.3	7.7	4.5	165.6	29.8	122.4	18.4	7.6	4.5
16.	12.5	190.7	41.6	137.3	19.9	8.6	5.4	192.1	39.1	135.9	20.0	8.5	5.4
18.	7.5	163.6	26.5	133.5	18.1	7.4	4.3	165.3	24.0	131.7	18.2	7.3	4.3
18.	10.0	192.8	36.1	149.2	19.7	8.3	5.2	194.7	33.2	147.3	19.8	8.2	5.2
20.	7.5	190.1	30.0	159.3	19.5	8.0	5.0	193.1	26.7	156.9	19.7	7.8	5.0
20.	10.0	223.0	40.4	177.0	21.2	8.8	6.0	225.5	36.7	174.5	21.3	8.7	6.0
24.	7.5	250.4	37.6	217.6	22.4	9.1	6.6	254.2	32.6	213.9	22.6	8.9	6.6
24.	10.0	288.9	49.7	239.1	24.0	10.0	7.7	292.9	44.1	235.1	24.2	9.8	7.7
							L =	80					
12.	12.5	132.7	32.3	89.3	17.1	7.4	3.8	133.3	31.1	88.8	17.1	7.4	3.8
12.	15.0	152.8	41.0	99.2	18.7	8.3	4.6	153.4	39.6	98.6	18.8	8.2	4.6
14.	12.5	160.9	36.7	112.2	18.5	8.0	4.4	161.8	35.0	111.2	18.6	7.9	4.4
14.	15.0	184.2	46.2	123.8	20.2	8.8	5.3	185.2	44.2	122.9	20.2	8.8	5.3
16.	12.5	190.8	41.3	137.2	19.9	8.6	5.1	192.2	38.9	135.9	20.0	8.5	5.1
16.	15.0	217.4	51.5	150.6	21.6	9.4	6.0	218.8	48.9	149.3	21.7	9.3	6.0
20.	12.5	256.0	51.1	194.0	22.8	9.7	6.6	258.5	47.2	191.5	22.9	9.6	6.6
22.5	10.0	263.8	45.6	214.4	23.0	9.5	6.6	267.0	40.9	211.2	23.1	9.4	6.6
25.	10.0	306.9	51.6	255.6	24.8	10.2	7.6	311.1	45.7	251.4	24.9	10.1	7.6
25.	12.5	347.4	64.8	277.6	26.4	11.1	8.8	351.8	58.3	273.2	26.6	10.9	8.8
							L =	85					
12.	15.0	152.9	40.8	99.2	18.7	8.3	4.3	153.4	39.6	98.6	18.8	8.2	4.3
14.	12.5	161.0	36.5	112.1	18.5	8.0	4.1	161.8	34.9	111.2	18.6	7.9	4.1
14.	15.0	184.3	45.9	123.7	20.2	8.8	4.9	185.2	29.6	122.8	20.2	8.8	4.9
16.	12.5	191.0	41.1	137.1	19.9	8.6	4.8	192.2	38.8	135.8	20.0	8.5	4.8
20.	12.5	256.3	50.7	193.7	22.8	9.7	6.2	258.6	47.0	191.4	22.9	9.6	6.2
22.5	12.5	300.7	57.2	233.7	24.6	10.4	7.2	303.9	52.3	230.5	24.8	10.2	7.2
25.	12.5	347.9	64.2	277.1	26.4	11.1	8.3	352.0	57.9	273.0	26.6	10.9	8.3

1 KIP PER FT.

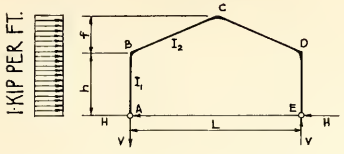


L = 90

Dimensions are in feet.
Loads are in kips.
Moments are in ft.-kips.

$\frac{I_2}{I_1} = 1$						$\frac{I_2}{I_1} = \frac{1}{2}$			
h	f	$-M_B$	$+M_C$	H_A	V_A	$-M_B$	$+M_C$	H_A	V_A
12	15.0	421.1	65.1	35.1	45	427.4	50.9	35.6	45
12	17.5	396.8	37.2	33.1	45	401.9	24.4	33.5	45
14	12.5	465.6	131.2	33.3	45	476.2	111.2	34.0	45
14	17.5	419.5	68.5	30.0	45	426.5	53.0	30.5	45
16	15.0	456.8	127.4	28.6	45	468.1	105.6	29.3	45
18	12.5	487.5	187.0	27.1	45	504.3	157.8	28.0	45
18	15.0	468.5	153.7	26.0	45	482.4	128.1	26.8	45
20	12.5	495.0	208.5	24.8	45	513.9	177.5	25.7	45
22.5	12.5	499.9	234.8	22.2	45	523.0	198.9	23.2	45
25	10.0	516.0	290.1	20.7	45	546.5	247.4	21.9	45
					L = 95				
12	17.5	442.6	40.1	36.9	47.5	448.0	26.7	37.3	47.5
14	12.5	519.0	146.6	37.1	47.5	531.1	122.8	37.9	47.5
14	17.5	467.9	75.4	33.4	47.5	475.6	58.1	33.9	47.5
16	15.0	510.2	139.6	31.9	47.5	522.1	116.5	32.6	47.5
18	12.5	545.0	205.1	30.3	47.5	562.9	174.3	31.3	47.5
20	12.5	553.1	229.4	27.7	47.5	573.6	196.0	28.7	47.5
22.5	15.0	542.1	224.6	24.1	47.5	563.6	188.8	25.1	47.5
25	12.5	563.0	283.6	22.5	47.5	591.7	240.6	23.7	47.5
					L = 100				
12	20.0	463.5	12.7	38.6	50	468.2	1.2	39.0	50
14	17.5	519.2	81.8	37.1	50	527.3	63.6	37.7	50
14	20.0	494.0	50.5	35.3	50	500.2	35.3	35.7	50
16	15.0	566.5	152.4	35.4	50	579.1	127.9	36.2	50
16	17.5	541.5	116.2	33.8	50	552.2	93.8	34.5	50
18	15.0	581.4	184.0	32.3	50	597.2	155.2	33.2	50
18	17.5	558.1	149.4	31.0	50	571.5	122.8	31.8	50
20	15.0	592.6	212.9	29.6	50	611.5	179.9	30.6	50
22.5	15.0	603.5	244.5	26.9	50	625.6	207.3	27.8	50
25	12.5	626.8	309.8	25.1	50	657.2	264.2	26.3	50

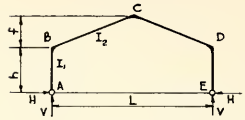
L=90



Dimensions are in feet
Loads are in kips.
Moments are in ft.-kips.

$\frac{I_2}{I_1} = 1.$								$\frac{I_2}{I_1} = \frac{1}{2}$					
h	f	$+M_B$	$-M_C$	$-M_D$	H_A	H_E	V_A	$+M_B$	$-M_C$	$-M_D$	H_A	H_E	V_A
12	15.0	152.9	40.7	99.1	18.7	8.3	4.1	153.4	39.5	98.6	18.8	8.2	4.1
12	17.5	173.1	50.1	108.9	20.5	9.0	4.8	173.7	48.7	108.3	20.5	9.0	4.8
14	12.5	161.0	36.4	112.0	18.5	8.0	3.9	161.9	34.8	111.1	18.6	7.9	3.9
14	17.5	207.9	55.9	135.1	21.9	9.6	5.5	208.7	54.0	134.3	21.9	9.6	5.5
16	15.0	217.7	51.0	150.3	21.6	9.4	5.3	218.9	48.7	149.1	21.7	9.3	5.3
18	12.5	222.9	45.5	164.1	21.4	9.1	5.2	224.6	42.7	162.4	21.5	9.0	5.2
18	15.0	252.7	56.5	179.3	23.0	10.0	6.1	254.4	53.4	177.6	23.1	9.9	6.1
20	12.5	256.5	50.3	193.5	22.8	9.7	5.9	258.7	46.8	191.3	22.9	9.6	5.9
22.5	12.5	301.0	56.8	233.4	24.6	10.4	6.8	304.0	52.1	230.3	24.8	10.2	6.8
25	10.0	307.8	50.4	254.7	24.8	10.2	6.8	311.6	45.0	250.9	25.0	10.0	6.8
							L=	95					
12	17.5	173.2	49.9	108.8	20.4	9.1	4.6	173.7	48.7	108.3	20.5	9.0	4.6
14	12.5	161.1	36.2	111.9	18.5	8.0	3.7	161.9	34.7	111.1	18.6	7.9	3.7
14	17.5	208.0	55.7	135.0	21.9	9.6	5.2	208.8	53.9	134.2	21.9	9.6	5.2
16	15.0	217.8	50.8	150.2	21.6	9.4	5.1	219.0	48.5	149.1	21.7	9.3	5.1
18	12.5	223.1	45.2	163.9	21.4	9.1	4.9	224.7	42.5	162.3	21.5	9.0	4.9
20	12.5	256.7	50.0	193.3	22.8	9.7	5.6	258.8	46.6	191.2	22.9	9.6	5.6
22.5	15.0	338.2	69.1	252.4	26.3	11.2	7.4	341.2	64.2	249.5	26.4	11.1	7.4
25	12.5	348.6	63.0	276.4	26.4	11.1	7.4	352.5	57.2	272.5	26.6	10.9	7.4
							L=	100					
12	20.0	193.5	59.9	118.5	22.1	9.9	5.1	194.0	58.6	118.0	22.2	9.8	5.1
14	17.5	208.1	55.6	134.9	21.9	9.6	5.0	208.8	53.8	134.2	21.9	9.6	5.0
14	20.0	231.7	66.4	146.3	23.5	10.5	5.8	232.4	64.6	145.6	23.6	10.4	5.8
16	15.0	217.9	50.6	150.1	21.6	9.4	4.8	219.0	48.4	149.0	21.7	9.3	4.8
16	17.5	244.6	61.5	163.4	23.3	10.2	5.6	245.8	59.2	162.3	23.4	10.1	5.6
18	15.0	253.0	55.9	179.0	23.1	9.9	5.4	254.6	53.1	177.5	23.1	9.9	5.4
18	17.5	283.0	67.6	194.0	24.7	10.8	6.3	284.5	64.5	192.5	24.8	10.7	6.3
20	15.0	289.9	61.5	210.1	24.5	10.5	6.1	292.0	57.8	208.1	24.6	10.4	6.1
22.5	15.0	338.5	68.7	252.2	26.3	11.2	7.0	341.3	64.0	249.3	26.4	11.1	7.0
25	12.5	349.0	62.5	276.0	26.5	11.0	7.0	352.7	57.0	272.3	26.6	10.9	7.0

1 KIP PER FT.

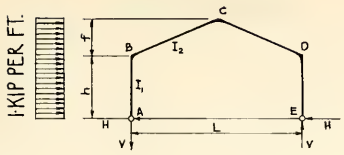


L=105

Dimensions are in feet.
Loads are in kips.
Moments are in ft.-kips.

$\frac{I_2}{I_1} = 1$							$\frac{I_2}{I_1} = \frac{1}{2}$				
h	f	$-M_B$	$+M_C$	$+M_X$	H_A	V_A	$-M_B$	$+M_C$	$+M_X$	H_A	V_A
14	20.0	544.8	54.9	164.8	38.9	52.5	551.9	37.5	150.3	39.4	52.5
14	25.0	494.8	0.0	141.3	35.3	52.5	499.9	-14.5	130.1	35.7	52.5
16	10.0	688.8	258.9	292.5	43.0	52.5	707.7	228.1	263.6	44.2	52.5
16	12.5	656.3	209.3	257.0	41.0	52.5	672.2	181.0	231.0	42.0	52.5
16	15.0	625.7	165.4	227.9	39.1	52.5	639.1	139.5	204.7	39.9	52.5
16	17.5	597.4	127.1	204.6	37.3	52.5	608.7	103.5	183.9	38.0	52.5
18	10.0	698.9	290.6	318.0	38.8	52.5	721.7	255.1	284.3	40.1	52.5
18	12.5	670.0	243.1	282.3	37.2	52.5	689.5	210.0	251.6	38.3	52.5
18	15.0	642.5	200.4	252.3	35.7	52.5	659.2	169.1	224.0	36.6	52.5
18	17.5	616.6	162.3	227.4	34.3	52.5	630.9	134.0	202.7	35.0	52.5
22.5	10.0	711.3	351.0	369.1	31.6	52.5	742.9	305.2	325.0	33.0	52.5
22.5	12.5	688.6	306.7	333.3	30.6	52.5	716.2	263.8	292.5	31.8	52.5
22.5	15.0	666.9	266.4	302.3	29.6	52.5	691.2	225.9	264.5	30.7	52.5
						L= 110					
14	15.0	663.9	137.5	221.1	47.4	55	674.8	114.9	201.2	48.2	55
14	17.5	629.8	95.5	197.9	45.0	55	638.8	75.2	180.6	45.6	55
14	20.0	598.5	58.7	179.6	42.8	55	606.1	40.4	164.9	43.3	55
16	12.5	721.8	227.1	279.6	45.1	55	738.6	197.1	252.1	46.2	55
16	15.0	687.9	179.3	248.1	43.0	55	702.0	152.0	223.5	43.9	55
16	17.5	656.7	137.4	222.7	41.0	55	668.6	112.5	200.9	41.8	55
16	20.0	627.6	100.5	202.2	39.2	55	637.7	77.6	182.6	39.9	55
18	12.5	737.2	263.6	306.9	41.0	55	757.7	229.0	274.7	42.1	55
18	15.0	706.7	217.1	274.4	39.3	55	724.3	184.9	245.1	40.2	55
18	17.5	677.9	175.7	247.4	37.7	55	693.0	145.9	220.9	38.4	55
18	20.0	650.9	138.5	225.0	36.2	55	663.8	111.1	201.1	36.9	55
20	12.5	748.3	296.5	332.6	37.4	55	772.6	257.0	295.5	38.6	55
20	15.0	720.8	251.1	299.4	36.0	55	741.9	214.3	265.5	37.1	55
20	17.5	694.5	210.4	271.4	34.7	55	712.8	176.1	240.4	35.6	55
22.5	12.5	758.2	332.7	362.1	33.7	55	787.4	287.4	319.1	35.0	55
22.5	15.0	734.0	288.9	328.5	32.6	55	759.7	245.9	288.3	33.8	55
22.5	17.5	710.4	249.5	300.0	31.6	55	732.9	209.3	263.1	32.6	55
25	20.0	701.6	249.6	301.6	28.1	55	725.7	206.3	262.0	29.0	55

L=105

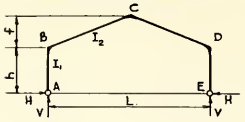


Dimensions are in feet
Loads are in kips
Moments are in ft.-kips

$\frac{I_2}{I_1} = 1.$							
h	f	$+M_B$	$-M_C$	$-M_D$	H_A	H_E	V_A
14.	20.0	231.7	66.3	146.3	23.6	10.4	5.5
14.	25.0	279.2	90.1	168.9	26.9	12.1	7.2
16.	10.0	165.0	30.9	123.0	18.3	7.7	3.2
16.	12.5	191.4	40.3	136.6	20.0	8.5	3.9
16.	15.0	218.0	50.4	150.0	21.6	9.4	4.6
16.	17.5	244.7	61.3	163.3	23.3	10.2	5.3
18.	10.0	193.8	34.5	148.2	19.8	8.2	3.7
18.	12.5	223.4	44.7	163.7	21.4	9.1	4.4
18.	15.0	253.1	55.7	178.9	23.1	9.9	5.2
18.	17.5	283.1	67.4	193.9	24.7	10.8	6.0
22.5	10.0	265.2	43.5	212.9	23.0	9.5	5.0
22.5	12.5	301.8	55.5	232.6	24.7	10.3	5.8
22.5	15.0	338.7	68.3	251.9	26.3	11.2	6.7
						L=	110
14.	15.0	184.7	45.2	123.3	20.2	8.8	3.8
14.	17.5	208.2	55.3	134.8	21.9	9.6	4.5
14.	20.0	231.7	66.3	146.3	23.6	10.4	5.3
16.	12.5	191.5	40.1	136.5	20.0	8.5	3.7
16.	15.0	218.1	50.2	149.9	21.6	9.4	4.4
16.	17.5	244.8	61.1	163.2	23.3	10.2	5.1
16.	20.0	271.7	72.7	176.3	25.0	11.0	5.9
18.	12.5	223.5	44.5	163.5	21.4	9.1	4.2
18.	15.0	253.3	55.5	178.8	23.1	9.9	5.0
18.	17.5	283.2	67.1	193.8	24.7	10.8	5.7
18.	20.0	313.3	79.5	208.7	26.4	11.6	6.6
20.	12.5	257.3	49.1	192.7	22.9	9.6	4.8
20.	15.0	290.2	60.9	209.8	24.5	10.5	5.6
20.	17.5	323.4	73.4	226.6	26.2	11.3	6.4
22.5	12.5	302.0	55.2	232.4	24.7	10.3	5.6
22.5	15.0	338.9	67.9	251.7	26.3	11.2	6.4
22.5	17.5	376.1	81.4	270.8	28.0	12.0	7.3
25.	20.0	472.9	105.1	339.6	31.4	13.6	9.2

$\frac{I_2}{I_1} = \frac{1}{2}$					
$+M_B$	$-M_C$	$-M_D$	H_A	H_E	V_A
232.4	64.5	145.6	23.6	10.4	5.5
279.9	88.2	168.2	27.0	12.0	7.2
166.0	29.2	122.0	18.4	7.6	3.2
192.5	38.4	135.6	20.0	8.5	3.9
219.1	48.3	148.9	21.7	9.3	4.6
245.8	59.0	162.2	23.4	10.1	5.3
195.2	32.3	146.8	19.8	8.2	3.7
224.8	42.3	162.2	21.5	9.0	4.4
254.6	53.0	177.4	23.1	9.9	5.2
284.6	64.4	192.4	24.8	10.7	6.0
267.8	39.8	210.4	23.2	9.4	5.0
304.5	51.4	229.9	24.8	10.2	5.8
341.5	63.7	249.2	26.4	11.1	6.7
185.4	43.7	122.6	20.2	8.8	3.8
208.9	53.7	134.1	21.9	9.6	4.5
232.4	64.6	145.6	23.6	10.4	5.3
192.5	38.3	135.5	20.0	8.5	3.7
219.1	48.2	148.9	21.7	9.3	4.4
245.9	58.9	162.2	23.4	10.1	5.1
272.7	70.4	175.3	25.0	11.0	5.9
224.9	42.2	162.1	21.5	9.0	4.2
254.7	52.8	177.3	23.1	9.9	5.0
284.7	64.3	192.3	24.8	10.7	5.7
314.8	76.5	207.2	26.5	11.5	6.6
259.1	46.1	190.9	23.0	9.5	4.8
292.1	57.5	207.9	24.6	10.4	5.6
325.3	69.7	224.7	26.3	11.2	6.4
304.6	51.2	229.8	24.8	10.2	5.6
341.6	63.5	249.1	26.4	11.1	6.4
378.8	76.7	268.1	28.1	11.9	7.3
476.5	98.6	336.0	31.6	13.4	9.2

1·KIP PER FT.

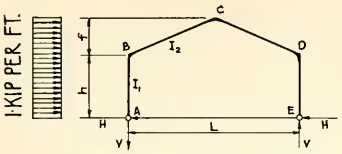


L=115

Dimensions are in feet
Loads are in kips.
Moments are in ft.-kips.

$\frac{I_2}{I_1}=1.$							$\frac{I_2}{I_1}=\frac{1}{2}$				
h	f	$-M_B$	$+M_C$	$+M_X$	H_A	V_A	$-M_B$	$+M_C$	$+M_X$	H_A	V_A
14.	15.0	726.6	148.3	239.9	51.9	57.5	738.1	124.6	219.1	52.7	57.5
14.	17.5	689.1	102.6	214.8	49.2	57.5	698.6	81.3	196.6	49.9	57.5
14.	20.0	654.8	62.6	195.0	46.8	57.5	662.8	43.3	178.9	47.3	57.5
16.	12.5	790.4	245.5	303.1	49.4	57.5	808.8	214.0	274.3	50.5	57.5
16.	15.0	753.2	193.4	268.9	47.1	57.5	768.0	164.8	243.3	48.0	57.5
16.	17.5	718.8	148.0	241.5	44.9	57.5	731.3	121.7	218.5	45.7	57.5
18.	12.5	807.6	285.1	332.6	44.9	57.5	829.2	248.5	298.6	46.1	57.5
18.	15.0	773.9	234.5	297.3	43.0	57.5	792.4	200.7	266.5	44.0	57.5
18.	17.5	742.3	189.3	268.1	41.2	57.5	758.1	158.2	240.3	42.1	57.5
20.	15.0	789.6	271.3	324.4	39.5	57.5	811.8	232.5	288.5	40.6	57.5
20.	20.0	733.1	187.0	268.3	36.7	57.5	749.8	153.4	238.5	37.5	57.5
22.5	15.0	804.5	312.0	355.6	35.8	57.5	831.5	267.1	313.6	37.0	57.5
22.5	17.5	778.4	269.2	324.6	34.6	57.5	802.1	226.9	285.8	35.7	57.5
25.	15.0	814.6	349.8	385.9	32.6	57.5	846.4	298.9	337.9	33.9	57.5
25.	17.5	791.3	307.9	354.3	31.7	57.5	819.7	259.6	309.4	32.8	57.5
25.	20.0	768.8	269.3	326.5	30.8	57.5	794.0	223.9	284.9	31.8	57.5
						L=120					
14.	15.0	792.2	159.3	259.3	56.6	60	804.2	134.5	237.6	57.4	60
14.	17.5	751.1	109.9	232.3	53.7	60	761.1	87.5	213.2	54.4	60
14.	20.0	713.6	66.6	211.0	51.0	60	722.0	46.2	194.0	51.6	60
16.	12.5	862.1	264.6	327.5	53.9	60	880.5	231.7	297.4	55.0	60
16.	15.0	821.3	208.3	290.8	51.3	60	836.8	178.2	263.8	52.3	60
16.	17.5	783.6	159.0	261.1	49.0	60	796.8	131.4	237.0	49.8	60
18.	12.5	881.1	307.4	359.3	49.0	60	903.8	269.0	323.6	50.2	60
18.	15.0	844.3	252.5	321.1	46.9	60	863.6	217.1	288.9	48.0	60
18.	17.5	809.5	203.7	289.7	45.0	60	826.1	170.1	260.3	45.9	60
20.	15.0	861.7	292.0	350.0	43.1	60	884.9	251.4	312.6	44.3	60
20.	20.0	799.6	200.8	289.6	40.0	60	817.3	165.5	258.2	40.9	60
22.5	15.0	878.2	336.0	383.7	39.0	60	906.6	288.6	339.4	40.3	60
22.5	17.5	849.4	289.7	350.4	37.8	60	874.4	245.3	309.6	38.9	60
25.	12.5	916.1	425.9	455.1	36.6	60	953.8	369.3	400.9	38.2	60
25.	20.0	839.2	289.5	352.1	33.6	60	865.7	241.8	308.4	34.6	60

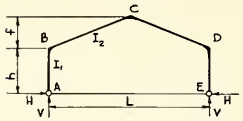
L=115



Dimensions are in feet
Loads are in kips.
Moments are in ft.-kips.

$\frac{I_2}{I_1}=1.$								$\frac{I_2}{I_1}=\frac{1}{2}$					
h	f	$+M_B$	$-M_C$	$-M_D$	H_A	H_E	V_A	$+M_B$	$-M_C$	$-M_D$	H_A	H_E	V_A
14.	15.0	184.7	45.1	123.3	20.2	8.8	3.7	185.4	43.7	122.6	20.2	8.8	3.7
14	17.5	208.2	55.1	134.8	21.9	9.6	4.3	208.9	53.6	134.1	21.9	9.6	4.3
14	20.0	231.8	66.0	146.2	23.6	10.4	5.0	232.5	64.4	145.5	23.6	10.4	5.0
16	12.5	191.6	40.0	136.4	20.0	8.5	3.5	192.5	38.2	135.5	20.0	8.5	3.5
16	15.0	218.2	50.1	149.9	21.6	9.4	4.2	219.1	48.2	148.9	21.7	9.3	4.2
16	17.5	244.9	61.0	163.1	23.3	10.2	4.9	245.9	58.9	162.1	23.4	10.1	4.9
18	12.5	223.6	44.3	163.4	21.4	9.1	4.0	224.9	42.0	162.1	21.5	9.0	4.0
18	15.0	253.4	55.2	178.6	23.1	9.9	4.7	254.7	52.7	177.3	23.2	9.8	4.7
18	17.5	283.3	66.9	193.7	24.7	10.8	5.5	284.7	64.2	192.3	24.8	10.7	5.5
20	15.0	290.4	60.6	209.6	24.5	10.5	5.3	292.2	57.4	207.8	24.6	10.4	5.3
20	20.0	356.9	86.3	243.1	27.8	12.2	7.0	358.8	82.5	241.3	27.9	12.1	7.0
22.5	15.0	339.1	67.6	251.5	26.3	11.2	6.1	341.7	63.4	249.0	26.4	11.1	6.1
22.5	17.5	376.3	81.1	270.6	28.0	12.0	7.0	378.9	76.5	268.0	28.1	11.9	7.0
25	15.0	390.7	74.9	296.8	28.1	11.9	7.0	394.1	69.5	293.4	28.3	11.7	7.0
25	17.5	431.8	89.4	318.2	29.8	12.7	7.9	435.2	83.5	314.8	29.9	12.6	7.9
25	20.0	473.1	104.6	339.4	31.4	13.6	8.8	476.6	98.3	335.9	31.6	13.4	8.8
							L=	120					
14	15.0	184.8	45.0	123.2	20.2	8.8	3.5	185.4	43.6	122.6	20.2	8.8	3.5
14	17.5	208.3	55.0	134.7	21.9	9.6	4.1	208.9	53.6	134.1	21.9	9.6	4.1
14	20.0	231.9	65.9	146.1	23.6	10.4	4.8	232.5	64.3	145.5	23.6	10.4	4.8
16	12.5	191.7	39.8	136.4	20.0	8.5	3.4	192.6	38.1	135.4	20.0	8.5	3.4
16	15.0	218.2	49.9	149.8	21.6	9.4	4.0	219.2	48.1	148.8	21.7	9.3	4.0
16	17.5	245.0	60.8	163.0	23.3	10.2	4.7	245.9	58.8	162.1	23.4	10.1	4.7
18	12.5	223.7	44.2	163.3	21.4	9.1	3.9	225.0	42.0	162.0	21.5	9.0	3.9
18	15.0	253.5	55.1	178.5	23.1	9.9	4.5	254.8	52.6	177.2	23.2	9.8	4.5
18	17.5	283.1	66.7	193.6	24.7	10.8	5.3	284.8	64.1	192.2	24.8	10.7	5.3
20	15.0	290.5	60.4	209.5	24.5	10.5	5.1	292.3	57.3	207.7	24.6	10.4	5.1
20	20.0	357.0	86.0	243.0	27.9	12.2	6.7	358.8	82.4	241.2	27.9	12.1	6.7
22.5	15.0	339.3	67.3	251.3	26.1	11.4	5.9	341.8	63.2	248.9	26.4	11.1	5.9
22.5	17.5	376.5	80.7	270.4	27.7	12.3	6.7	379.0	76.3	267.9	28.1	11.9	6.7
25.	12.5	350.1	60.8	274.9	26.5	11.0	5.9	353.3	56.0	271.7	26.6	10.9	5.9
25	20.0	473.4	104.2	339.1	31.4	13.6	8.4	476.8	98.1	335.8	31.6	13.4	8.4

1 KIP PER FT

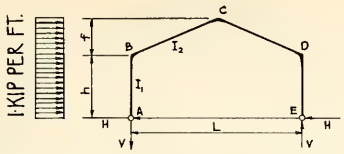


L = 125

Dimensions are in feet.
Loads are in kips.
Moments are in ft-kips.

$\frac{I_2}{I_1} = 1$							$\frac{I_2}{I_1} = \frac{1}{2}$				
h	f	$-M_B$	$+M_C$	$+M_X$	H_A	V_A	$-M_B$	$+M_C$	$+M_X$	H_A	V_A
16	12.5	937.2	284.1	352.6	58.6	62.5	956.2	250.1	321.5	59.8	62.5
16	17.5	851.3	170.5	281.5	53.2	62.5	865.1	141.7	256.3	54.1	62.5
18	12.5	958.0	330.2	386.8	53.2	62.5	981.7	290.1	349.6	54.5	62.5
18	17.5	879.7	218.4	311.9	48.9	62.5	897.0	184.2	281.5	49.8	62.5
20	12.5	973.5	371.2	418.6	48.7	62.5	1001.7	325.4	375.6	50.1	62.5
20	15.0	936.9	313.6	376.8	46.8	62.5	961.2	270.6	337.2	48.1	62.5
20	17.5	902.0	261.9	341.6	45.1	62.5	923.2	222.1	305.6	46.2	62.5
22.5	12.5	987.6	416.5	455.1	43.9	62.5	1021.3	364.0	405.3	45.4	62.5
22.5	17.5	923.6	311.1	377.1	41.1	62.5	949.7	264.5	334.4	42.2	62.5
25	12.5	997.1	457.5	489.3	39.9	62.5	1036.6	398.2	432.6	41.5	62.5
25	15.0	967.9	404.4	447.6	38.7	62.5	1003.0	348.3	394.7	40.1	62.5
25	17.5	939.8	355.5	410.9	37.6	62.5	971.0	302.4	361.6	38.8	62.5
27.5	12.5	1003.0	493.7	520.4	36.5	62.5	1048.1	428.1	457.3	38.1	62.5
27.5	17.5	951.5	396.4	443.3	34.6	62.5	988.1	336.7	387.2	35.9	62.5
30	12.5	1005.8	527.9	550.4	33.5	62.5	1056.5	456.1	480.9	35.2	62.5
30	17.5	960.1	433.3	473.4	32.0	62.5	1001.8	367.3	410.9	33.4	62.5
35	12.5	1006.3	587.6	604.1	28.8	62.5	1067.9	504.0	522.6	30.5	62.5
35	17.5	969.4	499.1	529.2	27.7	62.5	1021.3	421.2	454.5	29.2	62.5
						L = 130					
16	12.5	1014.9	305.0	379.3	63.4	65	1035.1	269.0	346.3	64.7	65
16	20.0	880.4	131.6	274.9	55.0	65	892.7	103.8	251.2	55.8	65
18	12.5	1038.0	354.2	415.6	57.7	65	1062.7	312.2	376.6	59.0	65
18	17.5	952.8	239.5	335.2	52.9	65	971.1	197.6	303.0	54.0	65
20	12.5	1055.2	397.8	449.3	52.8	65	1084.6	350.0	404.4	54.2	65
20	17.5	977.2	280.2	366.8	48.8	65	999.5	238.5	329.0	50.0	65
22.5	12.5	1070.8	446.4	488.4	47.6	65	1106.1	391.4	436.2	49.2	65
22.5	17.5	1001.0	332.7	404.5	44.5	65	1028.4	284.0	359.8	45.7	65
25	12.5	1081.4	490.4	525.0	43.3	65	1122.8	428.4	465.7	44.9	65
25	15.0	1049.6	433.2	479.2	42.0	65	1086.5	374.2	424.5	43.5	65
27.5	12.5	1088.1	529.3	558.3	39.6	65	1135.5	460.4	492.0	41.3	65
27.5	17.5	1031.9	424.3	475.3	37.5	65	1070.2	361.6	416.4	38.9	65
30	12.5	1091.9	565.3	589.9	36.4	65	1145.2	489.8	516.8	38.2	65
30	17.5	1041.5	463.9	507.5	34.7	65	1085.2	394.7	442.0	36.2	65
35	12.5	1093.0	629.3	647.3	31.2	65	1157.9	541.2	561.5	33.1	65
35	17.5	1052.2	534.1	566.9	30.1	65	1106.9	452.2	488.5	31.6	65

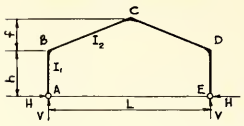
L=125



Dimensions are in feet
Loads are in kips.
Moments are in ft.-kips.

$\frac{I_2}{I_1} = 1.$								$\frac{I_2}{I_1} = \frac{1}{2}$					
h	f	$+M_B$	$-M_C$	$-M_D$	H_A	H_E	V_A	$+M_B$	$-M_C$	$-M_D$	H_A	H_E	V_A
16	12.5	191.7	39.7	136.3	20.0	8.5	3.2	192.6	38.1	135.4	20.0	8.5	3.2
16	17.5	245.0	60.6	163.0	23.3	10.2	4.5	246.0	58.7	162.0	23.4	10.1	4.5
18	12.5	223.8	44.0	163.2	21.4	9.1	3.7	225.0	41.9	162.0	21.5	9.0	3.7
18	17.5	286.8	60.0	190.2	24.9	10.6	5.0	288.0	57.6	189.0	25.0	10.5	5.0
20	12.5	257.7	48.5	192.3	22.9	9.6	4.2	259.4	45.7	190.7	23.0	9.5	4.2
20	15.0	290.6	60.1	209.4	24.5	10.5	4.9	292.4	57.1	207.7	24.6	10.4	4.9
20	17.5	323.8	72.6	226.2	26.2	11.3	5.6	325.5	69.3	224.5	26.3	11.2	5.6
22.5	12.5	302.6	54.3	231.8	24.7	10.3	4.9	304.9	50.7	229.5	24.8	10.2	4.9
22.5	17.5	376.7	80.4	270.2	28.0	12.0	6.4	379.1	76.1	267.8	28.1	11.9	6.4
25	12.5	350.4	60.4	274.7	26.5	11.0	5.6	353.4	55.8	271.6	26.6	10.9	5.6
25	15.0	391.2	74.1	296.3	28.1	11.9	6.4	394.3	69.1	293.2	28.3	11.7	6.4
25	17.5	432.3	88.5	317.7	29.8	12.7	7.2	435.5	83.1	314.5	29.9	12.6	7.2
27.5	12.5	400.9	66.9	321.0	28.3	11.7	6.4	404.9	61.1	317.0	28.5	11.5	6.4
27.5	17.5	490.7	97.0	368.7	31.6	13.4	8.1	494.9	90.3	364.5	31.7	13.3	8.1
30	12.5	454.3	73.6	370.7	30.1	12.4	7.2	459.2	66.6	365.8	30.3	12.2	7.2
30	17.5	551.9	105.9	423.1	33.4	14.1	9.0	557.1	97.6	417.9	33.6	13.9	9.0
35	12.5	569.4	88.2	480.6	33.8	13.7	9.0	576.8	78.2	473.2	34.0	13.5	9.0
35	17.5	682.5	124.7	542.5	37.0	15.5	11.0	690.4	112.9	534.6	37.2	15.3	11.0
							L=	130					
16	12.5	191.8	39.6	136.2	20.0	8.5	3.1	192.7	38.0	135.4	20.0	8.5	3.1
16	20.0	272.0	72.1	176.0	25.0	11.0	5.0	272.9	70.1	175.1	25.1	10.9	5.0
18	12.5	223.9	43.8	163.1	21.4	9.1	3.6	225.1	41.8	162.0	21.5	9.0	3.6
18	17.5	283.6	66.3	193.4	24.8	10.7	4.8	284.9	63.9	192.1	24.8	10.7	4.8
20	12.5	257.8	48.3	192.2	22.9	9.6	4.1	259.4	45.6	190.6	23.0	9.5	4.1
20	17.5	323.9	72.3	226.1	26.2	11.3	5.4	325.6	69.2	224.4	26.3	11.2	5.4
22.5	12.5	302.8	54.0	231.6	24.7	10.3	4.7	305.0	50.6	229.4	24.8	10.2	4.7
22.5	17.5	376.8	80.1	270.1	28.0	12.0	6.2	379.2	76.0	267.7	28.1	11.9	6.2
25	12.5	350.6	60.1	274.4	26.5	11.0	5.4	353.5	55.7	271.5	26.6	10.9	5.4
25	15.0	391.4	73.7	296.1	28.2	11.8	6.2	394.5	68.9	293.1	28.3	11.7	6.2
27.5	12.5	401.2	66.4	320.7	28.3	11.7	6.2	405.0	60.9	316.9	28.5	11.5	6.2
27.5	17.5	491.0	96.6	368.4	31.6	13.4	7.8	495.0	90.0	364.4	31.8	13.3	7.8
30	12.5	454.6	73.1	370.4	30.2	12.3	6.9	459.4	66.4	365.6	30.3	12.2	6.9
30	17.5	552.3	105.3	422.8	33.4	14.1	8.7	557.3	97.3	417.7	33.6	13.9	8.7
35	12.5	569.9	87.5	480.1	33.8	13.7	8.7	577.1	77.8	472.9	34.0	13.5	8.7
35	17.5	683.0	123.9	542.0	37.0	15.5	10.6	690.7	112.5	534.3	37.2	15.3	10.6

1 KIP PER FT.

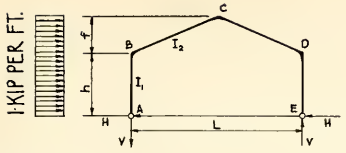


L = 135

Dimensions are in feet.
Loads are in kips.
Moments are in ft.-kips.

$\frac{I_2}{I_1} = 1$							$\frac{I_2}{I_1} = \frac{1}{2}$				
h	f	$-M_B$	$+M_C$	$+M_X$	H_A	V_A	$-M_B$	$+M_C$	$+M_X$	H_A	V_A
16	12.5	1096.1	326.0	406.5	68.5	67.5	1117.0	288.7	372.3	69.8	67.5
16	20.0	950.3	139.9	294.7	59.4	67.5	963.1	111.2	270.2	60.2	67.5
20	12.5	1140.2	425.3	481.0	57.0	67.5	1170.7	375.7	434.4	58.5	67.5
20	17.5	1055.4	299.3	392.8	52.8	67.5	1078.5	256.0	353.7	53.9	67.5
22.5	12.5	1157.3	477.3	522.8	51.4	67.5	1194.2	419.9	468.3	53.1	67.5
22.5	17.5	1081.4	355.4	433.1	48.1	67.5	1109.9	304.7	386.5	49.3	67.5
25	12.5	1169.4	524.1	561.6	46.8	67.5	1212.5	459.3	499.7	48.5	67.5
25	20.0	1068.7	354.5	434.7	42.7	67.5	1099.1	299.7	384.6	44.0	67.5
27.5	12.5	1177.0	565.6	597.1	42.8	67.5	1226.3	493.8	528.0	44.6	67.5
27.5	17.5	1115.7	452.8	508.1	40.6	67.5	1155.6	387.6	446.9	42.0	67.5
30	12.5	1181.4	604.1	630.8	39.4	67.5	1236.8	525.9	555.0	41.2	67.5
30	17.5	1126.4	495.1	542.4	37.5	67.5	1172.0	422.8	474.1	39.1	67.5
32.5	12.5	1183.5	638.9	661.7	36.4	67.5	1245.1	553.7	578.9	38.3	67.5
32.5	17.5	1134.0	534.1	574.9	34.9	67.5	1185.4	455.0	499.5	36.5	67.5
35	12.5	1183.5	672.1	691.7	33.8	67.5	1251.2	580.2	602.1	35.8	67.5
35	17.5	1138.6	570.2	605.7	32.5	67.5	1195.8	484.4	523.7	34.2	67.5
						L =	140				
18	12.5	1207.7	404.1	475.8	67.1	70	1234.5	358.8	433.7	68.6	70
18	17.5	1107.8	265.3	383.7	61.5	70	1127.5	226.6	349.1	62.6	70
20	12.5	1228.4	453.8	514.0	61.4	70	1260.2	402.1	465.4	63.0	70
20	15.0	1181.4	382.6	462.7	59.1	70	1209.0	334.3	418.2	60.5	70
20	17.5	1136.7	318.6	419.5	56.8	70	1160.8	273.6	378.9	58.0	70
22.5	12.5	1247.5	509.0	558.1	55.4	70	1285.8	449.3	501.4	57.2	70
22.5	17.5	1165.0	378.6	462.4	51.8	70	1194.9	325.6	413.7	53.1	70
25	12.5	1260.5	559.2	599.7	50.4	70	1305.6	491.7	535.1	52.2	70
25	17.5	1186.6	432.8	503.2	47.5	70	1222.2	372.2	446.9	48.9	70
27.5	12.5	1269.4	603.1	637.1	46.2	70	1320.8	529.3	566.0	48.0	70
27.5	15.0	1235.7	540.8	587.1	44.9	70	1282.0	469.3	519.1	46.6	70
27.5	17.5	1202.6	482.5	542.2	43.7	70	1244.1	414.6	478.5	45.2	70
30	12.5	1274.4	644.2	673.0	42.5	70	1332.4	562.0	593.5	44.4	70
30	17.5	1214.4	527.6	578.7	40.5	70	1262.1	452.1	507.3	42.1	70
32.5	15.0	1249.5	623.3	657.3	38.4	70	1308.2	537.4	574.6	40.3	70
32.5	17.5	1223.2	568.8	613.0	37.6	70	1276.8	486.2	534.4	39.3	70
35	12.5	1277.6	716.3	737.5	36.5	70	1348.3	620.3	644.0	38.5	70
35	15.0	1252.8	659.8	689.2	35.8	70	1317.6	567.2	599.8	37.7	70
35	17.5	1228.5	607.2	645.7	35.1	70	1288.1	517.9	560.3	36.8	70

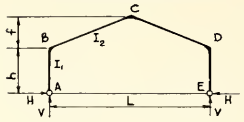
L=135



Dimensions are in feet
Loads are in kips
Moments are in ft.-kips.

$\frac{I_2}{I_1} = 1.$								$\frac{I_2}{I_1} = \frac{1}{2}$					
h	f	$+M_B$	$-M_C$	$-M_D$	H_A	H_E	V_A	$+M_B$	$-M_C$	$-M_D$	H_A	H_E	V_A
16	12.5	191.8	41.5	136.1	20.0	8.5	3.0	192.7	38.0	135.3	20.0	8.5	3.0
16	20.0	272.0	72.0	176.0	25.0	11.0	4.8	272.9	70.0	175.1	25.1	10.9	4.8
20	12.5	257.9	48.1	192.1	22.9	9.6	3.9	259.5	45.5	190.5	23.0	9.5	3.9
20	17.5	324.0	72.1	226.0	26.2	11.3	5.2	325.7	69.1	224.3	26.3	11.2	5.2
22.5	12.5	302.9	53.8	231.5	24.7	10.3	4.5	305.1	50.5	229.3	24.8	10.2	4.5
22.5	17.5	377.0	79.8	269.9	28.0	12.0	5.9	379.2	75.8	267.7	28.1	11.9	5.9
25	12.5	350.8	59.8	274.2	26.5	11.0	5.2	353.6	55.5	271.4	26.6	10.9	5.2
25	20.0	474.1	103.0	338.5	31.5	13.5	7.5	477.1	97.5	335.4	31.6	13.4	7.5
27.5	12.5	401.5	66.1	320.4	28.3	11.7	5.9	405.1	60.7	316.7	28.5	11.5	5.9
27.5	17.5	491.3	96.1	368.1	31.6	13.4	7.5	495.2	89.7	364.2	31.8	13.2	7.5
30	12.5	455.0	72.7	370.0	30.2	12.3	6.7	459.6	66.1	365.4	30.3	12.2	6.7
30	17.5	552.6	104.8	422.4	33.4	14.1	8.4	557.5	97.0	417.5	33.6	13.9	8.4
32.5	12.5	511.3	79.6	423.1	32.0	13.0	7.5	517.0	71.6	417.4	32.2	12.8	7.5
32.5	17.5	616.7	113.8	480.2	35.2	14.8	9.3	622.8	104.4	474.1	35.4	14.6	9.3
35	12.5	570.4	86.9	479.6	33.8	13.7	8.4	577.4	77.4	472.7	34.0	13.5	8.4
35	17.5	683.5	123.1	541.5	37.0	15.5	10.2	690.9	112.0	534.1	37.2	15.3	10.2
								L=140					
18	12.5	224.0	43.5	163.0	21.4	9.1	3.3	225.2	41.6	161.8	21.5	9.0	3.3
18	17.5	283.8	66.0	193.2	24.8	10.7	4.5	285.0	63.7	192.1	24.8	10.7	4.5
20	12.5	258.0	47.9	192.0	22.9	9.6	3.8	259.5	45.5	190.5	23.0	9.5	3.8
20	15.0	291.0	59.5	209.0	24.5	10.5	4.4	292.5	56.8	207.5	24.6	10.4	4.4
20	17.5	324.1	71.9	225.9	26.2	11.3	5.0	325.7	69.0	224.3	26.3	11.2	5.0
22.5	12.5	303.1	53.6	231.3	24.7	10.3	4.4	305.1	50.3	229.2	24.8	10.2	4.4
22.5	17.5	377.1	79.6	269.8	28.0	12.0	5.7	379.3	75.7	267.6	28.1	11.9	5.7
25	12.5	351.0	59.5	274.0	26.5	11.0	5.0	353.7	55.3	271.3	26.6	10.9	5.0
25	17.5	432.9	87.5	317.1	29.8	12.7	6.5	435.8	82.5	314.2	29.9	12.6	6.5
27.5	12.5	401.7	65.7	320.2	28.4	11.6	5.7	405.3	60.5	316.6	28.5	11.5	5.7
27.5	15.0	446.5	80.3	344.2	30.0	12.5	6.5	450.2	74.6	340.5	30.1	12.4	6.5
27.5	17.5	491.5	95.7	367.9	31.6	13.4	7.2	495.3	89.5	364.1	31.8	13.2	7.2
30	12.5	455.3	72.2	369.7	30.2	12.3	6.5	459.8	65.8	365.2	30.3	12.2	6.5
30	17.5	552.9	104.3	422.1	33.4	14.1	8.1	557.7	96.7	417.3	33.6	13.9	8.1
32.5	15.0	564.2	95.7	451.4	33.6	13.9	8.1	569.9	87.3	445.7	33.8	13.7	8.1
32.5	17.5	617.1	113.2	479.8	35.2	14.8	8.9	623.0	104.1	473.9	35.4	14.6	8.9
35	12.5	570.8	86.2	479.2	33.8	13.7	8.1	577.6	77.0	472.4	34.0	13.5	8.1
35	15.0	627.2	104.0	510.3	35.4	14.6	8.9	634.3	93.9	503.3	35.6	14.4	8.9
35	17.5	684.0	122.4	541.0	37.0	15.5	9.8	691.2	111.6	533.8	37.2	15.3	9.8

1 KIP PER FT

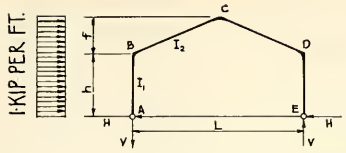


L=145

Dimensions are in feet
Loads are in kips.
Moments are in ft-kips.

$\frac{I_2}{I_1}=1$							$\frac{I_2}{I_1}=\frac{1}{2}$				
h	f	$-M_B$	$+M_C$	$+M_X$	H_A	V_A	$-M_B$	$+M_C$	$+M_X$	H_A	V_A
18	12.5	1297.3	430.5	507.6	72.1	72.5	1325.4	382.9	463.4	73.6	72.5
18	15.0	1241.9	351.7	453.5	69.0	72.5	1265.7	307.1	413.1	70.3	72.5
20	12.5	1319.9	483.3	548.1	66.0	72.5	1353.1	429.3	497.4	67.7	72.5
20	17.5	1221.0	338.7	447.3	61.1	72.5	1246.3	291.3	404.5	62.3	72.5
22.5	12.5	1340.7	542.0	594.9	59.6	72.5	1380.7	479.8	535.9	61.4	72.5
22.5	17.5	1251.7	402.5	492.7	55.6	72.5	1282.8	347.3	442.1	57.0	72.5
25	12.5	1355.4	595.1	638.8	54.2	72.5	1402.2	524.8	571.6	56.1	72.5
25	17.5	1275.3	460.2	536.0	51.0	72.5	1312.2	397.3	477.6	52.5	72.5
27.5	12.5	1365.2	641.7	678.4	49.6	72.5	1418.9	563.7	603.3	51.6	72.5
27.5	17.5	1292.8	519.1	577.4	47.0	72.5	1336.1	442.3	511.0	48.6	72.5
27.5	20.0	1257.9	454.5	534.3	45.7	72.5	1296.9	387.1	471.9	47.1	72.5
30	12.5	1371.0	685.4	716.5	45.7	72.5	1431.5	599.8	633.7	47.7	72.5
30	15.0	1338.3	620.7	663.3	44.6	72.5	1392.9	538.8	585.0	46.4	72.5
30	17.5	1306.0	560.7	615.8	43.5	72.5	1355.7	482.1	541.5	45.2	72.5
32.5	17.5	1315.5	604.8	652.5	40.5	72.5	1371.5	518.7	570.5	42.2	72.5
35	12.5	1375.2	762.0	784.9	39.3	72.5	1449.0	661.9	687.3	41.4	72.5
35	17.5	1321.9	645.3	686.9	37.8	72.5	1383.9	553.2	598.7	39.5	72.5
35	20.0	1295.8	592.4	644.5	37.0	72.5	1352.8	502.8	559.6	38.7	72.5
						L= 150					
18	12.5	1390.3	457.4	540.2	77.2	75	1419.1	408.5	494.7	78.8	75
18	15.0	1330.7	373.4	482.7	73.9	75	1355.3	328.2	441.5	75.3	75
18	20.0	1222.1	232.7	396.6	67.9	75	1240.5	193.8	362.6	68.9	75
20	12.5	1414.8	513.5	583.0	70.7	75	1449.1	457.7	530.6	72.5	75
20	17.5	1308.3	359.5	476.0	65.4	75	1334.3	310.6	431.8	66.7	75
22.5	12.5	1437.5	575.8	632.6	63.9	75	1479.0	511.2	571.3	65.7	75
22.5	17.5	1341.7	427.0	523.8	59.6	75	1373.9	364.7	471.3	61.1	75
25	12.5	1453.6	631.1	678.2	58.1	75	1502.1	559.5	609.6	60.1	75
25	15.0	1409.6	557.2	620.8	56.4	75	1453.0	487.7	555.3	58.1	75
25	17.5	1367.0	488.6	570.0	54.7	75	1405.5	423.1	509.2	56.2	75
27.5	12.5	1464.5	681.6	721.1	53.3	75	1514.0	610.0	652.1	55.1	75
27.5	17.5	1386.4	544.3	613.4	50.4	75	1431.3	470.8	544.5	52.1	75
30	12.5	1471.2	727.9	761.3	49.0	75	1533.9	639.0	675.4	51.1	75
30	17.5	1400.7	595.1	654.4	46.7	75	1452.4	513.3	577.0	48.4	75
32.5	12.5	1475.4	769.1	797.8	45.4	75	1544.9	672.8	704.2	47.5	75
32.5	17.5	1411.5	641.6	692.8	43.4	75	1469.8	552.0	607.6	45.2	75
35	12.5	1476.7	808.6	833.3	42.2	75	1553.4	704.5	731.9	44.4	75
35	17.5	1418.7	684.5	729.2	40.5	75	1483.4	587.4	636.3	42.4	75
35	20.0	1390.2	628.4	684.4	39.7	75	1449.7	535.1	596.0	41.4	75

L=145



Dimensions are in feet
Loads are in kips
Moments are in ft.-kips.

$\frac{I_2}{I_1} = 1.$								$\frac{I_2}{I_1} = \frac{1}{2}$					
h	f	$+M_B$	$-M_C$	$-M_D$	H_A	H_E	V_A	$+M_B$	$-M_C$	$-M_D$	H_A	H_E	V_A
18	12.5	224.1	43.4	162.9	21.5	9.0	3.2	225.2	41.6	161.8	21.5	9.0	3.2
18	15.0	253.9	54.3	178.1	23.1	9.9	3.8	255.0	52.2	177.0	23.2	9.8	3.8
20	12.5	258.1	47.7	191.9	22.9	9.6	3.6	259.6	45.4	190.4	23.0	9.5	3.6
20	17.5	324.2	71.7	225.8	26.2	11.3	4.8	325.8	68.9	224.2	26.3	11.2	4.8
22.5	12.5	303.2	53.4	231.2	24.7	10.3	4.2	305.2	50.2	229.2	24.8	10.2	4.2
22.5	17.5	377.3	79.3	269.6	28.0	12.0	5.5	379.4	75.5	267.5	28.1	11.9	5.5
25	12.5	351.1	59.2	273.9	26.5	11.0	4.8	353.8	55.2	271.2	26.7	10.8	4.8
25	17.5	433.1	87.2	316.9	29.8	12.7	6.2	435.9	82.4	314.1	29.9	12.6	6.2
27.5	12.5	401.9	65.4	320.0	28.4	11.6	5.5	405.4	60.3	316.5	28.5	11.5	5.5
27.5	17.5	491.8	95.3	367.6	31.6	13.4	7.0	495.4	89.3	364.0	31.8	13.2	7.0
27.5	20.0	537.1	111.4	391.1	33.3	14.2	7.8	540.8	105.0	387.3	33.4	14.1	7.8
30	12.5	455.6	71.8	369.4	30.2	12.3	6.2	459.9	65.6	365.1	30.3	12.2	6.2
30	15.0	504.2	87.4	395.8	31.8	13.2	7.0	508.8	80.6	391.3	32.0	13.0	7.0
30	17.5	553.2	103.8	421.8	33.4	14.1	7.8	557.8	96.4	417.2	33.6	13.9	7.8
32.5	17.5	617.5	112.6	479.4	35.2	14.8	8.6	623.2	103.8	473.7	35.4	14.6	8.6
35	12.5	571.3	85.7	478.8	33.8	13.7	7.8	577.8	76.7	472.2	34.0	13.5	7.8
35	17.5	684.5	121.7	540.5	37.1	15.4	9.5	691.5	111.3	533.5	37.3	15.2	9.5
35	20.0	741.6	140.9	570.9	38.7	16.3	10.4	748.7	129.7	563.8	38.9	16.1	10.4
							L=	150					
18	12.5	224.2	43.3	162.8	21.5	9.0	3.1	225.3	41.5	161.8	21.5	9.0	3.1
18	15.0	254.0	54.1	178.0	23.1	9.9	3.6	255.1	52.2	177.0	23.2	9.8	3.6
18	20.0	314.0	78.0	208.0	26.4	11.6	4.8	315.1	75.7	206.9	26.5	11.5	4.8
20	12.5	258.2	47.6	191.8	22.9	9.6	3.5	259.6	45.3	190.4	23.0	9.5	3.5
20	17.5	324.3	71.6	225.7	26.2	11.3	4.7	325.8	68.8	224.2	26.3	11.2	4.7
22.5	12.5	303.3	53.2	231.1	24.7	10.3	4.1	305.3	50.1	229.1	24.8	10.2	4.1
22.5	17.5	377.4	79.1	269.5	28.0	12.0	5.3	379.5	75.4	267.4	28.1	11.9	5.3
25	12.5	351.3	59.0	273.7	26.6	10.5	4.7	353.9	55.1	271.1	26.7	10.8	4.7
25	15.0	392.2	72.5	295.3	28.2	11.8	5.3	394.9	68.2	292.6	28.3	11.7	5.3
25	17.5	433.3	86.9	316.7	29.8	12.7	6.0	436.0	82.2	314.0	29.9	12.6	6.0
27.5	12.5	402.2	65.1	319.7	28.4	11.6	5.3	405.5	60.2	316.4	28.5	11.5	5.3
27.5	17.5	492.0	95.0	367.4	31.6	13.4	6.8	495.5	89.1	363.8	31.8	13.2	6.8
30	12.5	455.8	71.4	369.2	30.2	12.3	6.0	460.1	65.4	364.9	30.3	12.2	6.0
30	17.5	553.5	109.3	421.5	33.5	14.1	7.5	558.0	96.2	417.0	33.6	13.9	7.5
32.5	12.5	512.3	78.1	422.1	32.0	13.0	6.8	517.6	70.8	416.8	32.2	12.8	6.8
32.5	17.5	617.8	112.1	479.1	35.3	14.7	8.3	623.4	103.5	473.5	35.4	14.6	8.3
35	12.5	571.7	85.1	478.3	33.8	13.7	7.5	578.1	76.4	471.9	34.0	13.5	7.5
35	17.5	684.9	121.1	540.1	37.1	15.4	9.2	691.7	110.9	533.3	37.3	15.2	9.2
35	20.0	742.0	140.3	570.5	38.7	16.3	10.1	748.9	129.4	563.6	38.9	16.1	10.1

Analysis of Multi-Span Rigid Frames

by
DR. J. STERLING KINNEY*
Professor and Head,
Division of Structural Engineering
Rensselaer Polytechnic Institute

Selection of a Method of Analysis:—

Several different methods are available for analyzing continuous rigid frames. A listing would include the method of least work, slope deflection, moment distribution and the general method.

The writer prefers to discuss the general method, also known as the method of consistent distortions, in this presentation because it can be applied without significant alteration to any type of continuous single story rigid frame, with either prismatic or nonprismatic members, and regardless of irregularity in the outline of the frame. Solutions by the other methods mentioned become involved with large amounts of work preliminary to the main solution in evaluating properties of nonprismatic members and/or require a rather extensive knowledge of indeterminate structural analysis. The general method, on the other hand, is easy to understand and requires only that the analyst find various deflection components of the frame and solve simultaneous equations.

Heretofore the evaluation of the required deflection components has been accomplished by the method of virtual work or Castigliano's first theorem. Either method entails a great deal of work. In this discussion, however, a simplified method is presented which greatly reduces the labor involved in computing these deflection components and thus makes the use of the general method of analysis a feasible and preferable procedure. This method for computing deflections was developed independently by the author in 1946. It is an extension of the moment area principles into two dimensions. This extension results in the conjugate structure, which will be developed subsequently.

Brief Outline of the General Method: —

Since confusion often exists as to the particular method of analysis signified by a particular name, a brief illustration of the general method is in order. Consider the rigid frame of Fig. 1a. This frame is externally indeterminate to the third degree and consequently three condition equations, in addition to the three equations for static equilibrium, must be found for the solution. The frame is "cut back" to a statically determinate structure as shown in Fig. 1b, the redundant reaction components at A having been replaced by H, V, and M.

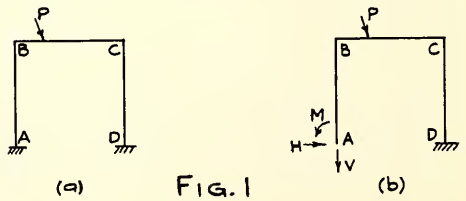


FIG. 1

In Figs. 2a, 2b, 2c and 2d respectively, are shown the deflection components of point A as caused by the real load P, a horizontal load of one kip acting at A, a vertical load of one kip acting at A, and a couple of one foot kip acting at A. The necessary three additional condition equations are obtained by expressing the fact that the final horizontal, vertical, and rotational deflection components of point A are zero, and these equations are written as follows:

*Author's Note: Dr. J. Sterling Kinney is a rare type of professional engineer gifted with almost everything, a mastery of theory of indeterminate structures, the ability to teach difficult subjects so as to make them your friends, a background of practical experience, a warmth of personality. Rigid frame design as taught to his senior students compares favorably with that taught as graduate work in many schools. Dr. Kinney's chapter, prepared especially as a donation to this book, is an outstanding contribution to engineering literature. M.P.K.

$$\Delta_H + H\delta_{HH} + V\delta_{HV} + M\delta_{HM} = 0$$
$$\Delta_V + H\delta_{VH} + V\delta_{VV} + M\delta_{VM} = 0$$
$$\theta_M + Ha'_{MH} + Va'_{MV} + Ma_{MM} = 0$$

The simultaneous solution of these equations will yield values for H, V and M. Thus the general method utilizes deflection condition equations. The various deflection components may be found by any method desired. The writer believes that the conjugate structure provides the easiest method for finding these deflection components.

Development of the Conjugate Structure:—Consider the frame of Fig. 3.

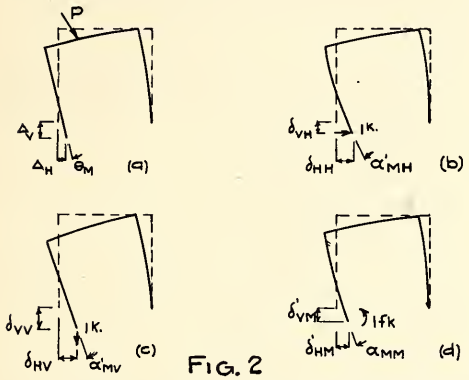


Fig. 2

If all of the frame is considered to be entirely rigid except the segment dx, then under the action of the load P, the flexural strain in the segment will be,

$$d\theta = \frac{M\ dx}{EI}$$

where M is the moment on the segment caused by P.

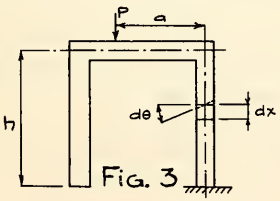


Fig. 3

The frame will then assume the shape shown below:

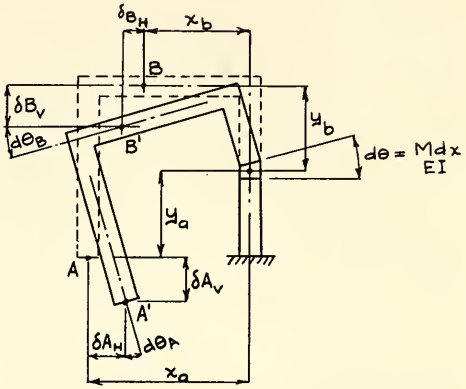


Fig. 4

It is apparent that:

$$\delta A_V = x_a d\theta = x_a \cdot \frac{M\ dx}{EI}$$

$$\delta A_H = y_a d\theta = y_a \cdot \frac{M\ dx}{EI}$$

$$d\theta_A = d\theta = \frac{M\ dx}{EI}$$

and that:

$$\delta B_V = x_b d\theta = x_b \cdot \frac{M\ dx}{EI}$$

$$\delta B_H = y_b d\theta = y_b \cdot \frac{M\ dx}{EI}$$

$$d\theta_B = d\theta = \frac{M\ dx}{EI}$$

Now consider a structure which is identical with the given frame as to the lengths of its members and their relative positions. This alternate structure, hereinafter called the conjugate structure, is located in a horizontal plane and the end corresponding to the point A is fixed. Fig. 5 results.

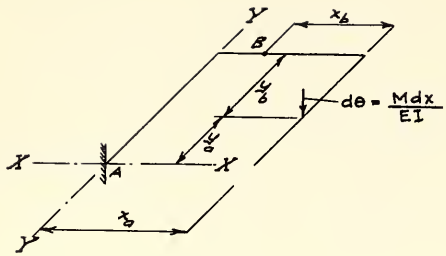


Fig. 5

If the flexural strain in the segment is considered as a vertical load acting on the conjugate structure at a point corresponding to the position of the segment in the original structure, as shown in Fig. 5 then, taking moments at A:

$$M_y = x_a \cdot d\theta = x_a \cdot \frac{M\ dx}{EI} = \delta A_V$$

$$M_x = y_a \cdot d\theta = y_a \cdot \frac{M\ dx}{EI} = \delta A_H$$

and also,

$$\text{Shear at A} = d\theta = \frac{M\ dx}{EI} = d\theta_A$$

and taking moments about B:

$$M_y = x_b \cdot d\theta = \delta B_V$$

$$M_x = y_b \cdot d\theta = \delta B_H$$

and also,

$$\text{Shear at B} = d\theta = d\theta_B$$

Now, if the frame is considered to be elastic throughout, the conjugate structure will be as shown in Fig. 6.

Note that when the M/EI diagram is of a shape such that the centroid is known the whole load (area of M/EI diagram) can be considered to act through this centroid when taking moments.

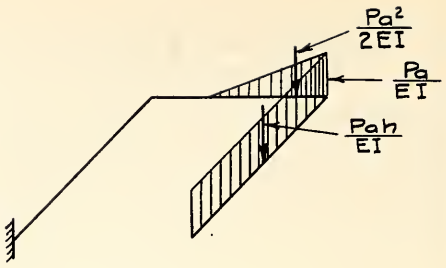


Fig. 6

Having seen how the conjugate structure is proportioned, and how it is used, it is now possible to formulate a group of principles as follows:

(1) The conjugate structure, for a given real structure, is identical to the real structure with regard to the lengths of the members and their relative position.

(2) The conjugate structure is positioned in a horizontal plane.

(3) The load, which acts in a vertical direction on the conjugate structure, is the M/EI diagram of the real structure, that is, the conjugate structure is loaded with the flexural strains of the real structure.

(4) If the flexural strain at a given section of the real structure is such as to cause tension on the outside fibers, then this flexural strain is represented as a downward load on the conjugate structure. If compression exists on the outside fibers of the real structure, the load on the corresponding section of the conjugate structure is up.

(5) The conjugate structure, under the action of the real structure flexural strains as loads and the reactions of the conjugate structure, must satisfy three equilibrium condition equations, specifically:

$$\Sigma M_X = 0$$

$$\Sigma M_Y = 0$$

$$\Sigma V = 0$$

The X and Y axes are as shown in Fig. 5.

(6) The shear at any section of the conjugate structure is the slope of the corresponding section of the real structure.

(7) The internal moment on any section of the conjugate structure is the deflection of the corresponding section of the real structure in a direction perpendicular to the lever arm used to find any particular moment. (If this statement seems ambigous, refer again to Fig. 4 and the explanation thereof.)

(8) The end of the conjugate structure corresponding to the end of the real structure which deflects always has a fixed support.

(9) If a section be passed through any point of the conjugate structure and if the portion of the conjugate structure to the right of the section tends to move down with respect to the part to the left of the section then the rotation of the corresponding point of the real structure is counterclockwise. Diagonal sections are passed from upper right to lower left through right hand vertical members and from upper left to lower right through left hand vertical members.

(10) If the moment at any point on a horizontal or inclined member of the conjugate structure, about an axis through that point parallel to the y—y axis, results in tension in the member top fibers normal to this axis, then the vertical deflection of the corresponding point on the real structure is down.

(11) If the moment at any point on a vertical or inclined member of the conjugate structure, about an axis through that point parallel to the x—x axis, results in tension in the member top fibers normal to this axis, and if the supported end of the member has an algebraically larger y coordinate than the unsupported end, the horizontal deflection component of the corresponding point of the real structure is toward the right, provided the fixed support of the conjugate structure is to the right, but will be toward the left if the fixed support of the conjugate structure is to the left. If, however, the supported end of the conjugate structure member has an algebraically smaller y coordinate than the unsupported end, then tension in the member top fibers as above described will signify a horizontal deflection component of the corresponding point on the real structure to the left if the conjugate structure fixed support is to the right, and a deflection to the right when the conjugate structure fixed support is to the left.

This last principle will be clarified by reference to Fig. 7. Fig. 7 (a) shows a real structure and Fig. 7 (b) the corresponding conjugate structure. The supported end of AB in the conjugate structure is at B and of BC is at C and, therefore, each member has a supported end with a y coordinate algebraically larger than the unsupported end. Consequently the tension on the top fibers normal to the x axis, which under the given loading will exist throughout AB and BC, signifies a horizontal deflection component to the right for all corresponding points of AB and BC on the real structure. The member CD on the conjugate structure, however, with its supported end at D falls into the opposite classification for which tension on the top side indicates, in this case a horizontal deflection component of corresponding points on the real structure toward the left. It is probable that some point along CD of the real structure will have zero horizontal deflection. The corresponding point on the conjugate structure will be the point at which the moment of the elastic loads, about an axis parallel to the x—x axis and through the point, will be zero. All sections between this point and C will have compression on the top fibers of the conjugate structure and corresponding sections of the real structure will deflect to the right, while all sections between this point and D will have tension on the top

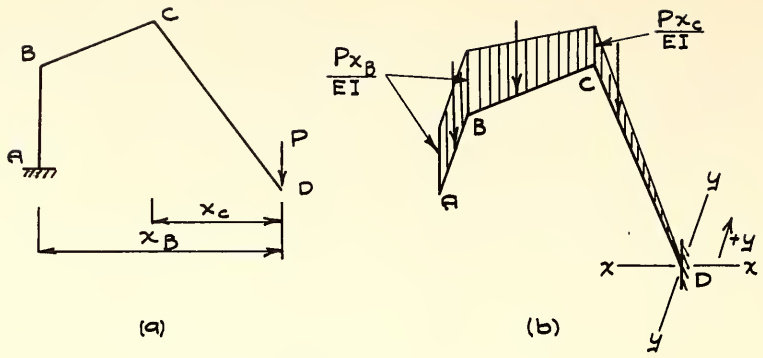


FIG. 7

fibers and corresponding sections of the real structure will deflect to the left.

Attention should be called to the fact that the sign conventions outlined in (9), (10), and (11) will apply only if the sign convention of (4) is followed. It should also be noted that in most cases it will be unnecessary to use the principles (9), (10), and (11) since structural sense will usually indicate the directions of the various deflection components.

Demonstration of the Conjugate Structure:—

The application of the method will now be demonstrated with several illustrative problems.

Problem 1: Using the conjugate structure, find the horizontal, vertical and angular deflection components of point A. E = 30,000 ksi.

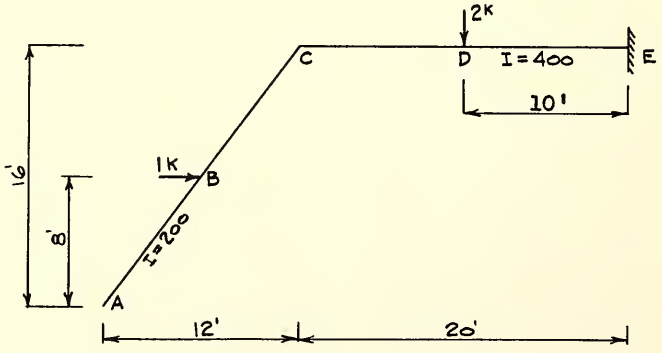


FIG. 8 (Problem 1)

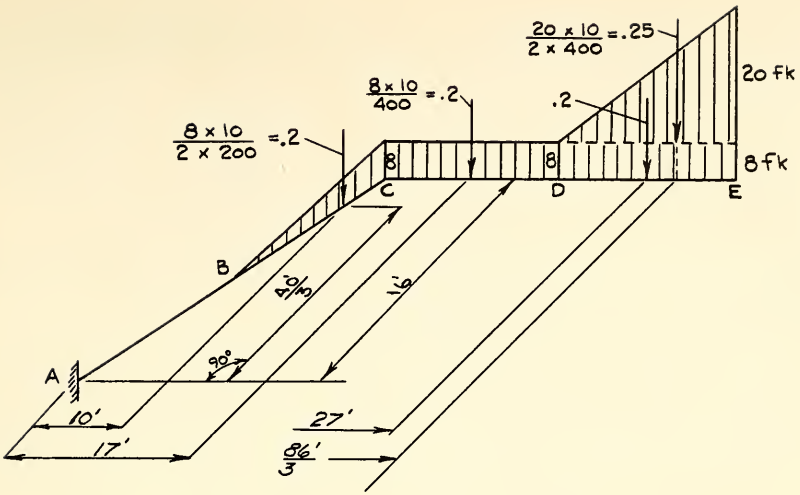


FIG. 9 (Problem 1)

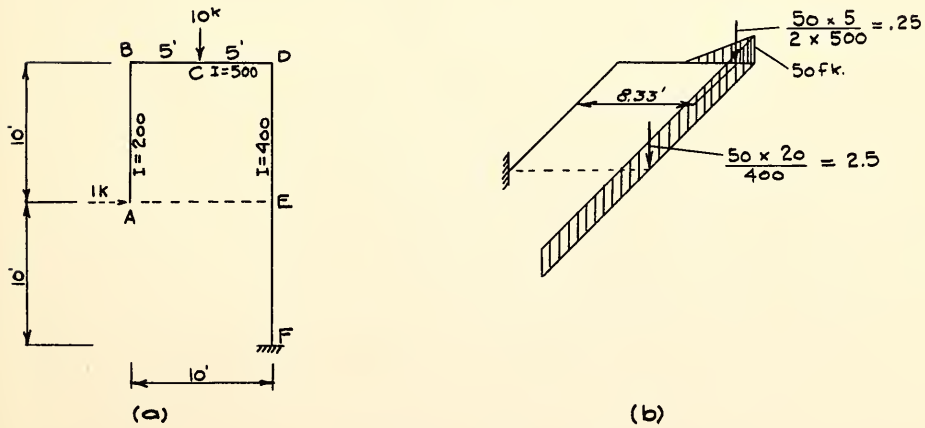


FIG. 10 (Problem 2)

The conjugate structure is:

$$E\triangle A_H = .2 \times \frac{40}{3} + .2 \times 16 + .2 \times 16 + .25 \times 16 = 13.07$$

$$\triangle A_H = \frac{13.07 \times 1728}{30{,}000} = .752\text{" to the right}$$
(Principle No. 11)

$$E\triangle A_V = .2 \times 10 + .2 \times 17 + .2 \times 27 + .25 \times \frac{86}{3} = 17.97$$

$$\triangle A_V = \frac{17.97 \times 1728}{30{,}000} = 1.033\text{" down}$$
(Principle No. 10)

$$E\theta_A = .2 + .2 + .2 + .25 = .85$$

$$\theta_A = \frac{.85 \times 144}{30{,}000} = .00408 \text{ radians counterclockwise}$$
(Principle No. 9)

Problem 2: Using the conjugate structure find the vertical, horizontal and rotational deflection components of point A as caused by the 10k load. Also find the horizontal deflection components of point A as caused by a 1k load acting to the right at A. E = 30,000 ksi.

$$E\triangle A_V = .25 \times 8.33 + 2.5 \times 10 = 27.08$$

$$\triangle A_V = \frac{27.08 \times 1728}{30{,}000} = 1.56\text{" down.}$$

$$E\triangle A_H = .25 \times 10 = 2.5$$

$$\triangle A_H = \frac{2.5 \times 1728}{30{,}000} = .144\text{" to the right.}$$

$$E\theta_A = 2.5 + .25 = 2.75$$

$$\theta_A = \frac{2.75 \times 144}{30{,}000} = .0132 \text{ radians counterclockwise}$$

For a 1[k] load acting horizontally to the right at A, the conjugate structure would be:

$$E\triangle A_H = .25 \times 6.67 + .2 \times 10 + 2 \times .125 \times 6.67 = 5.34$$

$$\triangle A_H = .307\text{" to the right.}$$

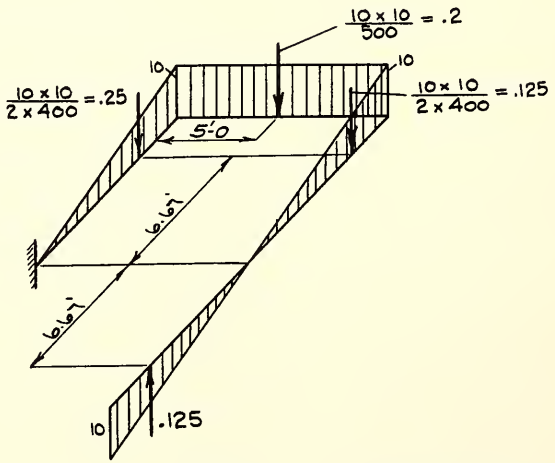


FIG. 11 (Problem 2)

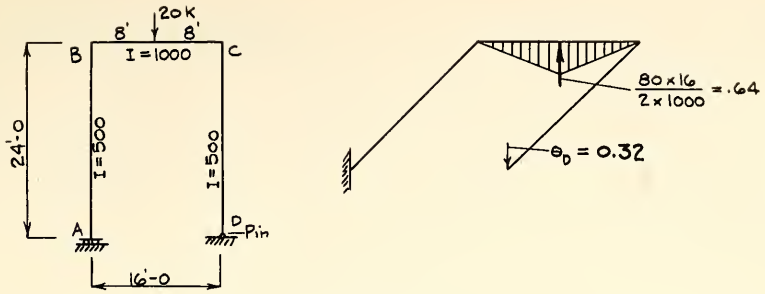


FIG. 12 (Problem 3)

Problem 3: Using the conjugate structure find:

(a) The horizontal deflection of point A caused by the 20^k load.

(b) The horizontal deflection of A caused by 1^k applied horizontally at A acting toward the left.

(a) In this case, point A has no vertical deflection component so, even though this end of the conjugate structure must be fixed, the moment about Y axis through A must be zero. This can be accomplished only by applying an external load θ_D at the free end of the conjugate structure, to give $\Sigma M_{AY} = 0$.

Assuming θ_D as down on the conjugate structure (as it obviously would have to be) the equation is:

$$\theta_D \times 16 - .64 \times 8 = 0$$

Therefore, $\theta_D = 0.32$

This value of θ_D, if multiplied by 144 and divided by E in ksi would give the slope of the right hand leg if desired. Note that this value of θ_D does not enter into the solution for $\triangle A_H$ since its y lever arm is zero in this case. If, however, the frame had unequal legs, it would have appeared in the equation for $\triangle A_H$.

$$E\triangle A_H = .64 \times 24 = 15.38$$

$$\triangle A_H = \frac{15.38 \times 1728}{30{,}000} = .884\text{" to the left.}$$

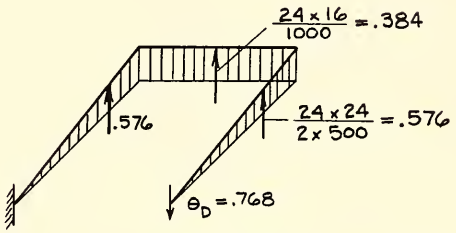


FIG. 13 (Problem 3)

(b)

$$\theta_D \times 16 - .384 \times 8 - .576 \times 16 = 0$$

$$\theta_D = .768$$

$$E\delta A_H = .384 \times 24 + 2 \times .576 \times 16$$

$$= 27.63$$

$$\delta A_H = \frac{27.63 \times 1728}{30{,}000} = 1.59\text{" to the left.}$$

Application to Multi-Span Frames:—

Consider the frame of Fig. 14 (a) which can be cut back to a statically determinate structure as shown in Fig. 14 (b). Redundant reactions at B and C can be assumed as indicated.

The conjugate structure for the span AB and for the load P is shown in Fig. 15.

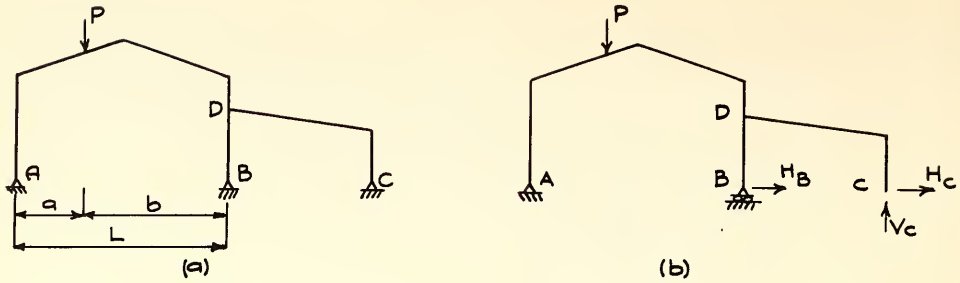


FIG. 14

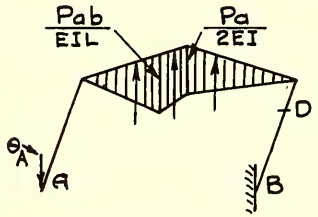


FIG. 15

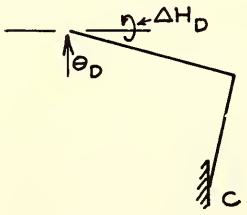


FIG. 16

The horizontal deflection of B and D and the rotation of D may be found therefrom. The conjugate structure for span BC is loaded at its unsupported end D with the rotation and horizontal deflection of D as shown in Fig. 16. The rotation of D is properly represented as a concentrated load in this figure and the horizontal displacement of D as a couple about an axis through D parallel to the x—x axis.

Thus the three required deflection components of B and C in Fig. 14 (b) and as caused by the real load may be found. In a similar way the deflection components as caused by one kip loads applied separately in the directions and at the points of application of the three redundant reactions may be found. The necessary deflection condition equations may then be solved.

Illustrative Analysis of a Multi-Span Frame:—

As an illustration of the analysis of a continuous rigid frame the structure of Fig. 17 (a) will be analyzed for a uniform load of 2 kpf of horizontal projection. It is assumed that the frame will have a constant moment of inertia.

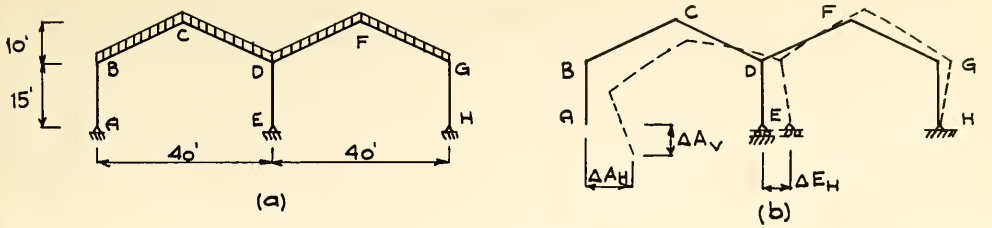


FIG. 17

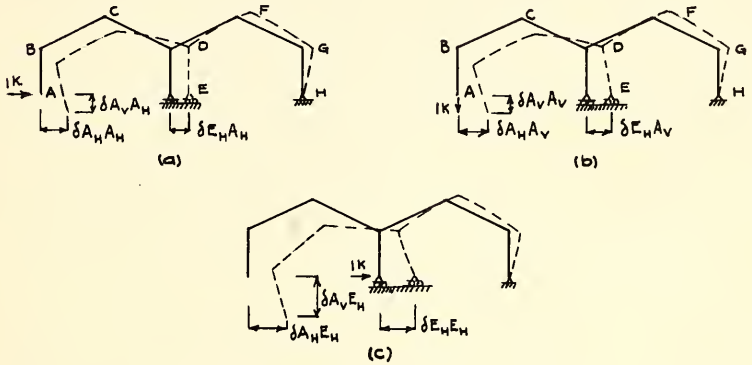


FIG. 18

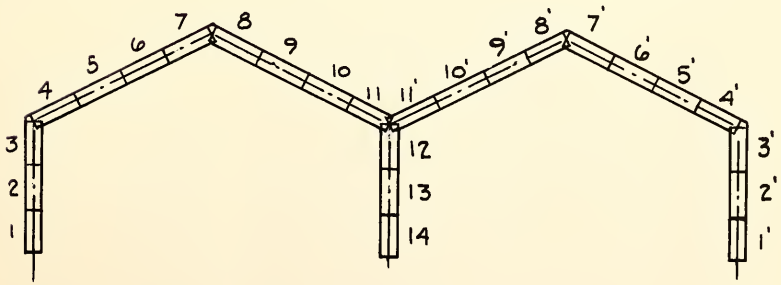


FIG. 19

The frame is first "cut back" to a statically determinate structure as shown in Fig. 17 (b). Deflections caused by the 2 kpf load are as shown by the dotted outline and as labeled. These deflections will first be evaluated. Then 1 k loads are applied individually and successively at A, first horizontally and then vertically, and then horizontally at E. These loads, and the resulting deflections, are shown in Figs. 18 (a), (b) and (c). These deflections will also be evaluated.

Finally three required deflection condition equations are written which simply state that, in the actual structure, the horizontal deflection at A, the vertical deflection at A, and the horizontal deflection at E are zero when the redundant reactions H_A, V_A and H_E are acting. These redundant reactions are assumed to act in the same direction as the unit forces shown in Fig. 18.

The three condition equations are:

$$H_A\delta A_H A_H + V_A\delta A_H A_V + H_E\delta A_H E_H + \triangle A_H = 0$$

$$H_A\delta A_V A_H + V_A\delta A_V A_V + H_E\delta A_V E_H + \triangle A_V = 0$$

$$H_A\delta E_H A_H + V_A\delta E_H A_V + H_E\delta E_H E_H + \triangle E_H = 0$$

It is first necessary to divide the frame into segments about five feet long. These segments are shown in Fig. 19. The column segments are 5 feet long and the segments in the sloping girders are 5.59 feet long.

The bending moments at the centers of the girder segments resulting from the 2 kpf load acting on the determinate structure of Fig. 17 (b) are shown in the table below as well as the relative values of the elastic weights of these segments. The actual value of the elastic weight for each segment would be $\frac{M \triangle S}{EI}$ where M is the moment at the center of each segment, $\triangle S$ is the segment length, I is the moment of inertia at the center of each segment and E is the modulus of elasticity. In the present case E and I are constant so the elastic load can be used as the relative value $M\triangle S$. The vertical reaction at E in Fig. 17 (b) is 160^k.

Segment	x_A	$M = \frac{2x^2}{2}$	$M = \frac{2x^2}{2} - 160\,(x-40)$	$\triangle S$	$M\triangle S$
4	2.5	6.25		5.59	35
5	7.5	56.25			314
6	12.5	156.25			873
7	17.5	306.25			1712
8	22.5	506.25			2830
9	27.5	756.25			4227
10	32.5	1056.25			5904
11	37.5	1406.25			7861
11'	42.5		1806.25— 400 = 1406.25		7861
10'	47.5		2256.25—1200 = 1056.25		5904
9'	52.5		2756.25—2000 = 756.25		4227
8'	57.5		3306.25—2800 = 506.25		2830
7'	62.5		3906.25—3600 = 306.25		1712
6'	67.5		4556.25—4400 = 156.25		873
5'	72.5		5256.25—5200 = 56.25		314
4'	77.5		6006.25—6000 = 6.25		35

The conjugate structure for span EH of Fig. 17 (b) is shown in Fig. 20. This is loaded with the elastic loads computed above for segments 11' through 4'.

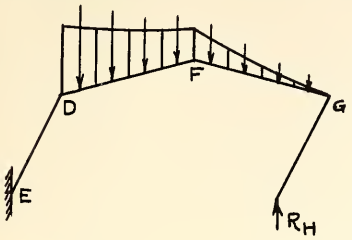


Fig. 20

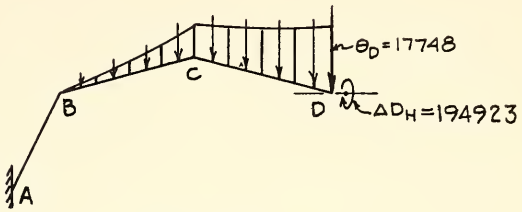


Fig. 21

Segment	x_E	y_A	$M\triangle S$	$M\triangle S \cdot x_E$	$M\triangle S \cdot y_A$	$M\triangle S \cdot (y_A - 15)$
11'	2.5	16.25	7861	19653	127741	9826
10'	7.5	18.75	5904	44280	110700	22140
9'	12.5	21.25	4227	52837	89824	26418
8'	17.5	23.75	2830	49525	67212	24762
7'	22.5	23.75	1712	38520	40660	14980
6'	27.5	21.25	873	24007	18551	5456
5'	32.5	18.75	314	10205	5887	1177
4'	37.5	16.25	35	1312	568	44
			23756	240339	461143	104803

$$R_H = \frac{240339}{40} = 6008 \qquad \triangle E_H = +\ 461143$$

$$\triangle D_H = 104803 + 6008 \times 15 = +\ 194923$$

$$\theta_D = 23756 - 6008 = +\ 17748$$

Vertical and horizontal deflection components of A ($\triangle A_V$ and $\triangle A_H$) in Fig. 17 (b) are found from the conjugate structure of Fig. 21.

Segment	x_A	y_A	$M\triangle S$	$M\triangle S \cdot x_A$	$M\triangle S\, y_A$
4	2.5	16.25	35	87	568
5	7.5	18.75	314	2355	5887
6	12.5	21.25	873	10912	18551
7	17.5	23.75	1712	29960	40660
8	22.5	23.75	2830	63675	67212
9	27.5	21.25	4227	116242	89824
10	32.5	18.75	5904	191880	110700
11	37.5	16.25	7861	294787	127741
				709898	461143

$$\triangle A_V = +\ 709898 + 17748 \times 40 = +\ 1{,}419{,}818$$

$$\triangle A_H = +\ 461143 + 17748 \times 15 + 194923 = +\ 922{,}286$$

Deflection components of A and E resulting from the 1[k] horizontal load at A as shown in Fig. 18 (a) are next computed. The relative elastic loads for this loading, which are symmetrical about the vertical centerline, are computed for the left half of the frame in the table below.

Segment	$M = y_A$	$\triangle S$	$M\triangle S$
1	2.5	5.0	12.5
2	7.5	5.0	37.5
3	12.5	5.0	62.5
4	16.25	5.59	90.8
5	18.75	5.59	104.8
6	21.25	5.59	118.8
7	23.75	5.59	132.8
8	23.75	5.59	132.8
9	21.25	5.59	118.8
10	18.75	5.59	104.8
11	16.25	5.59	90.8

The conjugate structure for the right span is similar to that shown in Fig. 20 but is loaded with elastic loads as computed in the table above. The necessary computations are shown in the following table.

Segment	x_E	y_A	$M\triangle S$	$M\triangle S \cdot x_E$	$M\triangle S \cdot y_A$	$M\triangle S\,(y_A - 15)$
1'	40.0	2.5	12.5	500	31	− 156
2'	40.0	7.5	37.5	1500	281	− 281
3'	40.0	12.5	62.5	2500	781	− 156
4'	37.5	16.25	90.8	3405	1475	+ 113
5'	32.5	18.75	104.8	3406	1965	+ 393
6'	27.5	21.25	118.8	3267	2524	+ 742
7'	22.5	23.75	132.8	2988	3154	+1162
8'	17.5	23.75	132.8	2324	3154	+1162
9'	12.5	21.25	118.8	1485	2524	+ 742
10'	7.5	18.75	104.8	786	1965	+ 393
11'	2.5	16.25	90.8	227	1475	+ 113
			1006.9	22388	19329	4227

$$R_H = \frac{22388}{40} = 559.7 \qquad \delta E_H A_H = + 19329$$

$$\delta D_H A_H = 4227 + 559.7 \times 15 = + 12623$$

$$a_D = 1006.9 - 559.7 = + 447.2$$

The conjugate structure for the left span is similar to Fig. 21. The elastic loads along the girders are similar to those used above with the addition of the concentration a_D and the couple

$\delta D_H A_H$ at the free end of the conjugate structure. The computations to obtain $\delta A_V A_H$ and $\delta A_H A_H$ are given below.

Segment	x_A	y_A	$M\triangle S$	$M\triangle S \cdot x_A$	$M\triangle S \cdot y_A$
1	0	2.5	12.5	....	31
2	0	7.5	37.5	....	281
3	0	12.5	62.5	....	781
4	2.5	16.25	90.8	227	1475
5	7.5	18.75	104.8	786	1965
6	12.5	21.25	118.8	1485	2524
7	17.5	23.75	132.8	2324	3154
8	22.5	23.75	132.8	2988	3154
9	27.5	21.25	118.8	3267	2524
10	32.5	18.75	104.8	3406	1965
11	37.5	16.25	90.8	3405	1475
			1006.9	17888	19329

$\delta A_V A_H = + 17888 + 447.2 \times 40 = + 35{,}776$

$\delta A_H A_H = + 19329 + 447.2 \times 15 + 12623 = + 38{,}660$

A load of 1^k applied vertically at A as in Fig. 18 (b) will result in the deflections shown therein. The relative elastic loads are computed below for the left half of the frame. Those for the right half of the frame are similar.

Segment	$M = x_A$	$\triangle S$	$M\triangle S$
4	2.5	5.59	14.0
5	7.5	5.59	41.9
6	12.5	5.59	69.9
7	17.5	5.59	97.8
8	22.5	5.59	125.8
9	27.5	5.59	153.7
10	32.5	5.59	181.7
11	37.5	5.59	209.6

The conjugate structure to determine $\delta E_H A_V$ is similar to that of Fig. 20 and the necessary computations are tabulated below.

Segment	x_E	y_A	$M\triangle S$	$M\triangle S \cdot x_E$	$M\triangle S \cdot y_A$	$M\triangle S\,(y_A - 15)$
4'	37.5	16.25	14.0	525	227	17
5'	32.5	18.75	41.9	1362	786	157
6'	27.5	21.25	69.9	1922	1485	437
7'	22.5	23.75	97.8	2201	2323	856
8'	17.5	23.75	125.8	2202	2988	1101
9'	12.5	21.25	153.7	1921	3266	961
10'	7.5	18.75	181.7	1363	3407	681
11'	2.5	16.25	209.6	524	3406	262
			894.4	12020	17888	4472

$$R_H = \frac{12020}{40} = 300.5 \qquad \delta E_H A_V = + 17{,}888$$

$\delta D_H A_V = + 4472 + 300.5 \times 15 = + 8{,}979$

$a_D = 894.4 - 300.5 = + 593.9$

The deflection components $\delta A_V A_V$ and $\delta A_H A_V$ are computed with a conjugate structure similar to Fig. 21. This is loaded with the elastic loads above computed for segments 4 through 11 plus a couple at the free end equal to $\delta D_H A_V$ as well as a concentration equal to a_D.

Segment	x_A	y_A	$M\triangle S$	$M\triangle Sx_A$	$M\triangle Sy_A$
4	2.5	16.25	14.0	35	227
5	7.5	18.75	41.9	314	786
6	12.5	21.25	69.9	874	1485
7	17.5	23.75	97.8	1712	2323
8	22.5	23.75	125.8	2831	2988
9	27.5	21.25	153.7	4227	3266
10	32.5	18.75	181.7	5905	3407
11	37.5	16.25	209.6	7860	3406
				23758	17888

$\delta A_V A_V = +\ 23758 + 593.9 \times 40 = +\ 47{,}514$

$\delta A_H A_V = +\ 17888 + 593.9 \times 15 + 8979 = +\ 35{,}776$

A load of 1 kip acting horizontally at E, as shown in Fig. 18 (c) will result in the relative elastic loads computed below.

Segment	$M = y_A$	$\triangle S$	$M\triangle S$
14	2.5	5.0	12.5
13	7.5	5.0	37.5
12	12.5	5.0	62.5
11'	16.25	5.59	90.8
10'	18.75	5.59	104.8
9'	21.25	5.59	118.8
8'	23.75	5.59	132.8
7'	23.75	5.59	132.8
6'	21.25	5.59	118.8
5'	18.75	5.59	104.8
4'	16.25	5.59	90.8
3'	12.5	5.0	62.5
2'	7.5	5.0	37.5
1'	2.5	5.0	12.5

The deflection $\delta E_H E_H$ is computed by means of a conjugate structure which is again similar to Fig. 20 but loaded with the relative elastic loads computed above.

Segment	x_E	y_A	$M\triangle S$	$M\triangle S \cdot x_E$	$M\triangle S \cdot y_A$	$M\triangle S\,(y_A - 15)$
14	..	2.5	12.5	....	31	....
13		7.5	37.5	....	281	....
12	..	12.5	62.5	....	781	....
11'	2.5	16.25	90.8	227	1476	114
10'	7.5	18.75	104.8	786	1965	393
9'	12.5	21.25	118.8	1485	2525	743
8'	17.5	23.75	132.8	2324	3154	1162
7'	22.5	23.75	132.8	2988	3154	1162
6'	27.5	21.25	118.8	3267	2525	743
5'	32.5	18.75	104.8	3406	1965	393
4'	37.5	16.25	90.8	3405	1476	114
3'	40.0	12.5	62.5	2500	781	−156
2'	40.0	7.5	37.5	1500	281	−281
1'	40.0	2.5	12.5	500	31	−156
		Σ (11' thru 1') =	1006.9	22388	20426	4231

$$R_H = \frac{22388}{40} = 559.7 \qquad \delta E_H E_H = +\ 20{,}426$$

$\delta D_H E_H = +\ 4231 + 559.7 \times 15 = +\ 12{,}627$

$a_D = 1006.9 - 559.7 = +\ 447.2$

The deflections $\delta A_V E_H$ and $\delta A_H E_H$ are found by use of a conjugate structure similar to Fig. 21 which is loaded at its free end with a couple equal to $\delta D_H E_H$ and a concentration equal to a_D.

$$\delta A_H E_H = 447.2 \times 15 + 12627 = + 19{,}335$$

$$\delta A_V E_H = 447.2 \times 40 = + 17{,}888$$

Attention is called to the fact that by Maxwell's reciprocal theorem $\delta A_H E_H = \delta E_H A_H$, $\delta A_V E_H = \delta E_H A_V$, and $\delta A_V A_H = \delta A_H A_V$. Comparison of the above computed relative values for these deflections will show that the above equalities exist. This is a valuable check on the correctness of the values for these deflections.

The computed relative values for all deflection components are now substituted in the deflection condition equations previously written.

$$+ 38660\ H_A + 35776\ V_A + 19335\ H_E + 922{,}286 = 0$$

$$+ 35776\ H_A + 47514\ V_A + 17888\ H_E + 1{,}419{,}818 = 0$$

$$+ 19329\ H_A + 17888\ V_A + 20426\ H_E + 461{,}143 = 0$$

These equations are solved simultaneously in the following table.

Equation	Operation	H_A	V_A	H_E	Constant	Equation Sum	Check Operation
(1)		+38660	+35776	+19335	+ 922,286	1,016,057	
(2)		+35776	+47514	+17888	+1,419,818	1,520,996	
(3)		+19329	+17888	+20426	+ 461,143	518,786	
(4)	(1)—1.0806 (2)	.....	−15568	+ 5	− 611,969	−627,532	−627,531
(5)	(1)—2.0001 (3)	.....	.....	−21519	− 46	− 21,565	− 21,567

From (5), for all practical purposes, $H_E = 0$. The symmetry of the structure indicates that this is the correct value for H_E.

From (4), $V_A = -39.31$ k. The minus sign indicates that the true direction of V_A is opposite to the direction of the 1 k vertical load assumed in Fig. 15 (b).

From (3), $H_A = +12.52$ k. The plus sign indicates that the true direction of H_A is the same as that of the 1[k] horizontal load in Fig. 15 (a).

The above redundant reactions having been determined, it is now possible to find the values for the remaining reactions. If a constant moment of inertia is to be used throughout the frame in the final design then the above analysis is sufficient and the most critical combination of moment and thrust at any section can be determined and the proper steel section for the entire frame computed therefrom. If, however, the frame is to be fabricated with a nonconstant moment of inertia, regardless of whether the variability is the result of nonprismatic members or of several prismatic members with different moments of inertia, the above analysis is preliminary. The critical moments obtained therefrom combined with thrusts if necessary, are used to determine critical section moduli at corresponding sections of the structure. The outline of the frame, and the resulting variation of moment of inertia, between these critical sections is usually such as to effect a smooth and graceful transition from one critical section to another. The resulting frame might be designated the first trial nonprismatic frame.

After the outline of the first trial nonprismatic frame has been established, as just described, the above preliminary analysis must be modified to determine the effect of the variation of moment of inertia upon the various

deflection components. This is quite easily done with the conjugate structure. It will be recalled that relative values of elastic weights have been used in the preceding analysis to simplify computations. In other words, instead of using the absolute values $M^{\triangle}S/EI$, the relative value $M^{\triangle}S$ was used since E and I were constant. Now, to include the effect of a varying moment of inertia, it is only necessary to use a new relative elastic weight, specifically $M^{\triangle}S/I$. This is easily accomplished by dividing all computations in the preceding tables, which involve $M^{\triangle}S$, by the value for I at the center of each segment and resolving the three deflection condition equations. Critical sections of the first trial nonprismatic frame are rechecked for adequacy on the basis of this second analysis. Revisions and additional analyses are made as necessary.

Conclusion:—

The above method of analysis has been presented in this book because it is universally applicable to single story continuous rigid frames regardless of irregularities, it is easy to understand, and in many cases is easier to apply than alternate methods. Its application does not require an extensive training in indeterminate structural analysis and so the method is particularly suited for inclusion in this book, which is intended as a design manual rather than a textbook.

The writer is fully aware of the fact that the analysis of the two span continuous rigid frame discussed heretofore can be accomplished more easily by moment distribution if the members are prismatic. As soon as nonprismatic members are introduced into the frame, however, the solution by moment distribution will be found to involve considerably more work than the method presented herein. An additional advantage of the general method arises from the fact that an entirely independent analysis can be made as a check on the results of a first analysis simply by picking a different set of redundant reactions from those used for the first analysis and making a second set of computations. This is an important advantage for the design engineer who works alone. In contrast, only one solution is possible by either moment distribution or slope deflection.

A Large Displacement Deformeter Apparatus for Stress Analysis with Elastic Models*

by
PROF. WM. J. ENEY**
Head, Dept. of Civil Eng. and Mechanics
Director, Fritz Engineering Laboratory
Lehigh University

INTRODUCTION

In 1922 the late Prof. George E. Beggs of Princeton University introduced the deformeter method for the analysis of statically indeterminate structures. In his apparatus special gages are used to induce minute deformations into a small model of the structure. From the resultant deflections, measured with micrometer microscopes, the solution of the problem follows. The model is generally fashioned out of sheets of celluloid and proportioned so as to elastically behave, insofar as is possible, like the structure simulated.

Professor Beggs' contribution established the science now known as Model Analysis and led to the development of other useful tools for the structural engineer. Perhaps because of the cost of the Beggs Deformeter equipment, together with the fact that for best results it should be operated under laboratory conditions using skilled techniques, the method has not had the general use which it deserves.

As with all great inventions, modifications suggested themselves to others so Otto Gottschalk contributed the Continostat, and Anders Bull the Brass Wire Model. Both of these devices have proved useful, particularly as aids to instruction, although both lack the inherent advantage of the type of model used by Beggs. Developments since 1938 include the Ruge-Schmidt Moment Indicator and the M. I. T. Moment Deformeter especially adapted to structural frames where the members are of constant section.

In 1935 the writer, aware of the ease with which intricate models could be assembled with sheet material, sought a modification of the Beggs method whereby the use of the micrometer-microscopes could be eliminated, and developed the deformeter apparatus described herein. This deformeter differs from the Beggs deformeter principally in that (1) much larger deformations are employed, (2) the magnitude of gage displacements may be varied to fit the model, and (3) deflections are measured with ordinary engineers' scales. Part of the apparatus, in its earliest form, was presented in *The Engineering News Record,* February 16, 1939. Since that time the equipment has been improved and used by many investigators both here and abroad with excellent results. It is not patented and can be produced by a competent machinist at a relatively low cost.

Special gages of two types are used to introduce controlled amounts of displacement in a model of the structure. These displacements are of such magnitude that the deflections of the model can be measured with sufficient accuracy with either a scale of 100 divisions per

*This is not a duplicate but an abridgement of an SESA article.

**Author's Note: It is probably safe to say that nowhere else in the world have so many steel rigid frames been tested as at the Fritz Engineering Laboratory. And that, incidentally, was in keeping with the original aim to make Lehigh University the finest research facility. A leader for many years in the field of structural testing, Lehigh has maintained its unique advantage because of their option on the entire frameworks and power plant of genial and dynamic "Big Bill" Eney, their brilliant Head Master—veritably an institution in himself.

Inventor of numerous instruments used with structural models the world over, Prof. Eney has been consulted by many in this country and abroad in connection with structural laboratories. In the author's opinion, he should be loaned out nationally to teach *"Eneyism"* in engineering schools. When I recall my cardboard model struggle in the design of a steel rigid frame in 1908 (model analysis was an unknown science then) and when I think further of the crudeness of a test which I conducted on a concrete rigid frame I built in 1910, there results a feeling of satisfaction that today we have sound theories of design and also model-making for translating the results of tests into sound theories.

But do we? Not quite so!

Whereas the theory of rigid frame design is now taught in all engineering undergraduate work, though in varying degrees, the structural behavior of such structures is not taught to undergraduates. With steel rigid frames now being designed and built daily and in many cases blindly for lack of such knowledge, I believe that model analysis should be made part of undergraduate work in all schools. This book would have been incomplete and deficient without a generous helping of *Eneyism* in two separate articles. It is hoped that Prof. Eney's bountiful contribution will serve towards more widespread study and teaching of this necessary and most fascinating subject. M.P.K.

inch or a micrometer barrel adapted to this use by suitable attachments. The apparatus is simple to operate, requires little special skill in using the measuring scales, and is especially adapted for use in a design office, for with the large displacements, control of room temperature is unnecessary.

The Principles of the Deformeter Method

The application of the Muller Breslau's Principle and Maxwell's Theorem of Reciprocal Deflections to the deformeter method has been presented by Professors Beggs, Timby and others. The fundamental deformeter relation is given by the equation

$$X_a = KP_c\frac{\Delta_{ca}}{\Delta_{aa}} \qquad (1)$$

in which, as shown in Fig. 1,

X_a = the unknown force component acting at point "a" of structure,

P_c = force acting on the structure at point "c,"

K = a constant dependent upon the dimensions of the model. If X_a is a thrust or shear force component, K has a value of unity. If X_a is the moment, K is equal to the number of units of length on the structure represented by one unit on the model.

Δ_{aa} = displacement introduced into the model at point "a" in the direction of the force component X_a,

Δ_{ca} = the deflection of the load point "c" parallel to the line of action of force P caused by the induced displacement Δ_{aa}.

Each unknown force component acting either on or within the structure can be found by determining on the model the ratio of the deflection of the point corresponding to the point of load on the structure, and measured parallel to the line of action of that load, to the displacement induced in the model along the line of action of the unknown force component.

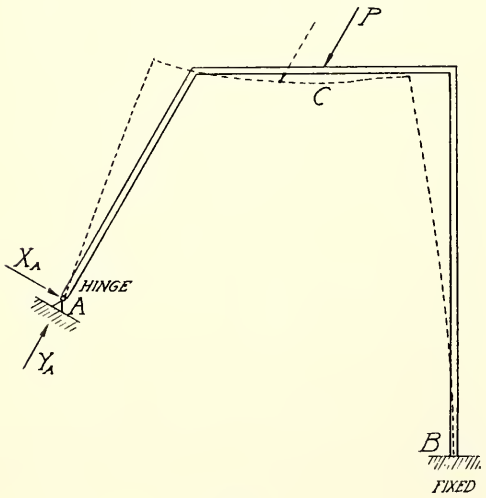


Fig. 1a.

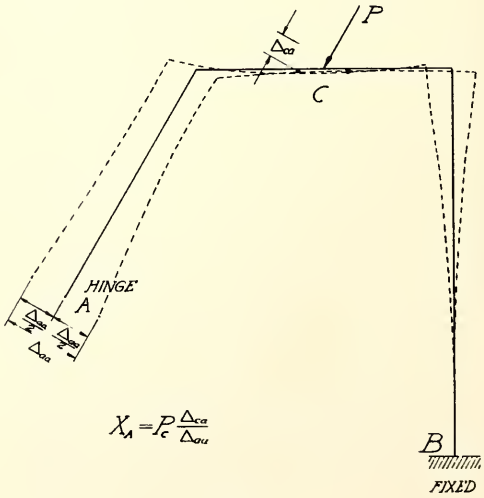


Fig. 1b.

Proportioning the Model

The ratio Δ_{ca}/Δ_{aa} (Eq. (1)) must be identical for the structure and model, thereby defining the conditions of similitude. A planar model will satisfactorily duplicate the elastic action of its prototype if it is geometrically similar and proportioned so that the moment of inertia of the model bears a fixed ratio to that of the structure at corresponding sections.

When the model is fabricated from a homogeneous sheet material of constant thickness, this is accomplished by making the width of the model proportional to the cubic root of the moments of inertia of the structure. Strict adherence with the conditions for similitude dictate that the cross-sectional areas of the model and structure be proportional, but no serious error results if this is not done, for generally the deformations due to shear and direct stress are negligible.

The selection of the material for the model depends primarily upon the shape of the structure. If the members are of constant moment of inertia, the brass wire model is excellent. The individual brass wires can be soldered together and bent to the desired shape and the model is ready for service. If the moments of inertia vary, this can be approximated by joining together short lengths of wires of appropriate diameter. Selection of the wire sizes should be based on the measured modulus of elasticity and moment of inertia and not on relative diameters alone.

If the structure involves curved members and variable moments of inertia, a planar model cut from seasoned sheets of cellulose acetate is best. This plastic is available in sizes 20" × 50" of uniform thickness up to 1/8". Methylmethacrylate plastics sold as Lucite and Plexiglas are not as suitable because the thickness of the sheet varies considerably. The model can be cut as a unit from one sheet or individual members can be joined together with either acetone and similar solvents or a body cement made by dissolving shavings of the plastic in acetone. These joints should age 48 hours before they are sufficiently elastic. Both the pure acetone and the body cement must be used sparingly as the solvent is rapidly absorbed by the plastic and diffused throughout, lowering the modulus of elasticity in that area. For the same reason, the thickness of the model should not be increased by cementing together strips of plastic.

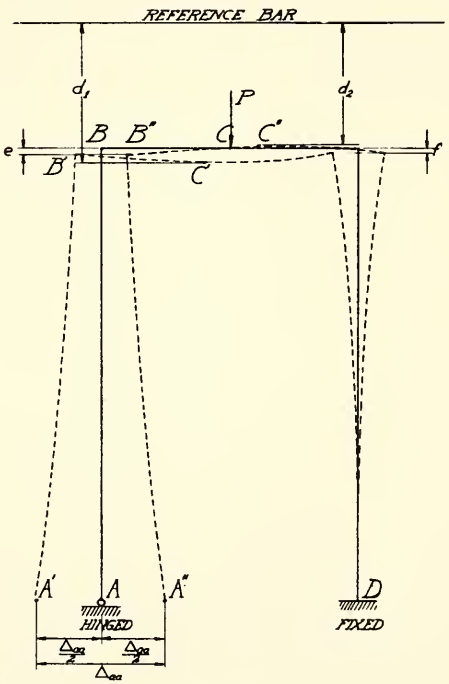


FIG. 2.

Cardboard of even the finest grade, although formerly extensively used, has not always proven satisfactory. The paper fibers are so oriented that the material has a grain and is therefore not isotropic. This can be detected by flexing a piece in the hands. The errors re-

sulting from this variation in the cardboard may not be large but are best avoided.

The care with which the cellulose acetate model is finished to exact dimensions depends upon the degree of accuracy required of the analysis. For preliminary studies the model could be used as carefully sawed on a band saw, eliminating all finishing except those areas where the dimensions are relatively small. If the greatest possible precision is desired, such as where a comparison with an analytical analysis is to be made, the model must be brought to an exact size.

Elimination of Geometric Errors Caused by Large Displacements

Mathematical examination of Eq. (1) discloses that only when the displacement Δ_{aa} is so small as to not change the geometric position of the model will the ratio Δ_{ca}/Δ_{aa} be identical to that of the structure when K equals unity. When the deflected position of the model and its prototype are not similar, geometric errors enter into the measured deflection Δ_{ca}. These errors can, however, be eliminated for all practical purposes so that large displacements can be employed to good advantage.

The principal source of geometric error is illustrated in its simplest form in Fig. 2. Here is shown, in exaggerated fashion, the elastic axis of a simple frame with rigid corners in its normal position and when equal but opposite displacements are induced at point A. When point A is displaced to the position A', the side member assumes the curved sloping position $A'B'$ and point B moves a distance "e" to point B'. The effect of this is to *increase* the apparent elastic deflection of point C by some portion of the movement "e" at point B. If A is now displaced to the right to point A'' such that AA' and AA'' are equal, the side member again shortens, taking the curved sloping position $A''B''$. The effect of this is to *decrease* the apparent elastic deflection of point C by an identical portion of the movement "e" at point B. If now the deflection of point C be taken as d_1-d_2 the effect of the movement "e" is eliminated and the deflection measured is that due to the elastic deformations alone. Here the deflection of point C is taken as the difference in two readings taken on a scale sliding along a reference bar and oriented parallel to the load which would act at point C.

Obviously this hypothesis is sound only when the proportional limit of the material has not been exceeded. When there is uncertainty, it is quite simple to vary the induced displacement Δ_{aa} and determine if the deflection Δ_{ca} varies proportionately.

Proof of accuracy of this hypothesis has been obtained both by analytical calculations and experimentally.

Functions of the Deformeter Gage

The displacements which must be introduced to determine the reaction forces shown acting on the frame of Fig. 5a are shown in Fig. 5b to 5f. This frame is hinged at A and fixed at B. The function of the deformeter apparatus is to afford a means to control these displacements and measure the deflection.

It should be noted that in Fig. 5b the displacement Δ_1 must be introduced in equal amounts from the normal position, allowing rotation but no vertical displacement. The deflection Δ_2 at the point of load must be the component parallel to the direction of load. The fact that the horizontal reaction H_A acts to the right (Fig. 5b) is established by observing that in Fig. 5b, as A is displaced to the right, point C moves in a direction opposite to that of the load which acts at this point on the prototype.

Note that in Fig. 5c the model must be allowed to rotate at A as the displacement Δ_3 is introduced, but in Fig. 5f no rotation can be permitted. The deflection of the model (Fig. 5f) must be entirely the effect produced when V_B, the force being sought, alone displaces the

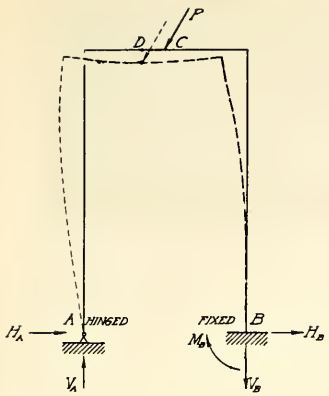


FIG. 5a.

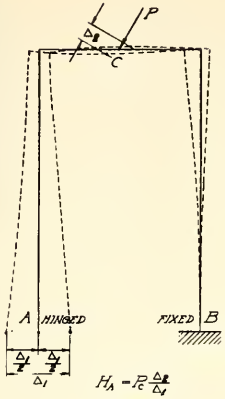


FIG. 5b.

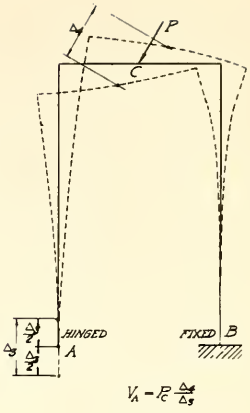


FIG. 5c.

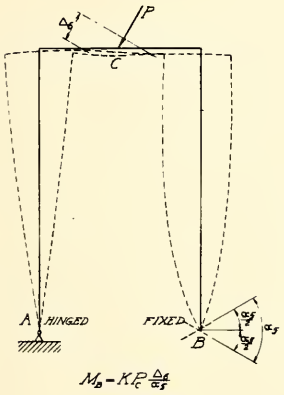


FIG. 5d.

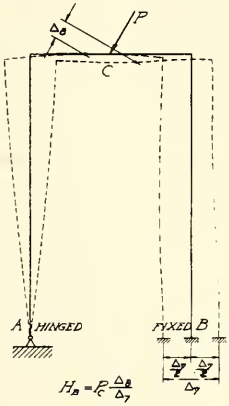


FIG. 5e.

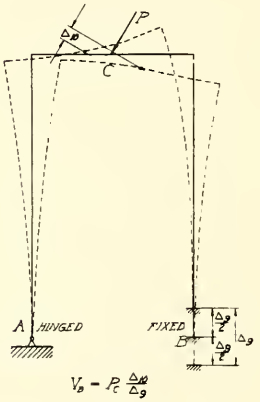


FIG. 5f.

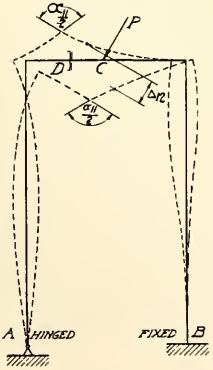


FIG. 5g.

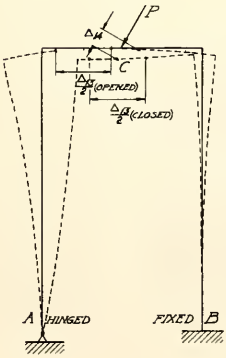


FIG. 5h.

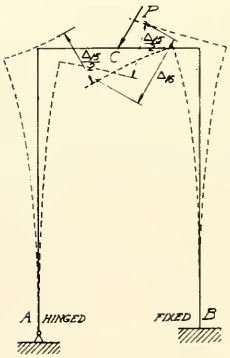


FIG. 5i.

model, but all other forces are simultaneously acting so that point B neither rotates nor displaces horizontally.

The moment M_B (Fig. 5a) is found by rotating point B as shown in Fig. 5d. It is observed that if this rotation is introduced in a clockwise direction, point C moves against the load, establishing the clockwise direction of the moment M_B shown in Fig. 5a. It should be further observed that if identical rotations α_5 were introduced into both the model and its prototype, which is K times larger, the deflection of point C on the prototype would be $K\Delta_6$, defined in Fig. 5d.

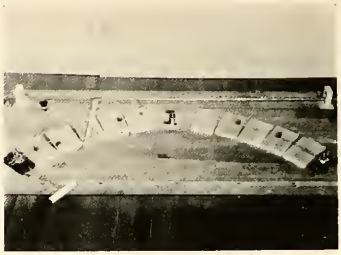
The shear, thrust, and moment at internal points are best determined directly with gages rather than by use of the experimentally determined reactions. Figs. 5g to 5i show the displacements which must be introduced and the deflections which are measured. The model must be cut to introduce these displacements.

The structural designer frequently must deal with moving loads and for this needs influence lines. These are obtained with the model by measuring the deflection of the several positions of the load when gage displacements are introduced.

DESCRIPTION OF DEFORMETER APPARATUS

Fig. 6 is a photograph of the deformeter apparatus and a celluloid model of an arch rib. The model is supported on steel balls to eliminate friction. Through the model may be seen small hollow curtain rings surrounding the steel balls, which serve to keep the balls under control. Small weights rest on the model over the balls to prevent buckling.

The deflection of the load points is measured by a scale oriented in the direction of the loads acting on the prototype. This scale, graduated 100 divisions per inch, is easily read with a watchmaker's loupe to an accuracy of $\pm 0.002''$.



The scale slides along a reference bar, permitting the measurement of the deflection of all load points at one setting of a gage. Each load point is marked by a cross cut on a small piece of white celluloid and filled with India ink.

Internal Deformeter Gage

Two types of gages are employed: an internal deformeter and a reaction gage. The details of an internal deformeter are shown in Fig. 9. This gage consists of a base plate of transparent material containing a pattern of accurately located holes for holding bars A and B in a displaced position relative to each other. The gage is attached to the model, normal to the axis on which the thrust acts, and is supported upon small steel balls. The model is clamped with straps to the two bars, which are pinned to the base plate in a normal position with a clear opening between them. The portion of the model between the two bars is then removed by cutting with a small saw blade. Four pins inserted through bars A and B into the normal holes of the base plate (Fig. 9b) hold the model in the normal position. Thereafter, whenever these pins are so placed, the model resumes the original position it had before the section was removed. When attaching the gage to the model, its orientation is simplified by the use of a template (Fig. 9c), for transferring to the model the location of the drilled holes through which the clamping screws are inserted.

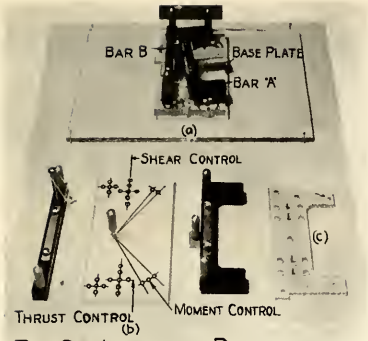


FIG. 9. INTERNAL DEFORMETER

Moment displacements are produced by removing the pins in bar *A*, placing one of them through the rotation ear and a second pin through the end of the U-shaped bar *A* into the base plate. This operation is then reversed, producing a rotation equally divided in both directions from the normal. The gage in Fig. 9a shows this operation.

Thrust displacements are produced by increasing and then decreasing the distance between bars *A* and *B*, maintaining the deformation by inserting the pins into the proper holes in the base plate. No lateral displacement of the bars relative to each other is permitted during this operation.

Shear displacements are produced by sliding the bars parallel with each other but at the same distance apart that they were in the normal position, and maintaining this displaced position by means of the pins (and holes in the base plate).

The pattern of holes in the base plate permits thrust and shear displacements of 0.400″, 0.500″, or 0.900″ and moment displacements of 0.050, 0.100, 0.200, 0.300 radians. The pattern is so simple that only by extreme carelessness could an operator actually use one displacement and think he was using another. The range of displacement enables the operator to vary the displacement to suit the model.

Reaction Gage

The details of a reaction gage are shown in Fig. 10. Essentially it consists of a control plate *D* and a movable bar *C* to which the model is attached with a clamping strap. The gage is placed on the board by first locating the aligning bar and then sliding the gage along it to obtain an exact setting. All tedious orientation of the gage is thereby eliminated. The gage is held fast to the aligning bar with a set screw and can also be fastened firmly to the drawing board.

Bar *C* can be moved parallel to the aligning bar, effecting a shear displacement in the model of 0.500″, 1.000″, 1.500″, or 2.000″. If the structure is hinged at the support, this is simulated in the model by placing one pin *E* through a sleeve *F* in bar *C*. This holds the bar in the displaced position but allows it to freely rotate. To eliminate friction, the supporting pads *G* are swung clear of bar *C* and the bar is supported on steel ball bearings which roll freely in the ball race of plate D, Fig. 10b. When an unusually flexible and slender hinged end model is used, the gages are operated as though the structure is fixed ended, as later described, in which case the model is not strapped to bar *C* but is allowed to freely rotate about sleeve *F* (Fig. 10c), or a smaller pin which can be placed in bar *C*.

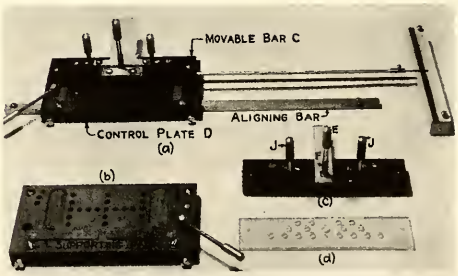


FIG. 10 REACTION GAGE

If the structure has a "fixed end," supporting pads are swung under bar *C*, raising it off the ball bearings into the position shown in Fig. 10a. When bar *C* is moved to effect the shear displacement, rotation is prevented by inserting one pin *E* placed through sleeve *F*, and a second pin *J* placed in the extreme left or right shear hole. Actually, a very negligible amount of rotation may take place because some clearance must be provided between pin *E* and sleeve *F* to avoid friction when the models are hinged ended. This rotation can be eliminated by inserting a third pin between the two pins previously placed, although this is unnecessary except when the model is unusually stiff. Reference to Fig. 10 shows the simple means of effecting these displacements.

Thrust displacements are effected in a manner similar to the shear displacement, except that the range is limited to a total movement of either 0.500″ or 1.000″ in the gage shown. However, gages more recently built have an additional thrust displacement of 1.500″. One pin is used in sleeve *F* if the model is "hinged," two pins if "fixed ended" with the pads under bar *C* and omitting pin *E* in sleeve *F* unless the model is very rigid.

Moment displacements are effected by inserting pin *E* through sleeve *F* and rotating bar *C*. Bar *C* is held in a rotated position either by inserting pin *H* (Fig. 10a) into an arrangement of holes producing a movement of 0.050, 0.100, 0.200, 0.300 radians or using the propulsion screws to obtain any amount of rotation up to 0.400 radians. If the latter method is used, the rotation is measured on a 100 divisions per inch scale located ten inches from the center of rotation. This scale and pointer are removable from the body of the gage and are only attached when the rotation has to be measured. No matter how stiff or flexible the model, a suitable amount of rotation can be introduced.

A template such as shown in Fig. 10d, containing a duplicate of the hole pattern of bar *C*, is useful for drilling the clamping screw holes in the base of the model so that it can be readily attached to bar *C*. Brass wire models are attached in a manner similar to the celluloid models, except that small flat plates must be soldered to the wire to insure firm attachment of the gages.

The reaction gages are designed so they can be adapted to the measurement of settlement stresses. A stiff arm can be clamped to bar *C*, all pins withdrawn, and with bar *C* supported on ball bearings, the force necessary to produce predetermined displacements can be measured.

The control holes in both gages must be located with great accuracy. Precision jigs are essential, and after the holes are drilled they must be carefully reamed to insure uniform fitting of the pins.

ANALYSIS OF A RIGID FRAME HIGHWAY BRIDGE

As an illustration of the use of the apparatus, an analysis of one of the steel rigid frames of the lake front ramp of the Main Avenue Bridge in Cleveland, Ohio, is presented. The details of the north frame are shown in Fig. 11. The dimensions of the model of this frame and the scale factor *K* are given in Fig. 12. The three-girder and two-column segments were cut from sheet celluloid 0.100″ thick and then cemented together at piers *B* and *C* (Fig. 12) to form the model. The column segments and the narrow portions of the girders were hand-filed, the remainder left as cut on the band saw. Reaction gages were placed at supports *E* and *F*. The simple supports at *A* and *D* were simulated by circular discs cemented to the girder and free to rotate or slide between fixed guides. An internal deformeter was inserted in the model successively at the location of each

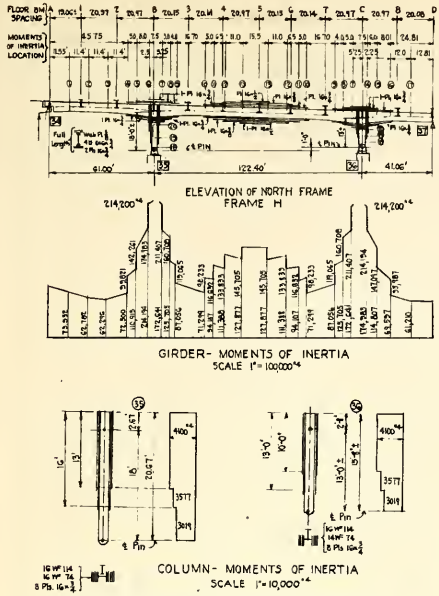
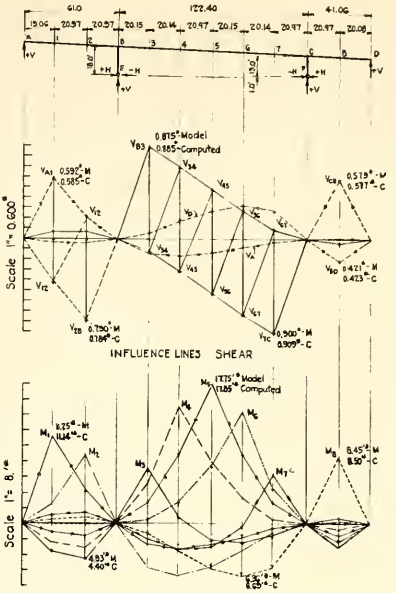


FIG.-11 DIMENSIONS OF STEEL PROTOTYPE FRAME



INFLUENCE LINES FOR MOMENT AT FLOOR BEAMS
FIG.- 13 INFLUENCE LINES MOMENT AND SHEAR

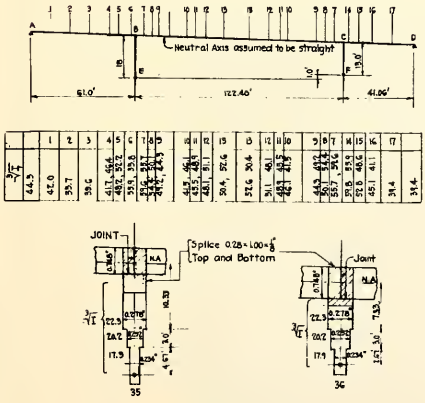

Neutral Axis assumed to be straight
JOINT
Splice 0.28 x L00 = 5/16
Top and Bottom
Joint
MODEL SCALES: LINEAR 1" = 40"
WIDTH 1" = 0.0125 x 5/16

FIG-12 DETAILS OF CELLULOID MODEL

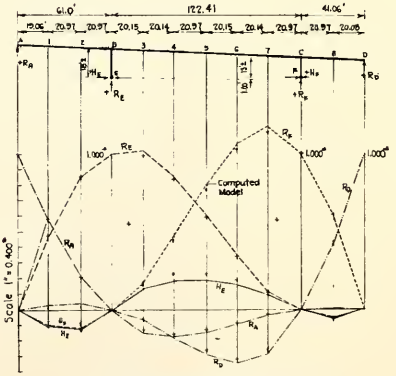


FIG-14 INFLUENCE LINES FOR REACTIONS
NORTH FRAME- H

of the eight intermediate floor beams. When this deformeter was removed, continuity of the model was restored by either clamping the severed ends together or by use of two clamping plates, or splicing with celluloid and an acetone solution.

Although all of the readings were taken by inexperienced students working in the Structural Models Laboratory at Lehigh University, and despite the fact that the model should have been more carefully prepared if a close check with calculated values was to be realized, nevertheless the agreement with the calculated values was excellent. The calculated influence lines for girder shear in the panels between floor beams and girder moment at the floor beams are given in Fig. 13. The corresponding values obtained with the model are shown by crosses plotted at the panel points. The calculated and model values are almost identical, as can be seen by comparing the data. The calculated influence lines for reactions appear in Fig. 14, where the model values are shown by crosses. Here the maximum disagreement occurs in the horizontal reaction. At pier 36, the girder web is 6'6" deep, or approximately half the length of the column. The calculated horizontal reaction, in this case, depends on the assumptions made relative to the stiffness of the joint. In the model a splice plate was placed at the juncture with the girders in an attempt to simulate the prototype more closely. There is no way of knowing if the calculated or the model values for the horizontal reactions are more nearly correct. These differences are of no significance, since the horizontal reaction has small effect on the moments in the girder as evidenced by the agreement of calculated and model moments in the girder at the floor beam (Fig. 13).

Use of Simple Cardboard Models to Obtain Design Data

by
PROF. WM. J. ENEY
Head, Dept. of Civil Eng. and Mechanics
Director, Fritz Engineering Laboratory
Lehigh University

The effect of such structural features as curved knees joining columns and roof members or crane runway brackets and stiffened columns on gabled frame action may be conveniently investigated with simple cardboard models. While carpet rubber, or cellulose acetate are more homogeneous than cardboard, the latter is easily cut out with a knife. The model can be laid on a piece of drawing paper, displacements induced and the position of the model recorded by pen points pricked through stations on the model. Figures 1 and 2 show two gabled frames whose models are given as figures 1a and 2a respectively. Arms at the supports held these points rigid or were used to control the displacements induced.

In figure 1a the right support was displaced 1.00 inch horizontally, 2.500 inches vertically and rotated through 0.155 radians. The corresponding deflections of one point "b" on the roof member are labelled Δ_{ba}. Similar deflections were recorded at many additional points along the frame. The influence lines were obtained from the equations

$$H = P\frac{\Delta_{ba}}{\Delta_{aa}} \qquad V = P\frac{\Delta_{ba}}{\Delta_{aa}} \qquad M = n\,P\,\frac{\Delta_{ba}}{\Delta_{aa}}$$

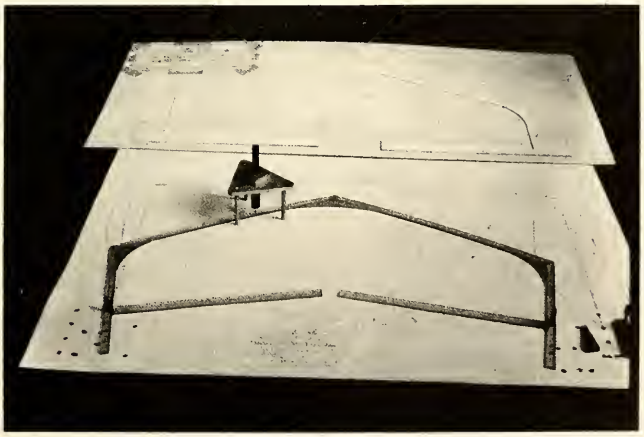
where $n = \dfrac{95.5 \times 12}{23.9} = 47.94$

Considerable time is saved by use of the diagrams given as figure 1c in which the displacement $\Delta_{aa} = 2.50$ inches is laid out parallel to the reaction and the vertical component 1.00 horizontally. The deflection Δ_{ba} is picked off with dividers and fitted vertically on the diagram. The divider is then pivoted and adjusted to the vertical reaction. This in turn is plotted on figure 1b and the influence areas are then readily planimetered.

Graphical diagrams can be used for the horizontal reaction figure 2d and the moment figure 2e.

Figures 1b and 2b show the influence lines of the curved knees and crane column.

The only equipment used is shown in Fig. 3.



FIXED FRAME
INFLUENCE LINES RIGHT REACTION
UNIT LOAD NORMAL TO FRAME
CARDBOARD MODEL

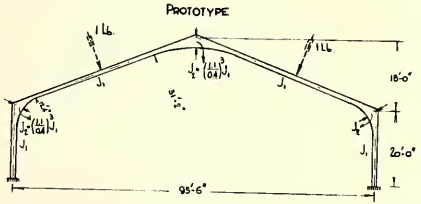


Fig. 1

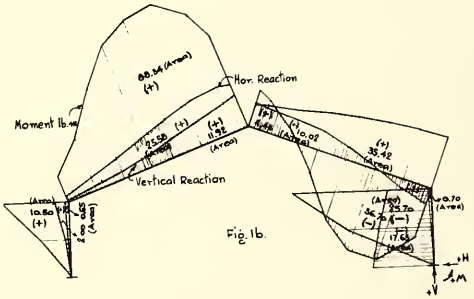


Fig. 1b.

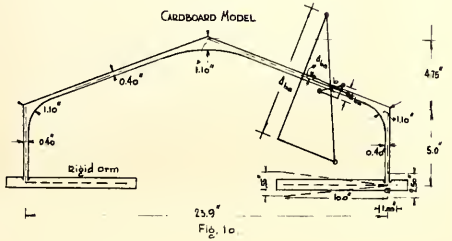


Fig. 1a

Scales:
Horizontal Comp. 1" = 1 lb.
Vertical Comp. 1" = 1 lb.
Moment 1" = 1 lb.in
o——o Represents the total displacement at "b" for each displacement induced at "a".

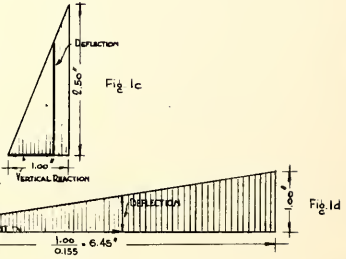
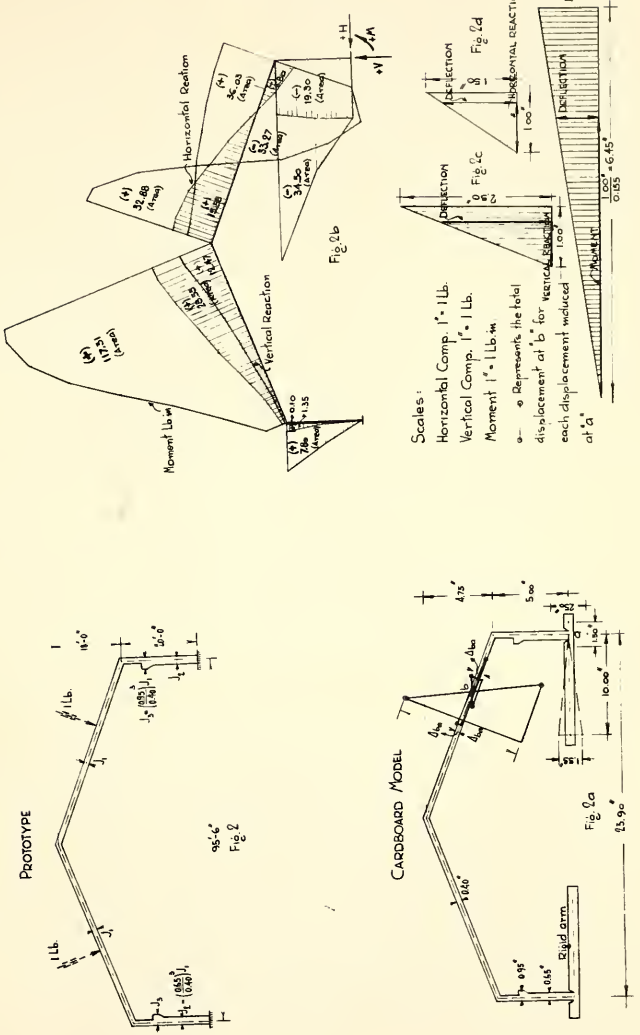


Fig. 1c
Fig. 1d

FIXED FRAME
INFLUENCE LINE RIGHT REACTION
UNIT LOAD NORMAL TO FRAME
CARDBOARD MODEL



Measured Stresses in a Two-Hinged Beam Arch

by
P. E. SONESON, B.S., M.S.E.*
Professor of Architectural Engineering
Purdue University

This article presents a comparison between the computed unit stresses of the design and the unit stresses from measured strains on a typical, two-hinged beam arch of Purdue University Field House at Lafayette, Indiana. The discussion shows how the design unit stresses, computed in the customary manner, differ from the unit stresses measured along the flanges at the leg and knee of a steel beam arch that supports a quarter-pitch roof and a spectators' balcony. This research was done in 1937 and part of 1938 as a joint project of the Purdue University Engineering Experiment Station and the School of Civil Engineering. Their permission to use this data has been given, and also by the American Society of Civil Engineers whose monthly publication, *Civil Engineering*, presented in April, 1942, a copyrighted article, by the writer, based on this data.

The investigation reported here includes these items:

1. Extensometer readings
   (a) before erection
   (b) after the building was completed
   (c) with a full audience on the balconies
2. Comparison of computed and measured unit stresses as a check on the method of design.

DESCRIPTION OF THE STRUCTURE

The Field House is 173 ft. by 303 ft. with side walls 52 ft. above grade and the peak of the end walls 84 ft. high. The masonry walls are trimmed with stone, and large areas of glass blocks admit light to the interior. The supporting structure is a frame of steel members, the most prominent being 12 beam arches of 170 ft. span and 23 ft. 1 inch on centers (see Figure 1.) The beam arch is a patented form of the two-hinged arch and is fabricated from rolled steel sections into a neat arch form with an I cross section of varying depth.

The turnbuckles of the tie rods were turned until the rod was fairly taut when jumped upon and the hinge bar in the bearing plate groove was not in contact with the side of the groove. The hinge is made of a steel bar, 1 inch by 2 inches, welded on the bottom of the arch leg base and fitting in a groove in the steel bearing plate of the concrete pier. This hinge is eccentric to the column center line, being about 8½ inches farther toward the outside flange of the leg; this produces a tendency to rotate the base of the column toward the inside of the building.

Offsetting this tendency are two things which restrain the free action of the arch:

1. For protection a mass of concrete was poured around each arch leg from top of pier to 6 inches above ground and 3 inches beyond the steel arch leg in all directions.
2. The exterior masonry walls (at both ends of the arch) are built so as to enclose the outer flange of the arch leg, with 8 inches of the wall thickness ex-

*Author's Note: The previous chapter was devoted to stress analysis with models. This one contains an unusual article on measured strains in a rigid frame structure with balcony from which the actual unit stresses were computed. While the article is based on research done in 1937 and 1938, it has considerable information of instructional value today. Because of that, it was selected for this Manual in the belief that it might serve as a text and reference for schools and for practicing engineers.

The original paper was far ahead of its time for general appreciation when projected and copyrighted by Prof. Soneson. Perhaps few persons noticed it then for few persons were rigid-framing. But I thought it too good to let it mold so I rang Prof. Soneson's bell. Amiable and cooperative, at Purdue since 1930 and an experienced structural designer, he smiled graciously at this book, outward bound for educational purposes. He rewrote his paper to fit io and released it with accompanying drawings. It is all yours now. Every one should see it. For this is the day of rigid frames, and most of Prof. Soneson's material definitely belongs to today.

Do not let the words "Beam Arch" in the title confuse you. American Bridge Co., who built this structure, used that term to distinguish their patented rigid frame from other types. But for family discussion and study, it is a rigid frame. M.P.K.

tending inside the face of the flange. Although a masonry wall is slightly elastic this tight connection undoubtedly prevents the arch from acting freely.

The balcony is supported by plate girders (with sloping upper flange) attached to the inner flange of each arch leg by a pair of vertical angles along the full depth of the connection and by a group of eight $1\frac{1}{4}''\phi$ bolts near the top of the balcony girder. This group of bolts is intended to resist the horizontal tendency of the upper part of the connection to separate from the flange of the arch leg. The steps of the balcony are of risers made of steel plates and angles and the treads are of 2 inch concrete placed over steel pan forms.

In alternate bays air conditioning machinery is located near the "knee" of the arch; this and steel runways produce small dead loads on the arch just above the junction of the arch leg and rafter portions of the span.

The arch legs were cambered with an inward slope and the peak of the arch cambered upward so that the dead load deformation would leave the arch legs vertical.

METHOD OF DESIGN

Design of the Purdue Field House was made in the office of Walter Scholer,* Architect, of Lafayette, Indiana. The computations were by M. E. Norman, then of the American Bridge Company, Chicago, and the consultant was the late C. E. Ellis, Professor Emeritus of Structural Engineering, Purdue University.

The original design was made by using a diagrammatic outline of the arch span and considering the moment of inertia of the cross section as constant.

The tie rod under the floor, connecting the two legs, sets the horizontal reactions at the same level and allows the vertical reactions to be statically determinate. The horizontal reaction was secured by applying $\Delta x = \frac{Myds}{EI} = 0$, in which Δx = horizontal movement of left hinge with respect to the right hinge, M = bending moment at a section, y = vertical distance from left hinge to a section, ds = an increment of length measured along the span outline, E = Young's modulus of elasticity, I = moment of inertia of a cross section. Values for shear, bending moment, and thrust at various sections then could be computed easily. Unit stress caused by bending was recognized as the dominant effect and a curve of the bending moment was made to assist in choosing a definite size of rolled WF steel beam for the arch in general. To this, flange plates were added on the legs, where additional resisting moment was required, and the web expanded at the peak and at both knees by curving the interior flange contour to make a graceful and stronger corner junction.

The original design was checked by dividing the chosen arch into segments and applying $\Delta x = \Sigma\frac{Myds}{EI} = 0$ to secure the value of the horizontal reaction for various loading of dead load, live load on balcony, and live load on the roof.

The terms are the same as previously explained; the center line of the arch rib was used as the arch contour and each cross section

*Now Walter Scholer & Associates, Architects

was cut at right angles to this contour of the arch rib centerline. The value of I was computed according to the area and depth of the cross section thus secured. Shears, bending moments and thrusts were calculated for several cross sections, and unit stresses in the flanges were computed to check the original design. The total unit stresses were taken as the sum of the axial and bending unit stresses; $s = P/A + Mc/I$.

The design procedure is for a free-standing, two-hinged arch but the plans and the construction show several items that depart from this simple condition. They are the following:

1. The exterior flange of the beam arch legs are embedded eight inches in the exterior masonry walls.
2. The hinge at the south end of the arch is 5′3″ lower than at the north end. The tie rod is horizontal and attaches to the south leg 5′3″ above the hinge.
3. A block of concrete is poured around the base of each leg to protect it from the soil; it extends 3″ beyond the steel and about 6″ above the ground level. This casing is larger at the south leg due to the lower base.
4. The hinge at each column base is eccentric to the centerline of the leg, being 8½″ farther toward the outside flange.

These indeterminate effects tend to nullify each other, especially items 1, 3 and 4; any additional bending effect due to item 2 will be relieved in part by the stretching of the tie rod and the restraint of the protective concrete covering. The allowable unit stresses were not reduced to allow a margin for stresses due to these items.

The stresses caused by temperature difference over the arch as a whole were found to be very small and are not included in the total stresses.

Since the foundations under the masonry walls are continuous it was considered that the weight of the masonry was transferred directly and was not supported on the exterior flange of the arch leg. In the computed stresses nothing is included for any vertical weight of the adjacent masonry.

MEASUREMENT OF STRAINS

The stations for taking extensometer readings were expected to be placed on the flanges at both ends of two arches in the region from

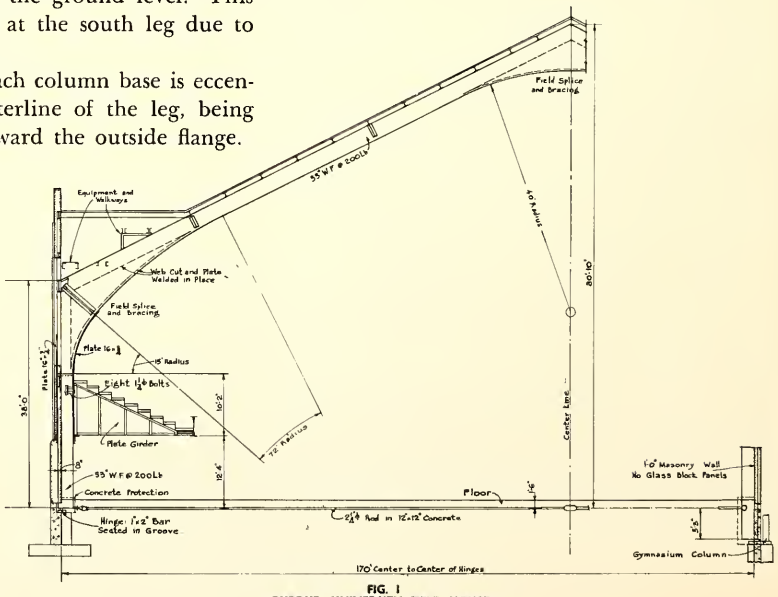


FIG. 1
PURDUE UNIVERSITY FIELD HOUSE

just below the balcony to the point where the inner curve of the knee meets the straight rafter portion; refer to Figure 2. The largest unit stresses were expected in this region and the varying cross section made computed stresses questionable. Due to congestion at the unloading area the stations on the second arch were incomplete; consequently, this data is for one arch only. Each station is a pair of small holes, twenty inches apart along the arch, made with an electric drill and No. 55 bit (0.052″ diameter); the hole was countersunk slightly with a larger bit.

Extensometer and temperature readings were taken as the steel lay on the ground; these constitute the zero load conditions to which later readings were compared. A standard bar, kept at constant temperature, was used as reference for the extensometer throughout the entire series of observations.

After the steel framing was in place two sets of wooden steps, previously designed and fabricated, were suspended from the roof beams in a position to give access to the extensometer stations for later observations (see view of interior).

The full dead load readings were taken with the same extensometer after the building was entirely completed. The temperature of the steel at each station was secured by taping thermometers to the steel surface for a period of at least fifteen minutes.

In a similar manner the extensometer readings for Dead+Balcony Live Loads were taken during basketball games when the audience filled the balcony. The audience prevented the use of the stations just above the top of the balcony (group marked C). In general the audience stayed in their seats and the impact effect, if any, was very small (this was verified by later measurements of the balcony deflection).

The original readings were taken on the steel

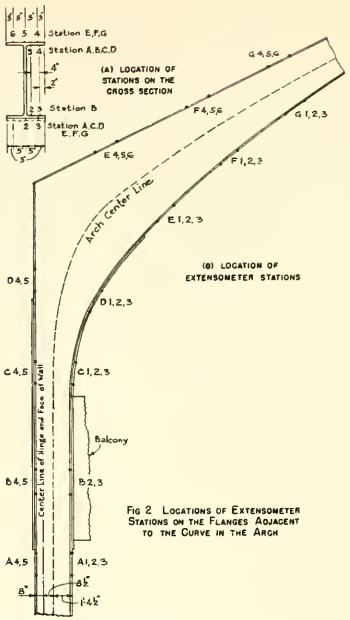


Fig 2 Locations of Extensometer Stations on the Flanges Adjacent to the Curve in the Arch

as it lay in the open, before erection, and the temperature varied from 30° to 35° F.; later readings were taken inside the finished building and the temperature ranged from 55° to 80° F. A basic temperature of 60° F. was selected and all readings were adjusted for the change in length of the steel within the 20 inch reach of the extensometer. The coefficient of expansion for steel was taken as 0.0000067, and the modulus of elasticity as 29000 kips per square inch. The instrument constant was 4.51.

The unit stresses were computed from the change between the adjusted extensometer readings for the different loadings:

$$\text{Unit stress} = \frac{\text{change}}{4.51 \times 20} \times 29000 \text{ kips per square inch.}$$

The measured unit stresses are the average of five series of observations at both ends of the arch under corresponding conditions of load.

COMPARISON OF MEASURED AND COMPUTED UNIT STRESSES

Dead Load:

In Figure 3 the computed and measured unit stresses are compared. It is easily noticed that the two ends of the arch do not yield identical results but that their general behavior is similar.

$$\text{Exterior flanges: } \frac{\text{measured}}{\text{computed}} = .63, \text{ average for north leg}$$
$$= .60, \text{ average for north rafter}$$
$$= .59, \text{ average for south leg}$$
$$= .44, \text{ average for south rafter}$$

On the interior flange the measured stresses are much nearer the computed value but no definite pattern is present; at station C the stresses are high and agree quite well. Erection stresses may account for some of the variation between similar points at the north and south ends. The most serious disagreement is at station A on the north end where the measured stress exceeds the computed by 20%; note that this is the largest measured stress at either end.

$$\text{Interior flanges: } \frac{\text{measured}}{\text{computed}} = .99, \text{ average for north leg}$$
$$= .86, \text{ average for north rafter}$$
$$= .98, \text{ average for south leg}$$
$$= .88, \text{ average for south rafter}$$

The difference between the comparative behavior of the outer and inner flanges is quite definite; the outer flange seems to be sort of "restrained" (stresses less than the computed) while at the inner flange the measured stresses more nearly agree with the computed values, being larger at some points and less at others. The writer feels that the masonry wall (around the outer flanges up to the "knee") is preventing the arch from behaving freely, as the design stresses expect it to do. If this is true the removal of the wall restraint would result in higher flange stresses, so that at several points on the inner flange the actual stresses would exceed the computed considerably.

From one point of view it might appear that some of the exterior wall weight should be considered as a load applied on the exterior flange of the arch legs. This would increase the compression stresses and serve to decrease slightly the gap between the measured and computed stresses in the exterior flange of the arch legs.

Dead Load and Balcony Live Load:

The computed stresses due to an audience on the balconies are the opposite of the dead load stresses, except for the stations under the balconies. The load on the balcony is seen as tending to rotate the arch legs inward and thereby to decrease the effect of the total dead load.

In Figure 4 the computed and measured unit stresses are compared. The measured stresses at Station C are not available because the presence of the audience prevented access to these stations.

As in the case of dead load the measured stresses along the exterior flanges are less than the computed but the relation between them has altered; none of these is high in amount or important in the design.

$$\text{Exterior flanges: } \frac{\text{measured}}{\text{computed}} = .76, \text{ average for north leg}$$
$$= .54, \text{ average for north rafter}$$
$$= .53, \text{ average for south leg}$$
$$= .40, \text{ average for south rafter}$$

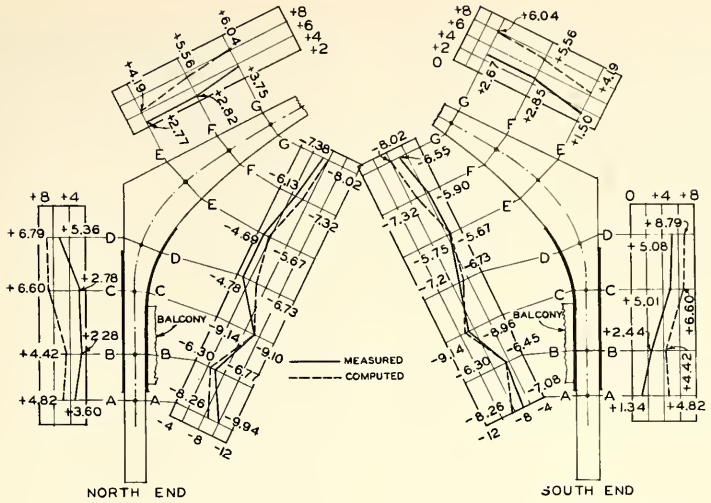


FIG. 3. COMPARISON OF MEASURED AND COMPUTED DEAD-LOAD STRESSES ONLY

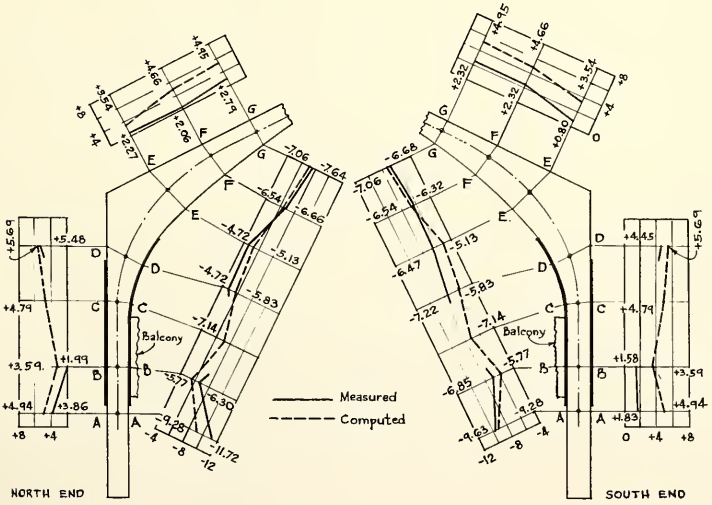


FIG. 4. COMPARISON OF MEASURED AND COMPUTED STRESSES FOR DEAD LOAD AND BALCONY LIVE LOAD

On the interior flange the measured stresses exceed the computed at eight out of twelve places, ranging from 81% to 126% of the latter. For this load condition the measured stresses are a larger percentage of the computed stresses than for dead load. It thus appears that the added live load is not causing the expected opposite stresses; instead of causing tension stresses on the interior flange above the balconies the audience load is causing more compression, that is, it is adding to the dead load stresses.

Interior flanges: $\frac{\text{measured}}{\text{computed}}$ =1.06, average for north leg
=1.01, average for north rafter
=1.16, average for south leg
=1.06, average for south rafter

These balcony live load stresses are not large but they indicate that the arch behavior under this load is not according to expectations. It is noted that at both ends, for stations above D on both flanges the balcony live load causes compression stresses, probably because the inward rotational effect of the balcony load tends to bend the knee, as a whole, into a more nearly straight line instead of bending each cross section like a beam.

These Dead + Balcony Live Load stresses bear out the same relations as noted under Dead Loads; that is, the measured stresses on the interior flange are a high percentage of the computed values while those on the exterior flanges are a lower percentage. The restraint of the exterior masonry wall is the most reasonable source of this condition and its removal would lead surely to higher stresses in both flanges. The interior flange stresses would then be still greater than the computed values, perhaps dangerously so at station A where the present 26% excess is found.

As noted under Dead Load, to consider part of the surrounding masonry as a load on the exterior flange of the arch leg would help to close the gap between the measured and computed unit stresses by causing additional computed compression stress in the arch legs. The relation between the masonry wall and the arch leg is indeterminate and too difficult for any analysis more serious than a guess.

No discussion regarding the actual location of the neutral axis, or the line of zero stress, is included because the serious restraint of the masonry wall and the absence of observations on the web make any rational statements impossible. The writer might surmise, from the higher stresses on the interior flange, that the line of load transmittal around the knee is inside the arch rib centerline. A careful and complete laboratory study would be required to locate the true neutral axis. In "Rigid Frames for Bridges and Buildings," published by the American Institute of Steel Construction, the author, Mr. Harold D. Hussey, states that research has brought out, among several important points, ". . . that the path of zero stress (and the neutral axis in bending) is quite close to the inner corner of the knee. It also has shown that the structural action outside of the knee follows closely the commonly accepted theory of arches."

CONCLUSIONS

From these measured strains the writer believes that the following general conclusions may be drawn:

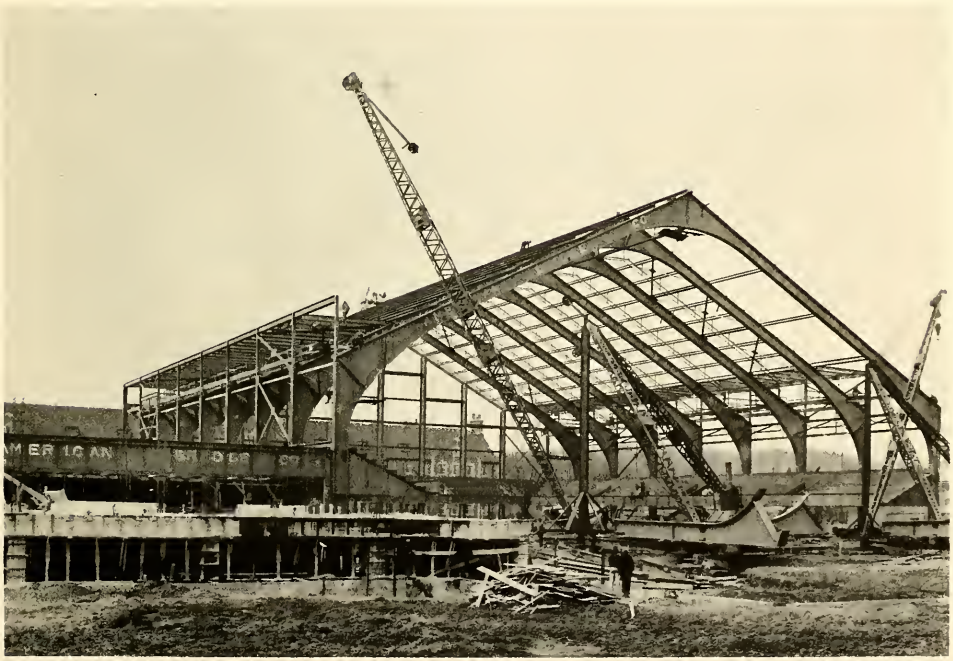
1. The general design procedure of considering the arch free from the several restraints of the adjacent construction does not lead to serious discrepancies between computed and actual stresses.

2. The exterior masonry wall built around the exterior flange of the arch "leg" serves to restrain the elastic action of the arch and thus to prevent the interior flange stresses from exceeding the computed values to a serious extent.

3. The inner flange stresses more nearly agree with the computed values than do the exterior flange stresses; if the arch were "free standing," as the design procedure usually considers it to be, the interior flange stresses might exceed the computed values considerably.

4. The balcony load effect tends to "straighten" the arch knee or bend it more nearly toward a straight line; this produces compression stresses in both flanges around the knee instead of tension in the interior flange and compression in the exterior flange.

5. The effect of roof loads is far greater than that of the balcony loads; the balcony live load stresses could be disregarded without danger because they are very small.



Erection of Beam Arches by American Bridge

PART II

89 Contents Part II
90 City Auditorium, Denver, Colo., 215 ft. span
Paul J. Foehl
94 Rigid Frame Bridge, Mount Pleasant, N.Y.
Arthur G. Hayden, Consulting Civil Engineer
96 Two-Span Welded Rigid Frame Bridge on Wilbur Cross Parkway, Connecticut
J. F. Willis, Engineer of Bridges & Structures
105 Ross Powerhouse, City of Seattle, Wash., Stepped Column Rigid Frames
Dr. John L. Savage, Consulting Engineer
115 Allen County War Memorial Coliseum, Fort Wayne, Ind., 224 ft. span
Ralph E. Coblentz, Chief Engineer
123 Main Ave. Bridge, Cleveland, O.
Fred L. Plummer
126 Transit Shed, Long Beach, Cal., 200 ft. span
R. R. Shoemaker, Chief Harbor Engineer
131 Philips Electrical Works, Eindhoven, Holland, 3 hinged frames, 132 ft. span
J. R. Bouten, Director and Architect
135 Boston Navy Yard, Extension to Structural Shop, 102 ft. span
J. F. Jelley, Rear Admiral, Chief, Bureau of Yards & Docks
A. Amirikian, Head Designing Engineer, Bureau of Yards & Docks
141 Oregon State College Pavilion, Corvallis, Ore., 222 ft. span
Prof. Glenn W. Holcomb, Oregon State College
Miles K. Cooper & Rowland S. Rose, Structural Engineers
151 An Architect Speaks for Welded Rigid Frames
Prof. George M. McConkey, University of Michigan
154 N. Y. Subways, 4 span welded Rigid Frames
Lincoln Arc Welding Foundation
156 Photos of Varied Rigid Frame Structures
American Bridge Division, U. S. Steel Co.
164 Miscellaneous Photos of Rigid Frame Structures

215 Foot Steel Rigid Frames for the City of Denver Auditorium

by

PAUL J. FOEHL*
Chief Engineer
Midwest Steel & Iron Works Co.,
Denver, Colo.

The City of Denver, Colorado, decided in 1940 to build a 221-foot by 267-foot annex adjoining their city auditorium. The architect for this structure, Mr. G. M. Musick, liked the idea of employing rigid frames for the roof support. That such a bold structure (for its day) was entirely feasible was concurred by the writer and Mr. R. M. Ross, who were associated at that time for the purpose of preparing the structural design for this building. The writer subsequently designed the rigid frames, and construction was started in the spring of 1941.

This annex is an arena-type building, having a central floor area completely surrounded by a balcony seating structure. The exterior walls are brick, the lower portions being faced with terra cotta. The roof consists of precast concrete tiles supported on steel beam purlins spaced 8 feet apart, which in turn are supported on twelve steel rigid frames spaced 20'-4" apart. It was recognized by the engineers that a somewhat larger spacing of the rigid frames would have been desirable, but the architect preferred to maintain the same spacing as that for the columns in the adjoining auditorium building so that connecting doors could be located in any bay.

The rigid-frame span is 214'-6" from center to center of supporting pins, certainly one of the largest rigid frame spans at that time. The total rise from center of pins to the top of the crown is 93'-10½". The structure, however, was detailed and fabricated with a built-in camber, which increased the height at the center 5½" and decreased the span from corner to corner of haunches 2¾", bringing about the result that the legs of the frame lean inward 1⅜" while in the fabricated position. The height of the legs is 66'-9½" from the pin to the outer corner of the haunch. The legs taper from an over-all depth of 2'-1⅞" at the pin to 6'-1⅞" at the haunch. The remainder of the rigid frame has a uniform depth of 6'-1⅞" over-all, except for the increase caused by the curved inner flanges at the haunches and at the crown. The radius of the inner flange at the haunch is ten feet. This is somewhat smaller than subsequently recommended by the American Institute of Steel Construction in its publication "Single Span Rigid Frames in Steel" by John D. Griffiths (1948), but the writer had to rely mainly on his own judgment at that time, and in doing so provided five stiffeners at each haunch. The radius of the inner flange at the crown is 68 feet. While this produces a pleasing esthetic effect it also increases the depth and strength of the section so that a hole 2½ feet wide and 4½ feet high could be placed in the web at the center of the span. This is required in order to allow passage along the catwalks supported between the frames at their crowns. The catwalks have a steel grating floor and handrails on each side. There are gates in these handrails at the approximate center of each bay, and a "bosun's chair" can be attached to a small I trolley beam in each bay. A portable hand winch is used to lower

*Author's Note: In 1940—when Paul J. Foehl was commissioned to design these unusually large span frames—not many steel rigid frame jobs had been built. There were practically none that size to serve as guides and there was little design data of practical value. To undertake this project took superior judgment, courage and initiative, all of which Paul Foehl possessed in abundance. Comparatively young then, he rose rapidly to a niche of professional prominence and since 1946 has been Chief Engineer of the Midwest Steel & Iron Works Co.

Although no longer new, this structure is presented here as indicative of the problems of a pioneering engineer and also as the barometer of a mind which is always moving forward—as I know personally. Mr. Foehl states in his paper that if he were to design the same structure today, he would probably do away with the hinges. He does not state—but I know personally that he is on top in his knowledge of welding and construction of welded rigid frames. M.P.K.

Front View (Note Holes at Center of Frames for Catwalk)

a man in the chair to any light fixtures for the purpose of servicing them.

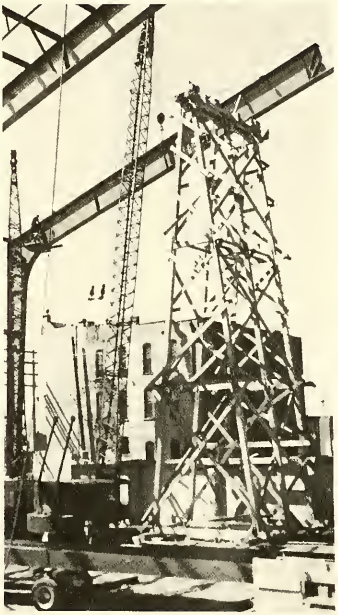
The legs of the rigid frame extend downward through the main floor to the basement floor level and are supported on 4½-inch steel pins, which in turn are supported on welded steel bases resting on wood pile and concrete foundations. The horizontal thrust is resisted by a single 2⅜-inch tie rod with 3-inch upset ends. These rods, somewhat larger than required in order to reduce their stretch under load, were brought tight by the use of two turnbuckles in each length. There is considerable doubt in the writer's mind at the time of this writing (1952) that he would use pin supports on a structure of this size again; instead, he would probably rest the structure directly on base plates, similar to ordinary column base plates.

The rigid frames are composed throughout of half-inch web plates and flanges built up of two 8 × 6 × ¾ angles and one 24 × 11/16 cover plate all riveted together. Each frame was erected in five pieces, using timber falsework and two cranes. Field splices occur at the upper tangent points of the haunches and at each tangent point of the bottom flange at the crown. Two frames were erected at a time. A leg section was set in place and held in its upright position with one crane while the second crane lifted the 78-foot rafter section, which weights 16½ tons and is the heaviest piece of

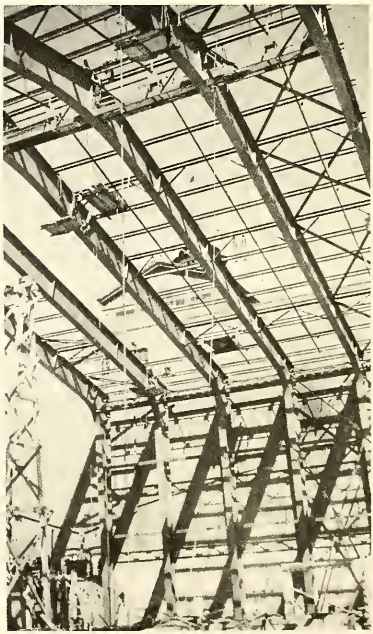
the structure. The upper end of this rafter section was supported on one of the timber towers; and when the lower end was spliced to the leg section, both cranes were then free to follow the same procedure on the second of the pair of rigid frames, and then to place the bracing and purlins between them. The cranes next moved over to erect the opposite ends of the frames in the same manner. The crown sections were erected last; then the timber towers were moved to the location of the next pair of rigid frames. Each rigid frame weighs about 67 tons, including base shoes and tie rod.

The structure stands free of the old building; a flexible joint is employed where the roof connects to the old building to allow for the inherent movement of rigid frames. At the opposite side of the building the new exterior brick wall is connected to the frames at the eave and at a point about half way between the eave and the top of the foundation wall in order to stabilize the wall against wind loads. The movement of the walls, induced by the frames, was not considered to be detrimental, since the brick work has concrete reinforcing rods imbedded in it. No noticeable damage to the wall has been observed. The steel balcony framing, however, is completely free standing.

The structural analysis was made by the method of moment distribution. The corner moment is about 4200 foot-kips, the center moment is about 3300 foot-kips, and the tie-rod load is about 71 kips for a uniform roof load of 1700 pounds per linear foot. This loading includes the weight of the rafters, but not of the legs of the rigid frame. A rough investigation of wind loads and unbalanced snow loads indicated such loadings are not consequential.



Erecting A Rafter



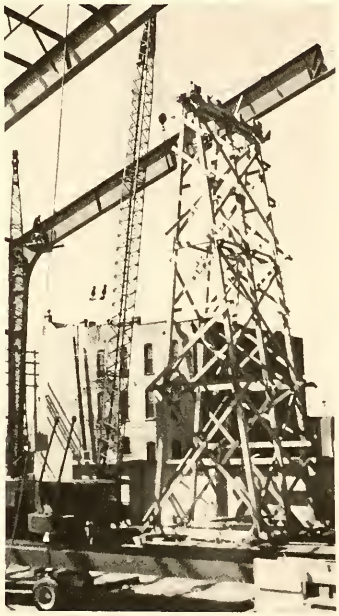
View of Bracing

BEVEL EDGE
OF F
CONT WELD

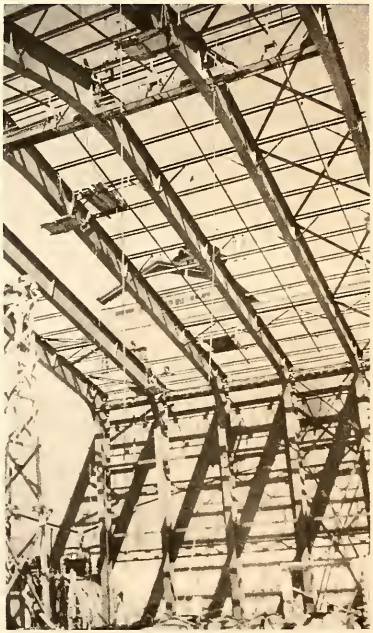
the structure. The upper end of this rafter section was supported on one of the timber towers; and when the lower end was spliced to the leg section, both cranes were then free to follow the same procedure on the second of the pair of rigid frames, and then to place the bracing and purlins between them. The cranes next moved over to erect the opposite ends of the frames in the same manner. The crown sections were erected last; then the timber towers were moved to the location of the next pair of rigid frames. Each rigid frame weighs about 67 tons, including base shoes and tie rod.

The structure stands free of the old building; a flexible joint is employed where the roof connects to the old building to allow for the inherent movement of rigid frames. At the opposite side of the building the new exterior brick wall is connected to the frames at the eave and at a point about half way between the eave and the top of the foundation wall in order to stabilize the wall against wind loads. The movement of the walls, induced by the frames, was not considered to be detrimental, since the brick work has concrete reinforcing rods imbedded in it. No noticeable damage to the wall has been observed. The steel balcony framing, however, is completely free standing.

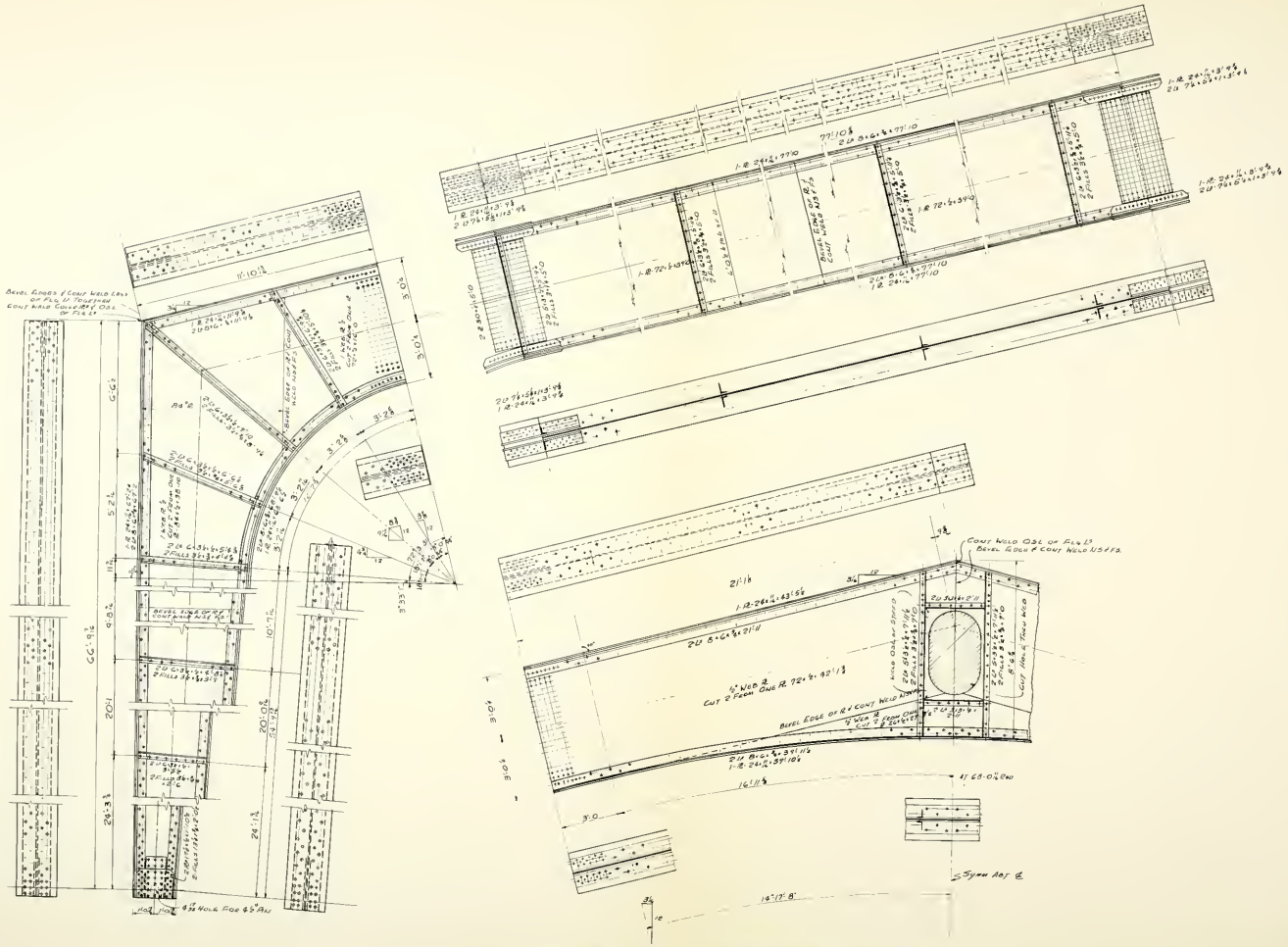
The structural analysis was made by the method of moment distribution. The corner moment is about 4200 foot-kips, the center moment is about 3300 foot-kips, and the tie-rod load is about 71 kips for a uniform roof load of 1700 pounds per linear foot. This loading includes the weight of the rafters, but not of the legs of the rigid frame. A rough investigation of wind loads and unbalanced snow loads indicated such loadings are not consequential.



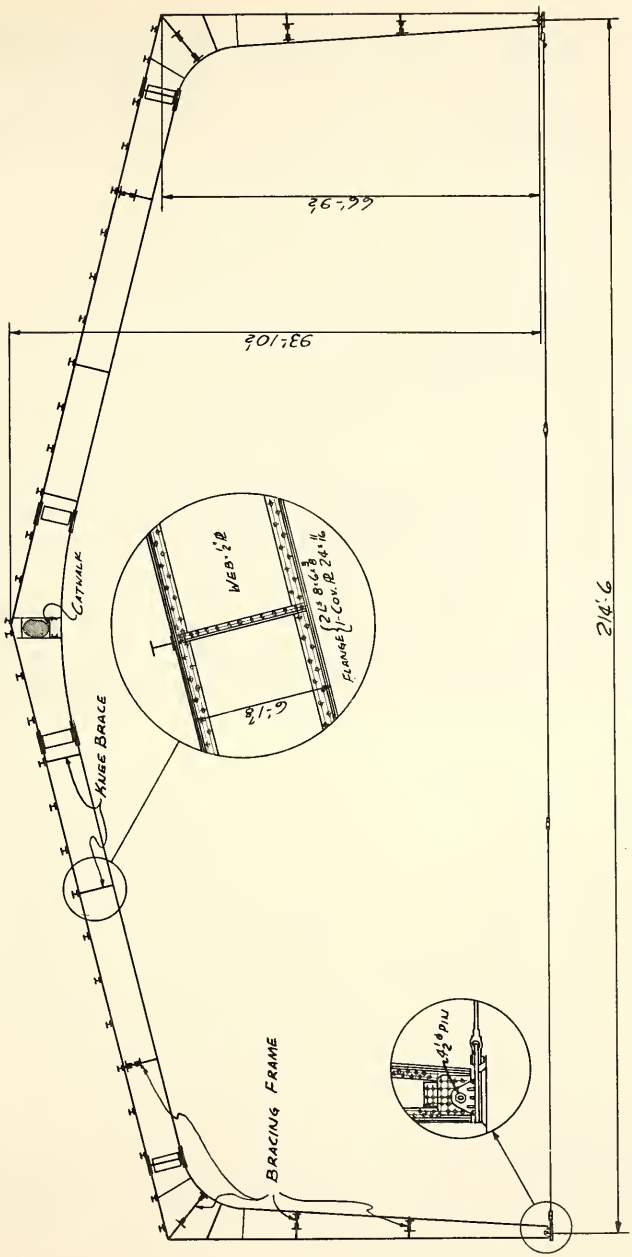
Erecting A Rafter



View of Bracing



DETAILS—CITY AUDITORIUM, DENVER, COLO.



CROSS SECTION-CITY AUDITORIUM, DENVER

Rigid Frame Bridge, Mount Pleasant, New York (built 1928)

DESIGNED BY ARTHUR G. HAYDEN
(Formerly Designing Engineer, Westchester County Park Commission)
Consulting Civil Engineer
St. Michaels, Maryland

(Notes from Author's Personal Diary)

He who blesses most is blest
And God and man shall own his worth
Who toils to leave as his bequest
An added beauty to the earth.
Whittier

In 1929 this steel rigid frame bridge was awarded first prize by the American Institute of Steel Construction as the best short-span steel bridge completed in the United States during that year. I visited it but not to admire its beauty. I also visited the larger Palmer Avenue Bridge at Yonkers in 1931, span 115 ft., and others to study the knees. My sole purpose was acquaintanceship. I talked quietly to each knee. The first one remained completely silent. But I knew that it was performing unlike the horizontal girder although I could not quite translate its behavior into words.

It was a mystery to me and so was I to the knee. But later the stiffeners began to talk, their voices in unison. A pattern of internal forces in action unfolded and I learned that the voice of the knee is the same in any language once you learns its behavior.

Compression from radial forces and bending, a terrific crushing power around the inner bend, a tendency for the web to buckle—the stiffeners staunch in resistance. The web trying to shear and again the stiffeners on guard.

Around the outside curve, a fierce pull and tension aiming to tear the horizontal girder away from the vertical leg. Then—the climax! Iron men at work in the compression area, they close ranks and bundle up; down comes the neutral axis leaving the tension partners above more space to stretch without "busting a gut." Team action is now completed, a perfect bulwark of internal defense against external forces; the pressure eases off beyond the knees, the steel tapering away from them with the decreased pressures and smaller bending moments.

What an epic show of strength, men against steel with steel bending to man's will! I had tried it. But Hayden put it over big. Earlier in 1924, when I was searching the earth for anything on knees, Prof. Swain, wizard of stress analysis of that era, thought that nothing was to be had. Then in 1926, when I went into No Man's land and designed a welded steel rigid frame, I was excoriated by my client, and flayed as a Consultant. Every one was afraid. No one knew knees. No one understood rigid frames. And that is why, in 1929, I thought that Hayden deserved a Distinguished Service Cross. Although I was not against the award of the beauty prize to him by the AISC and I could quote poet Whittier to acclaim Hayden's bequest to the earth of beauty in steel, my award would have been for bravery which belongs to the ages. Beauty in architecture is relative with time as in the other arts.

Today, a simple welded bridge, naked and streamlined, devoid of excess metal would be more natural and stand a better chance of wining a beauty prize than a riveted one with its mess of armor and rivets as for a battleship. Originality and creativeness leave legacies to posterity and Hayden's rigid frame bridges, successful and safe, stirred up the brave. Others ventured forth with rigid frame designs in the early thirties, still pioneers, but supported at least psychologically by Hayden's bridges.

He would have welded his knees, even in those days. Vision beyond the horizon!

He would have used square knees, which are more adaptable to bridges. But there were others in authority to satisfy.

So great was his philosophy of design that he was able to take the unknown in round knees, knead it in wisdom and reason, develop his own theory and above all convince his principals of its soundness.

I cherish his personal correspondence about knees, not all of which has been told here. But you do not need that to learn. His original structures record his accomplishments, as does his fine book "The Rigid Frame Bridge" (John Wiley & Sons). Because of this book, I have not shown any of Hayden's own design data as with other contributors. Every one should have it for study and inspiration. He belongs to the great. And you can always learn from the great.

M.P.K.

Two Span Welded Rigid Frame Bridge

Wilbur Cross Parkway, Town of Hamden, Connecticut

JOHN F. WILLIS*

Engineer of Bridges & Structures
State of Connecticut
State Highway Department

*Author's Note: The two-span all-welded highway bridge you see pictured here is a monument to simplicity, a symbol of the optimum in modern engineering and architectural design. Design data and drawings follow.

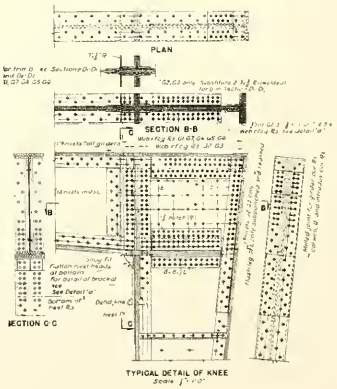
Planned in 1940 at a time when welding was scoffed at by many bridge engineers, when rigid frames were little known, this structure became just another milestone for John F. Willis in an already distinguished record of accomplishment in modern highway bridge design. Not revealed are the pioneering and creative spirit and the deep sense of design philosophy which made that possible. They are enshrined in more than forty-odd similar structures which Willis conceived and built, attesting to a record of progress in welded rigid frame bridge design not surpassed in number or quantity by any other state.

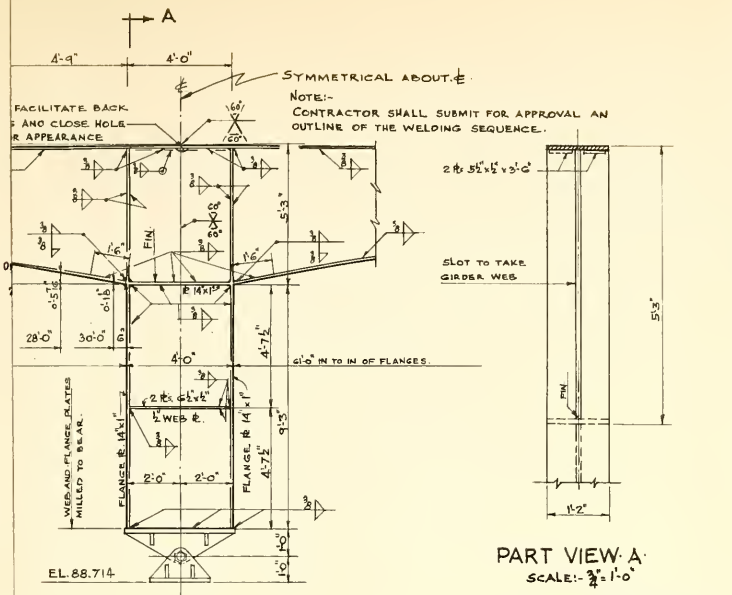
In the late twenties when Mr. Willis was in charge of bridge design, it was not beneath his pride and dignity to call in a noted engineering teacher to bring him and his staff up-to-date on design procedures. They were different in his college days. (Hayden's book later also contributed towards simplicity in design.)

That same attitude for progress was noted in his study and use of welded bridge construction long before others.

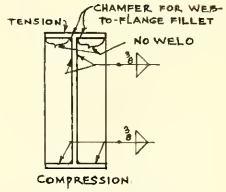
For those interested in a comparison between welded and riveted highway bridge design, photo herewith of the Cross-Bronx Expressway, New York City, designed in 1945 is an example, its rivets resembling a shell-pocked area with fortifications of splice and reinforcing plates. And this riveted structure, poor thing, simply can't help itself, even though it is the best designed bridge of its kind. Riveted construction cannot do otherwise.

Both the State of Connecticut and the engineering profession have benefited by the spirit and achievement of this pioneering and progressive highway bridge engineer—truly one among the greatest. M.P.K.

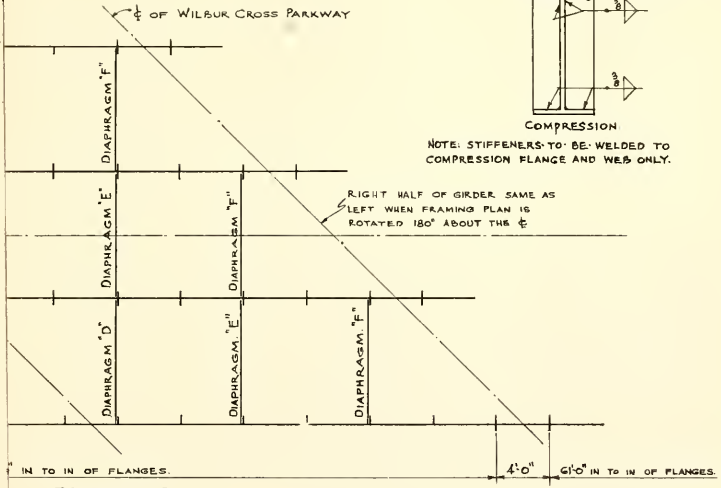




PART VIEW A
SCALE:- 3/4" = 1'-0"



NOTE: STIFFENERS TO BE WELDED TO COMPRESSION FLANGE AND WEB ONLY.


℄ OF WILBUR CROSS PARKWAY
DIAPHRAGM "F"
DIAPHRAGM "E"
DIAPHRAGM "F"
RIGHT HALF OF GIRDER SAME AS LEFT WHEN FRAMING PLAN IS ROTATED 180° ABOUT THE ℄
DIAPHRAGM "D"
DIAPHRAGM "E"
DIAPHRAGM "F"
" IN TO IN OF FLANGES.
4'-0"
61'-0" IN TO IN OF FLANGES.

FRAMING PLAN
SCALE:- 3/16" = 1'-0"

TE:-
L WELDING TO CONFORM TO THE LATEST
ECIFICATIONS OF THE AMERICAN
LDING SOCIETY.

VO-SPAN WELDED BRIDGE, CONNECTICUT

Two Span Welded Rigid Frame Bridge

Wilbur Cross Parkway, Town of Hamden, Connecticut

JOHN F. WILLIS*

Engineer of Bridges & Structures
State of Connecticut
State Highway Department

*Author's Note: The two-span all-welded highway bridge you see pictured here is a monument to simplicity, a symbol of the optimum in modern engineering and architectural design. Design data and drawings follow.

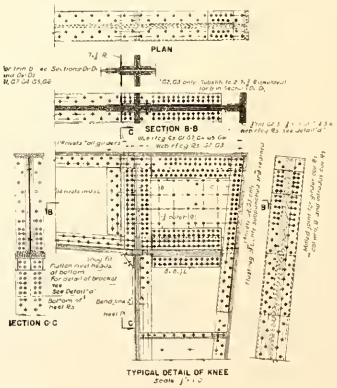
Planned in 1940 at a time when welding was scoffed at by many bridge engineers, when rigid frames were little known, this structure became just another milestone for John F. Willis in an already distinguished record of accomplishment in modern highway bridge design. Not revealed are the pioneering and creative spirit and the deep sense of design philosophy which made that possible. They are enshrined in more than forty-odd similar structures which Willis conceived and built, attesting to a record of progress in welded rigid frame bridge design not surpassed in number or quantity by any other state.

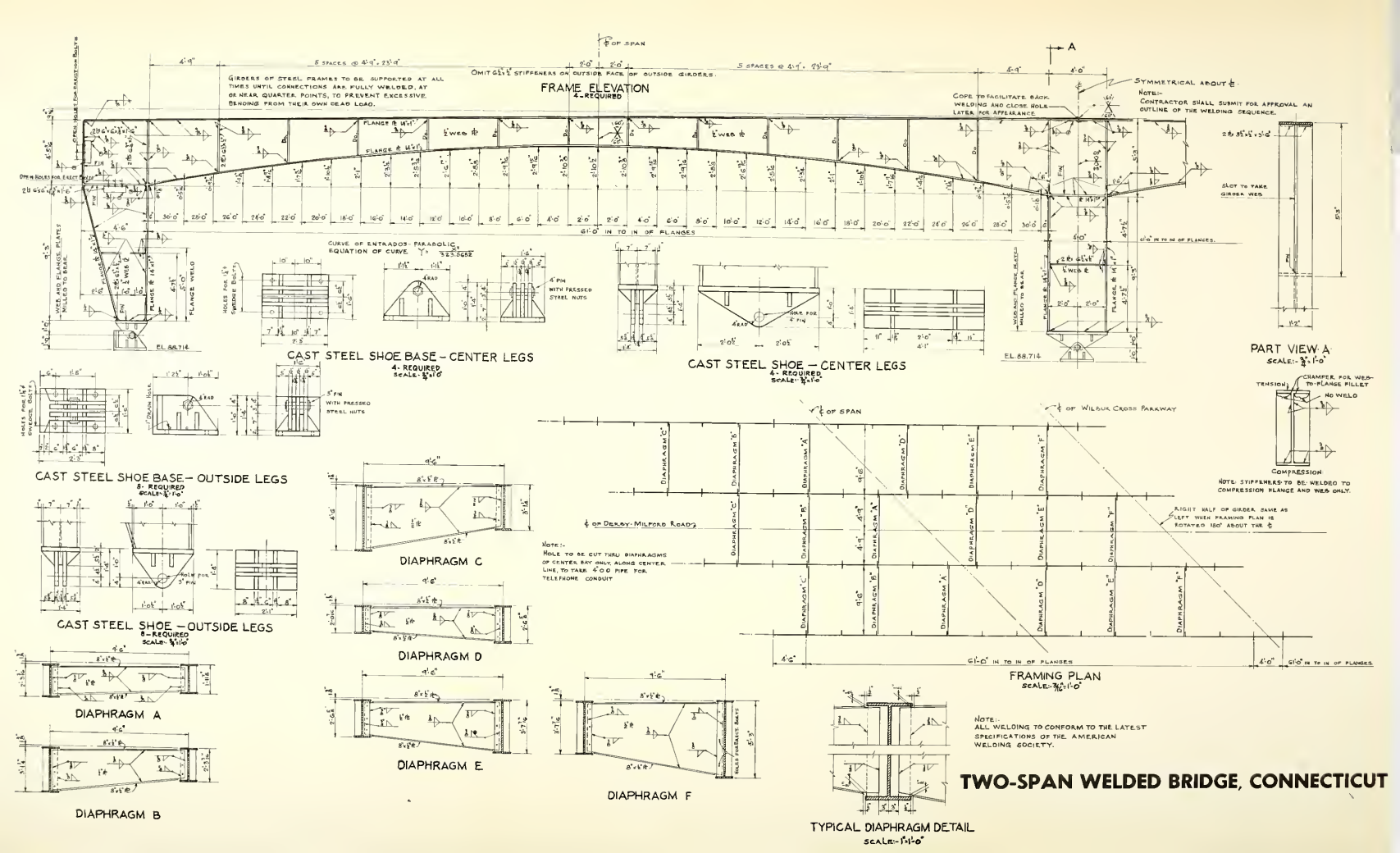
In the late twenties when Mr. Willis was in charge of bridge design, it was not beneath his pride and dignity to call in a noted engineering teacher to bring him and his staff up-to-date on design procedures. They were different in his college days. (Hayden's book later also contributed towards simplicity in design.)

That same attitude for progress was noted in his study and use of welded bridge construction long before others.

For those interested in a comparison between welded and riveted highway bridge design, photo herewith of the Cross-Bronx Expressway, New York City, designed in 1945 is an example, its rivets resembling a shell-pocked area with fortifications of splice and reinforcing plates. And this riveted structure, poor thing, simply can't help itself, even though it is the best designed bridge of its kind. Riveted construction cannot do otherwise.

Both the State of Connecticut and the engineering profession have benefited by the spirit and achievement of this pioneering and progressive highway bridge engineer—truly one among the greatest. M.P.K.




FRAME ELEVATION
4- REQUIRED
GIRDERS OF STEEL FRAMES TO BE SUPPORTED AT ALL TIMES UNTIL CONNECTIONS ARE FULLY WELDED, AT OR NEAR QUARTER POINTS, TO PREVENT EXCESSIVE BENDING FROM THEIR OWN DEAD LOAD.
OMIT 6½"x ½" STIFFENERS ON OUTSIDE FACE OF OUTSIDE GIRDERS.
5 SPACES @ 41'-9" = 208'-9"
8 SPACES @ 41'-9" = 334'-0"
61'-0" IN TO IN OF FLANGES
Note:- CONTRACTOR SHALL SUBMIT FOR APPROVAL AN OUTLINE OF THE WELDING SEQUENCE.
COPE TO FACILITATE BACK WELDING AND CLOSE HOLE LATER FOR APPEARANCE.
SYMMETRICAL ABOUT ℄
PART VIEW A
SCALE:- 3/4"=1'-0"
SLOT TO TAKE GIRDER WEB
CHAMFER FOR WEB-TO-FLANGE FILLET
NO WELD
TENSION
COMPRESSION
NOTE: STIFFENERS TO BE WELDED TO COMPRESSION FLANGE AND WEB ONLY.
CAST STEEL SHOE BASE - CENTER LEGS
4- REQUIRED
SCALE:- 3/4"=1'-0"
CAST STEEL SHOE - CENTER LEGS
4- REQUIRED
SCALE:- 3/4"=1'-0"
CAST STEEL SHOE BASE - OUTSIDE LEGS
8- REQUIRED
SCALE:- 3/4"=1'-0"
CAST STEEL SHOE - OUTSIDE LEGS
8- REQUIRED
SCALE:- 3/4"=1'-0"
℄ OF SPAN
℄ OF WILBUR CROSS PARKWAY
℄ OF DERBY-MILFORD ROAD
DIAPHRAGM A
DIAPHRAGM B
DIAPHRAGM C
DIAPHRAGM D
DIAPHRAGM E
DIAPHRAGM F
Note:- HOLE TO BE CUT THRU DIAPHRAGMS OF CENTER BAY ONLY ALONG CENTER LINE, TO TAKE 4'-0 PIPE FOR TELEPHONE CONDUIT
RIGHT HALF OF GIRDER SAME AS LEFT WHEN FRAMING PLAN IS ROTATED 180° ABOUT THE ℄
61'-0" IN TO IN OF FLANGES
FRAMING PLAN
SCALE:- 3/16"=1'-0"
TYPICAL DIAPHRAGM DETAIL
SCALE:- 1"=1'-0"
Note:- ALL WELDING TO CONFORM TO THE LATEST SPECIFICATIONS OF THE AMERICAN WELDING SOCIETY.

TWO-SPAN WELDED BRIDGE, CONNECTICUT

*Aut
here
engin

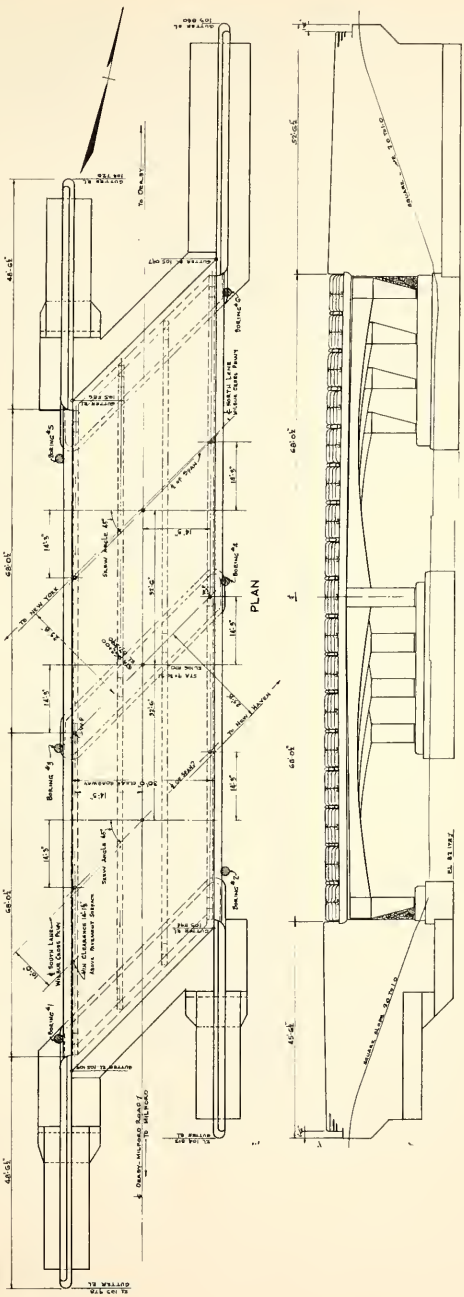
engin
anotl
accor
pione
made
tures
weldc
any

it w
teach
were
towar

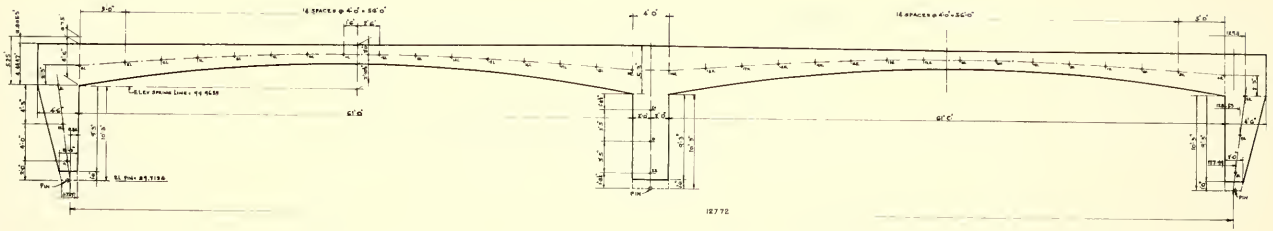
weldc

way
City,
area
struct
desig

benef
highw



GENERAL NOTES:—
Designed for 1920 Loading, Connecticut State Highway Department Specifications
Class C Loading, Impact Allowance 30%.
Girder Section of Steel Beams to be supported at all times at or near the Stiffeners, except until the connections are completed which shall be made as soon as practicable after the placing of the steel.
Load of 75 Tons (one wheel of Roller) may be placed on the bridge when the structural steel is fully erected before the back wall is poured.



PT.	A	$S=\frac{I}{C}$	MAX. POS. MOM. STRESSES						MAX. NEG. MOM. STRESSES					
			THRUST	MOM. FT.#	$\frac{P}{A}$	$\frac{M}{S}$	COMP.#/□"	TENS.#/□"	THRUST	MOM. FT.#	$\frac{P}{A}$	$\frac{M}{S}$	COMP.#/□"	TENS.#/□"
1	40.6	403.3							87550	191540	2160	5700	7850	3540
2	47.1	651.1							86960	594160	1850	10950	12800	9100
3	54.0	945.3							86220	1023780	1600	13000	14590	11400
4	54.0	945.3	1900	37330	40	470	510	440	90680	1057980	1680	13430	15110	11750
5	49.2	736.1	10950	212010	220	3460	3680	3230	90280	742090	1840	12100	13930	10260
6	46.0	606.2	17330	317440	380	6280	6660	5910	88170	521330	1920	10320	12240	8400
7	43.4	505.5	22800	393850	530	9350	9880	8820	84720	331410	1950	7870	9820	5910
8	41.4	430.9	28020	444520	680	12380	13060	11700	80450	174780	1940	4870	6810	2920
9	40.0	380.1	32320	466180	810	14720	15530	13910	71240	54250	1780	1710	3500	- 70
10	39.2	351.2	34280	456060	880	15580	16460	14710						
11	38.9	343.3	39610	419090	1020	14650	15670	13630						
12	39.3	356.3	40050	369090	1020	12430	13450	11410						
13	40.3	390.4	80450	314770	2000	9680	11670	7680						
14	41.8	446.9	75830	278470	1810	7480	9290	5660	32200	144330	770	3880	4650	3110
15	44.0	527.7	71240	214440	1620	4880	6500	3260	34170	333750	780	7600	8370	6810
16	46.7	635.2	66880	123710	1430	2340	3770	910	34150	559920	730	10580	11310	9850
17	50.1	772.7	62870	6630	1260	100	1360	- 1150	32410	821120	650	12750	13400	12100
18	54.0	944.3							32410	1116220	600	14190	14790	13590
19	58.5	1154.5							29370	1448810	500	15060	15560	14560
19.5	51.0	813.4	109430	356430	2150	5260	7410	3110	143830	356430	2820	5260	8080	2440

$$I=\frac{d(d^2+162d-324)+216}{24}$$

$$S=\frac{d(d^2+162d-324)+216}{12d}$$

$$A=27+\frac{d}{2}$$

MAX. SHEARS							
	1	2	3	4	5	6	7
DEAD LD.	34320	34320	34320	41140	33150	27500	21900
POS. LIVE LD.	38480	38480	38480	54860	50080	45650	41110
NEG. LIVE LD.	- 7320	- 7320	- 7320	- 4050	- 6190	- 8160	- 10510
60° T. RISE	17870	17870	17870	6880	6880	6880	6880
90° T. FALL	- 26800	- 26800	- 26800	- 10320	- 10320	- 10320	- 10320
MAX. POS. S.	90680			102890	90110	80040	69890
MAX. NEG. S.	+200						1070
WEB A.	13.62	20.11	27.0	27.0	22.18	18.99	16.39
UNIT SHEAR	6660	4510	3360	3810	4060	4220	4260
V.	3030	1900	1320	1500	1680	1800	1880
2-1/16" WELD=800	3790	2380	1650	1870	2090	2250	2350
	8	9	10	11	12	13	14
DEAD LD.	16320	10770	5230	- 300	- 5840	- 11380	- 16940
POS. LIVE LD.	36470	32630	28550	24370	20290	16470	13040
NEG. LIVE LD.	- 13470	- 16970	- 20800	- 24880	- 29060	- 33140	- 36080
60° T. RISE	6880	6880	6880	6880	6880	6880	6880
90° T. FALL	- 10320	- 10320	- 10320	- 10320	- 10320	- 10320	- 10320
MAX. POS. S.	59670	50280	40660	30950	21340	11980	2980
MAX. NEG. S.	- 7470	- 16530	- 25890	- 35510	- 45220	- 54840	- 63330
WEB A.	14.38	12.97	12.16	11.93	12.3	13.26	14.82
UNIT SHEAR	4150	3880	3350	2980	3680	4140	4270
V.	1870	1780	1550	1390	1700	1890	1920
2-1/16" WELD=800	2340	2230	1940	1730	2130	2370	2400
	15	16	17	18	19	19.5	
DEAD LD.	- 22520	- 28140	- 33790	- 39500	- 45270	0	
POS. LIVE LD.	10030	7430	5210	3330	1810	34770	
NEG. LIVE LD.	- 39660	- 43820	- 47270	- 50570	- 54500	- 34770	
60° T. RISE	6880	6880	6880	6880	6880	0	
90° T. FALL	- 10320	- 10320	- 10320	- 10320	- 10320	0	
MAX. POS. S.						34770	
MAX. NEG. S.	- 72500	- 82280	- 91390	- 100390	- 110090	- 34770	
WEB A.	16.97	19.71	23.05	26.98	32.0	20.0	
UNIT SHEAR	4270	4170	3970	3720	3500	1450	
V	1870	1770	1620	1460	1310	590	
2-1/16" WELD=800	2330	2210	2030	1830	1640	730	

$V = \frac{Vm}{I} = \frac{7V(d-1)}{I}$

SUMMARY

	M	H	V	M	H	V	M	H	V
	1			5			9		
DEAD LD.	- 73440	34320	48040	-229050	34320	33150	88440	34320	10770
POS. LIVE LD.	+ 15190	-7320	-3590	155570	3430	13570	231560	24800	2870
NEG. LIVE LD.	- 81440	38480	32630	-322720	38090	28520	45230	19050	10000
60° T. RISE	- 36670	17870	6880	-190330	17870	6880	- 97460	17870	6880
90° T. FALL	+ 55010	-26800	-10320	285490	-26800	-10320	146180	-26800	-10320
MAX. POS. M.				212010	10950	36400	466180	32320	3310
MAX. NEG. M.	- 191540	90670	87550	-742090	90280	68550	- 54250	71240	27650
	2			6			10		
DEAD LD.	- 229340	34320	47450	-118600	34320	27500	116410	34320	5230
POS. LIVE LD.	46360	- 7320	- 3590	183360	9800	11600	231620	26760	- 960
NEG. LIVE LD.	-252970	38480	32630	-234280	35980	24350	- 15120	10680	5190
60° T. RISE	- 111860	17870	6880	-168450	17870	6880	- 72020	17870	6880
90° T. FALL	167780	-26800	-10320	252680	-26800	-10320	108030	-26800	-10320
MAX. POS. M.				317440	17330	28780	456060	34280	- 6050
MAX. NEG. M.	- 594160	90670	86960	-521330	88170	58730			
	3			7			11		
DEAD LD.	-396790	34320	46710	- 28920	34320	21900	123910	34320	- 300
POS. LIVE LD.	79550	- 7320	- 3590	204270	15280	9260	226600	32090	2770
NEG. LIVE LD.	-435240	38480	32630	-156840	32530	20270	- 5550	- 2800	- 1500
60° T. RISE	-191740	17870	6880	-145660	17870	6880	- 45710	17870	6880
90° T. FALL	287610	-26800	-10320	218490	-26800	-10320	68570	-26800	-10320
MAX. POS. M.				393850	2280	20830	419090	39610	- 7860
MAX. NEG. M.	-1023780	90670	86220	- 331410	84720	49040			
	4			8			12		
DEAD LD.	- 393060	34320	41140	40060	34320	16320	111020	34320	- 5840
POS. LIVE LD.	105700	- 5620	15710	221470	20500	6300	230280	32530	530
NEG. LIVE LD.	-448450	38480	32600	- 92840	28260	16450	- 21820	3430	- 6190
60° T. RISE	-216460	17870	6880	-122000	17870	6880	- 18530	17870	6880
90° T. FALL	324690	-26800	-10320	-182990	-26800	-10320	27790	-26800	-10320
MAX. POS. M.	37330	1900	46530	444520	28020	12290	369090	40050	-15630
MAX. NEG. M.	-1057980	90680	80630	-174780	80450	39650	0		

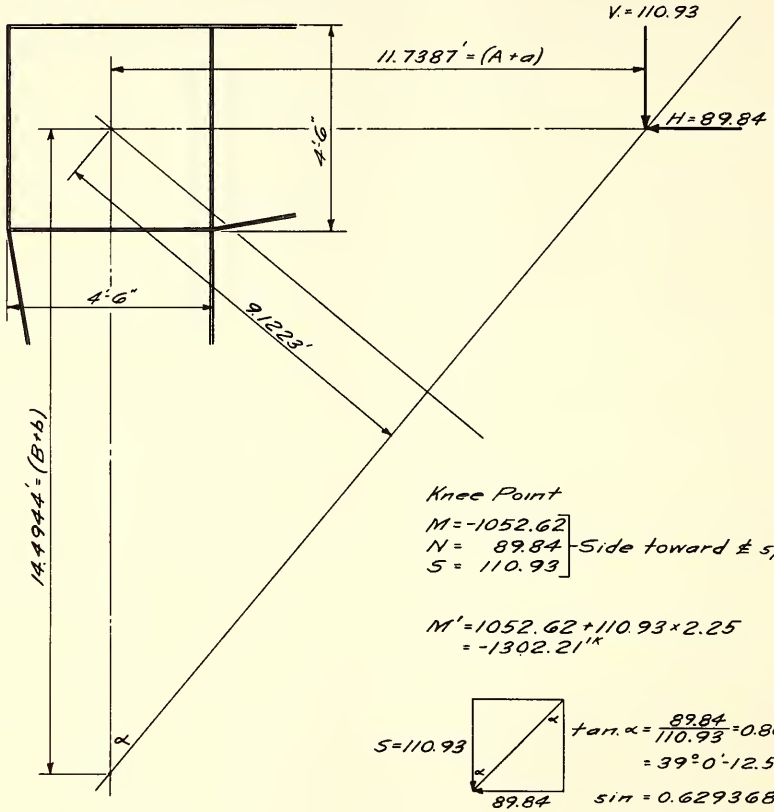
SUMMARY

	M	H	V	M	H	V	RESULTANT	H	V
	13			17			L. + R. REACTION		
DEAD LD.	77640	34320	- 11380	- 261670	34320	- 33790		34320	48320
POS. LIVE LD.	227580	28260	- 3290	137600	10680	- 14550	MAX.	38480	32600
NEG. LIVE LD.	- 56370	15280	- 10510	- 363390	24890	- 29060	H.	- 7320	- 3590
60° T. RISE	9550	17870	6880	130710	17870	6880		17870	- 6880
90° T. FALL	- 14330	- 26800	- 10320	- 196060	- 26800	- 10320		- 26800	- 10320
MAX. POS. M.	314770	80450	- 7780	6630	62870	- 41460	126220	90680	87800
MAX. NEG. M.				- 821120	32410	- 73170	34410	200	34410
	14			18			CENTER REACTION		
DEAD LD.	23800	34320	- 16940	- 398680	34320	- 39500		0	98110
POS. LIVE LD.	216140	23640	- 6720	99510	7080	- 16430	MAX.	34770	26860
NEG. LIVE LD.	- 110350	24680	- 16970	- 472730	24890	- 29060	H.	- 34770	26860
60° T. RISE	38520	17870	6880	163200	17870	6880		0	- 13760
90° T. FALL	57790	- 26800	- 10320	- 244800	- 26800	- 10320		0	20640
MAX. POS. M.	278470	75830	- 16780	0			116520	34770	111210
MAX. NEG. M.	- 144330	32210	- 44230	- 1116220	32410	- 78380	149710	- 34770	145610
	15			19			L. + R. REACTION		
DEAD LD.	- 50610	34320	- 22520	- 556910	34320	- 45270		34320	48320
POS. LIVE LD.	196680	19050	- 9730	59870	4040	1810	MAX.	23080	54860
NEG. LIVE LD.	- 180590	26640	- 20800	- 597020	21850	- 33140	V.	- 7320	- 3590
60° T. RISE	68370	17870	6880	196590	17870	6880		17870	6880
90° T. FALL	- 102560	- 26800	- 10320	- 294880	- 26800	- 10320		- 26800	- 10320
MAX. POS. M.	214440	71240	- 25370	0			133340	75270	110060
MAX. NEG. M.	- 333750	34170	- 53640	- 1448810	29370	- 88730	34410	200	34410
	16			19.5			CENTER REACTION		
DEAD LD.	- 145700	34320	- 28140	0	0	96330		0	98110
POS. LIVE LD.	170310	14690	- 12330	356430	34770	26860	MAX.	1690	70080
NEG. LIVE LD.	- 265580	26630	- 24890	- 356430	- 34770	26860	V.	0	0
60° T. RISE	99090	17870	6880	0	0	- 13760		0	- 13760
90° T. FALL	- 148640	- 26800	- 10320	0	0	20640		0	20640
MAX. POS. M.	123710	66880	- 33580	356430	34770	109430	188840	1690	188840
MAX. NEG. M.	- 559920	34150	- 63340	- 356430	- 34770	143830	84350	0	84350

Sheet No. 1

Investigation of Knee by Method of Bureau of Standards

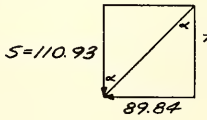
Scale $\frac{3}{8}$" = 1'0"



Knee Point

M = -1052.62
N = 89.84
S = 110.93
} Side toward ℄ span

$M' = 1052.62 + 110.93 \times 2.25$
$= -1302.21$ ft-k



$\tan \alpha = \frac{89.84}{110.93} = 0.80988$
$= 39°0'-12.5"$
sin = 0.629368
cos. = 0.777111

$N = 142.75$

$\frac{M}{N} = \frac{1302.21}{142.75} = 9.1223$

Sheet No. 2

KNEE INVESTIGATION

- Boundary on Lower Leg -

Depth of leg immediately adjacent to knee = 4'-6" = 54"

Isolated portion of web $2a = 2b = 54" - 2\times1 = 52"$

$a = b = 26" = 2.17'$

Proportion at V — Carried by each flg. at lower leg @ $y = b = \alpha = \frac{A_f}{2A_f + A_w} = \frac{14}{28+26} = \frac{14}{54} = 0.259$

Proportion at M — Carried by each flg. at lower leg @ $y = b = \gamma = \frac{I_f}{2I_f + I_w} = \frac{9833}{19666+5859}$

$= \frac{9833}{25525} = 0.385$

- Boundary on Upper Leg -

Prop. part of H carried by inner and outer flg. of upper leg @ $x = a$ $\quad K = \lambda = \frac{A_f}{2A_f + A_w} = 0.259$

Prop. part of M carried by inner and outer flg. of upper leg @ $x = a$ $\quad \mu = \nu = \frac{I_f}{2I_f + I_w} = 0.385$

$M_o = V(A+a) = H(B+b)$

$= 110.93 \times 11.7387 = 89.84 \times 14.4944$

$= 1302.17$

$M = M_o - Va + H(K-\lambda)b = M_o - Va$

$= 1302.17 - 110.93 \times 2.17$

$= 1061.45$

$F_x = -(1-K-\lambda)H - \left(\frac{\mu}{1-K+\lambda} - \frac{\nu}{1+K-\lambda}\right)\frac{M}{b} = -(1-2K)H = -43.30$

$F_y = -(1-2\alpha)V = -53.47$

$F_{xy} = V - \alpha V - \frac{\gamma}{a}(M_o - Hb) = 110.93 - 28.73 - 196.44 = -114.24$

$F_{yx} = H - \lambda H - \frac{VM}{(1+K-\lambda)b} = H - \lambda H - \frac{\nu M}{b} = 89.84 - 23.27 - 188.32 = -121.75$

$M_x = -\left(1 - \frac{\mu}{1-K+\lambda} - \frac{\nu}{1-K-\lambda}\right)M = -(1-2\mu)M = -244.13$

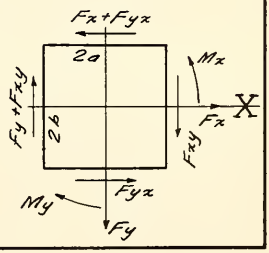
$M_y = -(1-2\gamma)(M_o - Hb) = -0.230 \times 1107.21 = -254.66$

For Equilibrium

$-M_x - F_{xy} \times 2a - F_{yx} = M_y - F_{yx} \times 2b - F_x b$

$-244.13 + 494.66 + 116.03 = -254.66 + 527.18 + 93.96$

$+366.56 = +366.48$



Sheet No. 3

KNEE INVESTIGATION

Investigation of inside corner — $\begin{cases} x = +2.17 \\ y = +2.17 \end{cases}$

$$\sigma_x = \frac{1}{4abt}\left(F_x + \frac{3M_x}{b^2}y\right)(a+x)$$

$$\sigma_x = \frac{1}{4\times 2.17^2 \times 0.0417}\left(-43.30 - \frac{3\times 244.13}{2.17^2}\times 2.17\right)(4.33)$$

$$\sigma_x = 1.2732\ (-43.30 - 337.51)\ 4.33 = -2099.39\ ^k/_{in^2} = -14579\ ^\#/_{in^2}$$

$$\sigma_y = \frac{1}{4abt}\left(F_y + \frac{3M_y}{a^2}x\right)(b+y)$$

$$\sigma_y = 1.2732\left(-53.47 - \frac{3\times 254.66}{2.17^2}\times 2.17\right)(4.33)$$

$$\sigma_y = 1.2732\ (-53.47 - 352.06)\ 4.33 = -2235.67\ ^k/_{in^2} = -15525\ ^\#/_{in^2}$$

$$\tau_{xy} = -\frac{1}{4abt}\left[M_x - a\,(2F_{xy} + F_y) + F_y x + F_x y - \frac{3}{2}M_y\left(1 - \frac{x^2}{a^2}\right) - \frac{3}{2}M_x\left(1 - \frac{y^2}{b^2}\right)\right]$$

$$\tau_{xy} = -1.2732\ [-244.13 + 611.83 - 121.75 - 114.24 - 0 - 0]$$

$$\tau_{xy} = -1.2732 \times 131.71 = 167.69\ ^k/_{in^2} = 1165\ ^\#/_{in^2}$$

$$\text{Max. normal stress (comp.)} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-14579 - 15525}{2} \pm \sqrt{\left(\frac{-14579 + 15525}{2}\right)^2 + 1165^2}$$

$$= -15052 \pm 1165 = -16217\ ^\#/_{in^2} \text{ max. comp.}$$

$$\text{Max. shear stress @ } x = -\frac{b_3}{b_4}\quad y = -\frac{c_3}{d_4}\qquad \tau_{max.} = -\left(b_2 - \frac{b_3^2}{b_4} - \frac{c_3^2}{d_4} + \frac{1}{2}\times\frac{b_3^2}{b_4} + \frac{1}{2}\times\frac{c_3^2}{d_4}\right)$$

$$b_2 = \frac{1}{4abt}\left[M_x - a\,(2F_{xy} + F_y) - \frac{3}{2}(M_x + M_y)\right] = 1.2732 \times 1115.89 = 1420.75$$

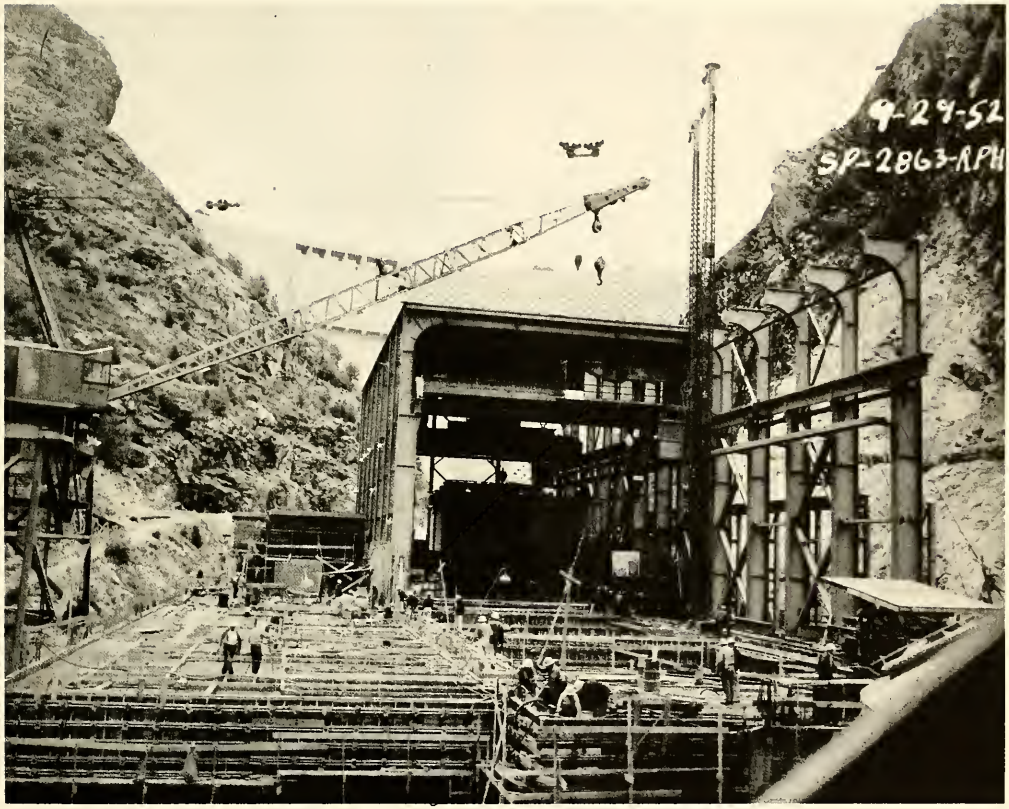
$$b_3 = \frac{F_y}{4abt} = -53.47 \times 1.2732 = -68.08 \qquad c_3 = \frac{F_x}{4abt} = -43.30 = 1.2732 = 55.13$$

$$b_4 = \frac{3M_y}{4a^3bt} = -\frac{3\times 254.66}{3.70} = -206.48 \qquad d_4 = \frac{3M_x}{4ab^3t} = \frac{-3\times 244.13}{3.70} = 197.94$$

$$T_{max.} = -(1420.75 + 22.45 + 15.35 - 11.23 - 7.68) = -1439.64\ ^k/_{in^2} = 9998\ ^\#/_{in^2}$$

Ross Powerhouse, City of Seattle, Washington

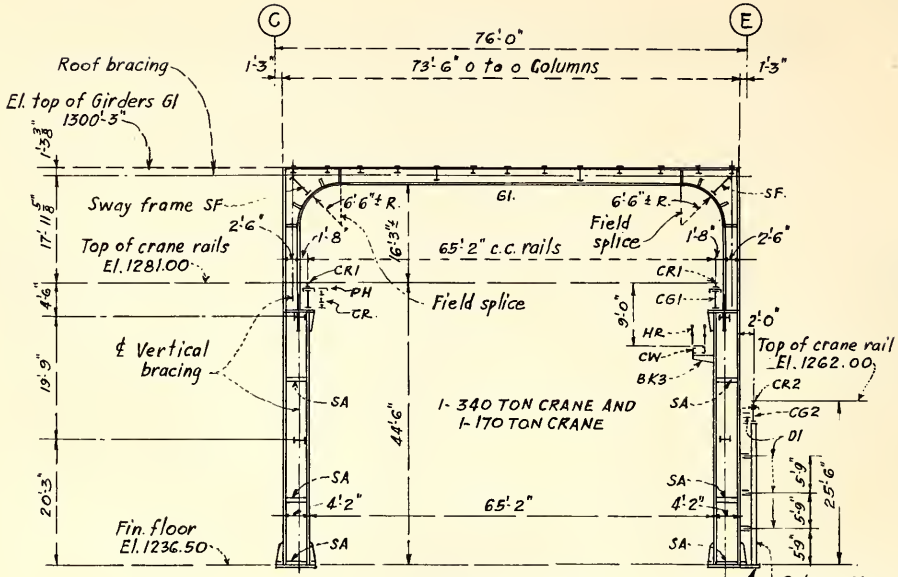
DR. JOHN L. SAVAGE*
Consulting Engineer
Denver, Colo.



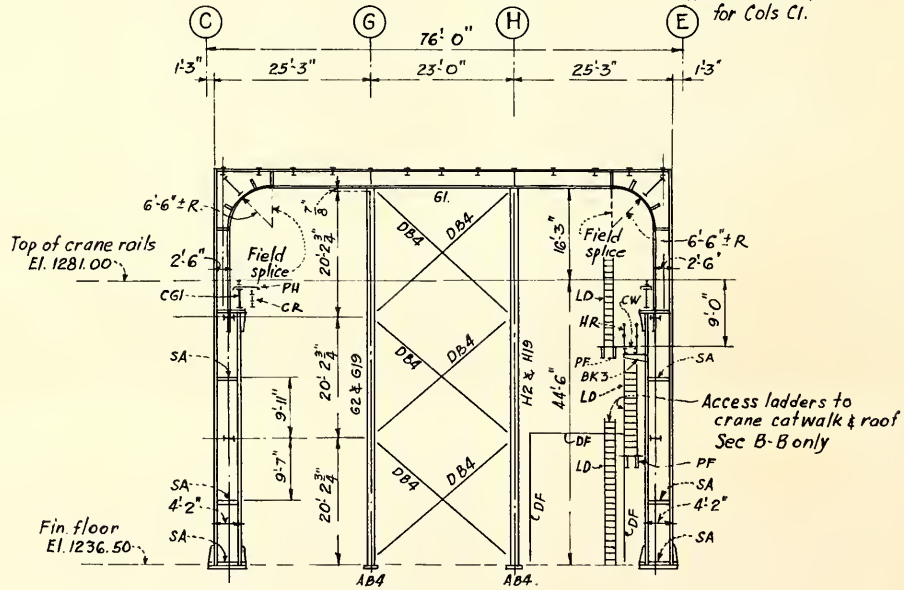
*Author's Note: Take a globe, revolve it and you will traverse the area of operations covered by the world-famed John L. Savage, Consultant on dams, irrigation and hydro-electric projects.

William H. Wolf of Denver, Colorado, brilliant and creative engineer in charge of the structural design, here again reveals the exceptional ability he so aptly demonstrated when formerly employed by the author on design of rigid frame subways for New York City. He was ably assisted by John G. Anderson who has had a rich background on steel design.

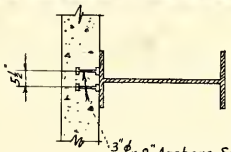
This entire structure is most modern in concept. It is unusual for a powerhouse in that it contains welded stepped-column rigid frames. Another outstanding feature is the dual-crane capacity, one giant crane of 340 tons and one of 170 tons. The design data reproduced here has been assembled so compactly by its designers that it can serve as an extraordinary source for reference and study. M.P.K.



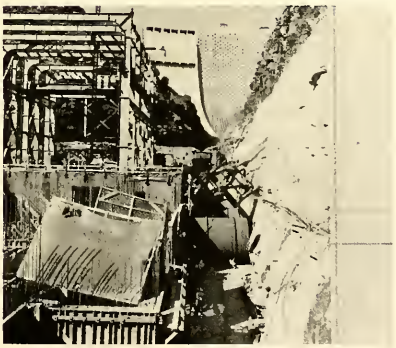
SECTION A-A
TYPICAL FOR FRAMES 3 TO 18 INCL.
EXCEPT OMIT COL. C1 AT LINE 3.



SECTION B-B
FRAMING ELEVATION LINE 2 & LINE 19.

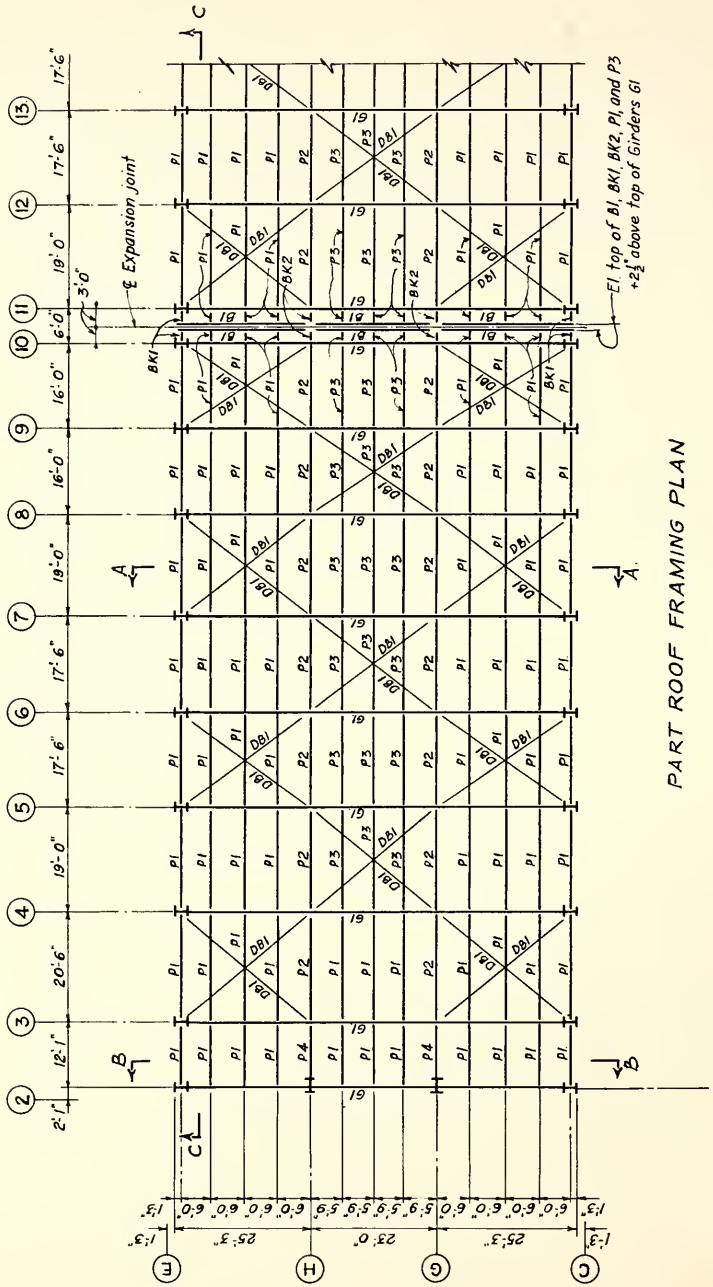


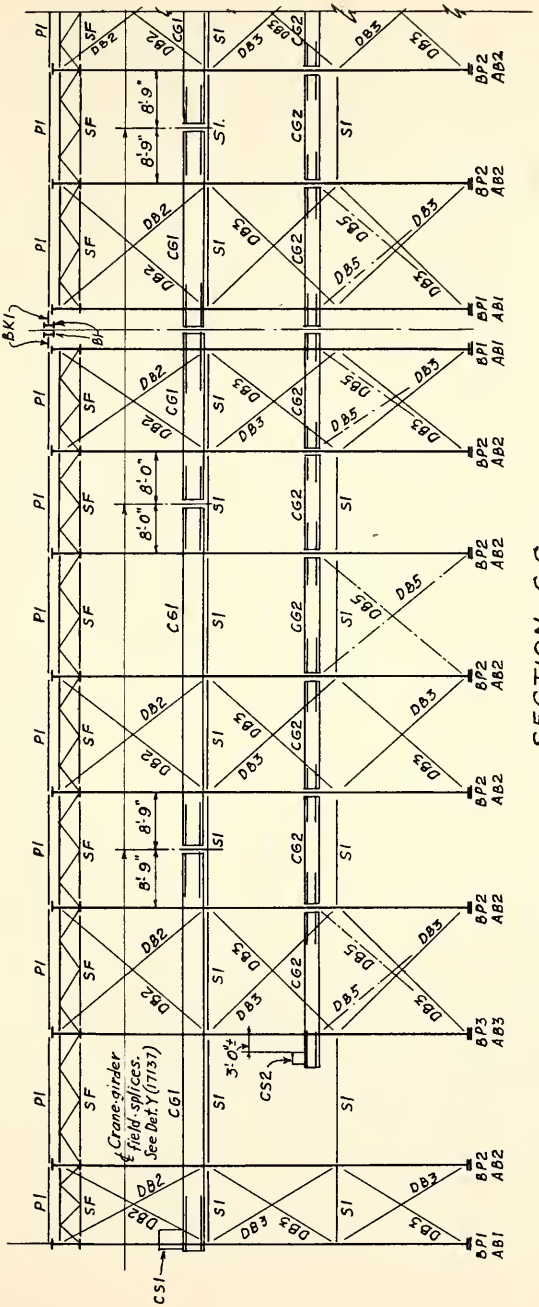
TYPICAL ANCHOR DETAIL FOR CONCRETE WALL TO COL.

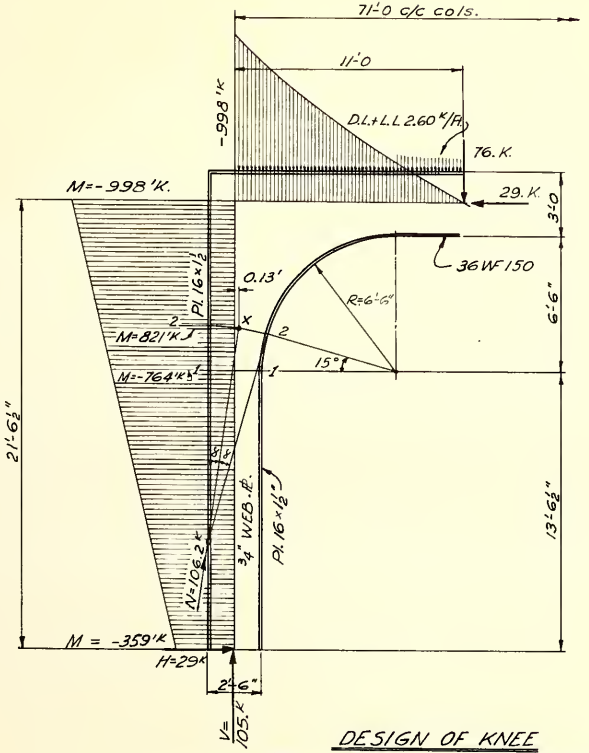


SCHEDULE AND SIZE OF MAIN MATERIAL

DESCRIPTION	DESIGN MARK	SIZE OF MATERIAL
Purlin	P1	12 WF 27
Purlin	P2	16 WF 36
Purlin	P3	10 WF 21
Purlin	P4	36 WF 150
Roof beam	B1	12 WF 27
Roof bracket	BK1	See Det Sec. H-H (17157)
Roof bracket	BK2	See Det Sec. K-K (17137)
Roof girder	G1	36 WF 150
Diagonal bracing	DB1	1-L 3x3x 5/16
Sway frame	SF	See Sec. P-P (17136)
Conductor rail	CR	40 Lb. A.S.C.E. rails
Crane column	C1	10 WF 54
Crane girder	CG1	1-36 WF 230 1-Pl. 26 x 5/8 2-L 4 x 4 x 5/8
Crane girder	CG2	1-24 WF 100 1-15 I. 33.9
Crane rail	CR1	175 Lb. Lorain rail or similar
Crane rail	CR2	100 Lb. A.S.C.E. rail.
Crane stop	CS1	36 WF 230
Crane stop	CS2	24 WF 100 (Details sim. to CS1)
Diagonal bracing	DB2	2-TT 4 x 4 x 3/8
Diagonal bracing	DB3	16 WF 36
Strut	S1	16 WF 36
Diagonal bracing	DB4	10 WF 21
Column	H2, G2 H19, G19	27 WF 145
Column	E3, E5, E6, E7, E8, E9, E12, E13, E14, E15, E16, E18, C3, C5, C6, C7, C8, C9, C12, C13, C14, C15, C16, C18.	UPPER PORTION 2-Flg. pl. 16 x 1 1/2 1-Web pl 27 x 3/4 LOWER PORTION 2-Flg. pl. 22 x 1 3/4 1-Web pl (above) 46 1/2 x 1 1/4 1-Web pl. (below) 46 1/2 x 3/4
Column	E4, E17, C4, C17	UPPER PORTION 2-Flg pl. 16 x 1 1/2 1-Web pl. 27 x 3/4 LOWER PORTION 2-Flg pl. 22 x 2 1-Web pl (above) 46 x 1 1/4 1-Web pl. (below) 46 x 3/4
Column	E2, E10, E11, E19, C2, C10, C11, C19	UPPER PORTION 2-Flg pl. 16 x 1 1-Web pl. 28 x 1/2 LOWER PORTION 2-Flg. pl 18 x 1 1/2 1-Web pl. 47 x 3/4
Base plate	BP1	1-Pl. 36 x 4 1/2 x 5'-8"
Base plate	BP2	1-Pl. 44 x 5 1/2 x 5'-8"
Base plate	BP3	1-Pl. 48 x 6 x 5'-8"
Anchor bolt	AB1	3"ϕ x 10'-0"
Anchor bolt	AB2	3 1/2"ϕ x 10'-0"
Anchor bolt	AB3.	3 1/2"ϕ x 10'-0"
Anchor bolt	AB4	1 1/4"ϕ x 2'-6"
Anchor bolt.	AB5	1"ϕ x 2'-6"
Diaphragm	D1	12" WF 27
Stiff. angle	SA	2 L's 6 x 4 x 1/2
Diag. Bracing	DB5	2 TT 3 1/2 x 3 x 5/16
Strut.	S2	16 WF 58


℄ Expansion joint
El. top of B1, BK1, BK2, P1, and P3 +2½" above top of Girders G1
PART ROOF FRAMING PLAN


SECTION C-C
(FRAMING ELEVATION E-LINE WALL)
(CRANE CATWALK AND ACCESS LADDERS NOT SHOWN)
(DIAGONAL BRACING DB5 FOR COL. C1 ONLY)
℄ Crane-girder field-splices. See Det. Y (17137)
CS1
CS2
3'-0½"
8'-9"
8'-0"
BK1
B1

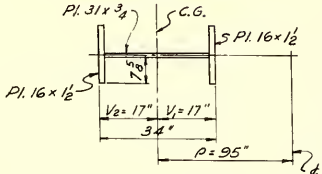


DESIGN OF KNEE

(SEE FIG. 1 FOR CRITICAL LOADING COMBINATION)

"Design of Rigid Frame Knees" by Fredrich Bleich D. Eng. published by A.I.S.C. 1943

STRESS AT SEC. 2-2



$A = 71.2\square''$
$I_{cg.} = 14{,}560\ ''^4$
$\alpha = 7°30'$
$N = -106.2^K$
$M_x = -821^{IK} + (0.13')(105^K) = -808^{IK}$
$\rho = (6'-6") + 17" = 95"$
$V_1 = -17"$
$V_2 = +17"$
f = Average Stress on inside flange

SECTION 2-2

$$f = \frac{1}{\cos\alpha}\left[\frac{N}{A} - \frac{M_x}{A\rho} - \frac{M_x V_1}{I} \times \frac{\rho}{\rho + V_1}\right] \quad \text{see Bleich}$$

$$f = \frac{1}{0.991}\left[\frac{-106.2}{71.2} - \frac{-808^{IK}(12)}{71.2 \times 95} - \frac{-808^{IK}(12)(-17)}{14{,}560} \times \frac{95}{95+(-17)}\right]$$

$$f = 1.01\ [-1.49 + 1.44 - 13.78]$$

$f = -13.83\ K/0" + 20\%$ for rotation

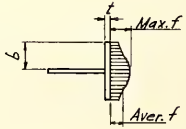
$f = -13.83 - 2.77 = -16.6\ K/0"$

Max. fiber stress

$$f_{max.} = \frac{-16.6^K}{0.878} = 18.9\ K/0"$$

$f'max. = 18.9^K/0" \times 1.238 = 23.4\ K/0"$

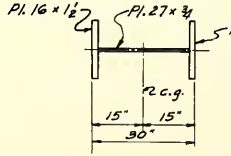
$f'max.$ = Stress perpendicular to the edges of the flange. In order to reduce this bending stress, welded ribs are provided to support the outstanding legs of the flange to the web.



$$\frac{b^2}{Rt} = \frac{(7.625)^2}{78(1.6)} = 0.50 \quad v = 0.878 \quad u = 1.238$$

See tables in Bleich

STRESS AT SEC. 1-1



$A = 68.3\square''$
$I = 10{,}980\ in.^4$
$N = 105^K$
$M = -764'^K$

$$f = \frac{P}{A} \pm \frac{Mc}{I}$$

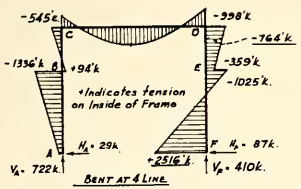
$$= \frac{105^K}{68.3} \pm \frac{764'^K(12") \times 15"}{10{,}980''^4}$$

$$= 1.54\ K/0" \pm 12.5\ K/0" = 14.04\ K/0"$$

Add 20% for rotation $= 2.81\ K/0"$

Total $= 16.85\ K/0"$

SECTION 1-1

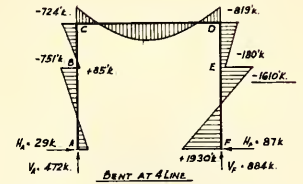


BENT AT 4 LINE.

LOADING COMB. FOR MAX. ANCHOR BOLT TENSION & CRITICAL STRESS AT BOTTOM OF KNEE AND BASE

D.L. + L.L. ROOF + VERT. CRANE + IMPACT + LAT. CRANE + LONG. CRANE.

FIGURE 1

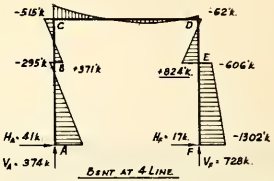


BENT AT 4 LINE.

LOADING COMB. FOR CRITICAL STRESS IN LOWER COL. AT A, B, E & F

D.L. + L.L. ROOF + VERT. CRANE + IMPACT + LAT. CRANE + LONG. CRANE.

FIGURE 2



BENT AT 4 LINE.

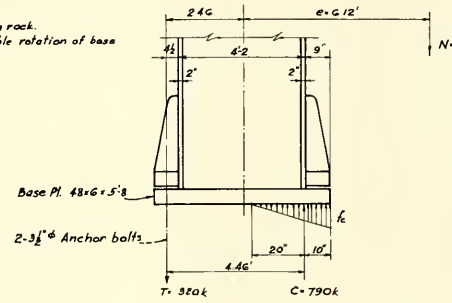
LOADING COMB. FOR CRITICAL STRESS IN UPPER COL. AT B & E

D.L. + VERT. CRANE + IMPACT + LAT. CRANE

FIGURE 3

BASIC ASSUMPTIONS & DESIGN DATA

Base at A & F assumed 100% fixed. Powerhouse on rock.
Moments at haunch increased 20% to allow for possible rotation of base
Crane girder designed as continuous member.
(Rotation at supports)
(No settlement at supports)
(Max. uplift at any support = 1/8")



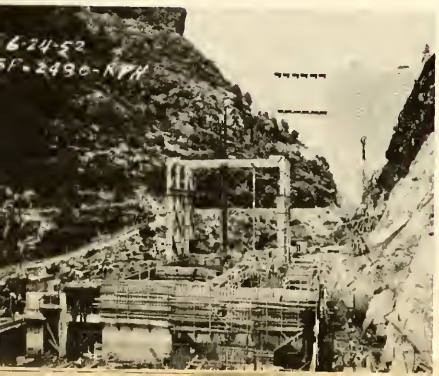
DESIGN OF BASE
(4 LINE BENT)

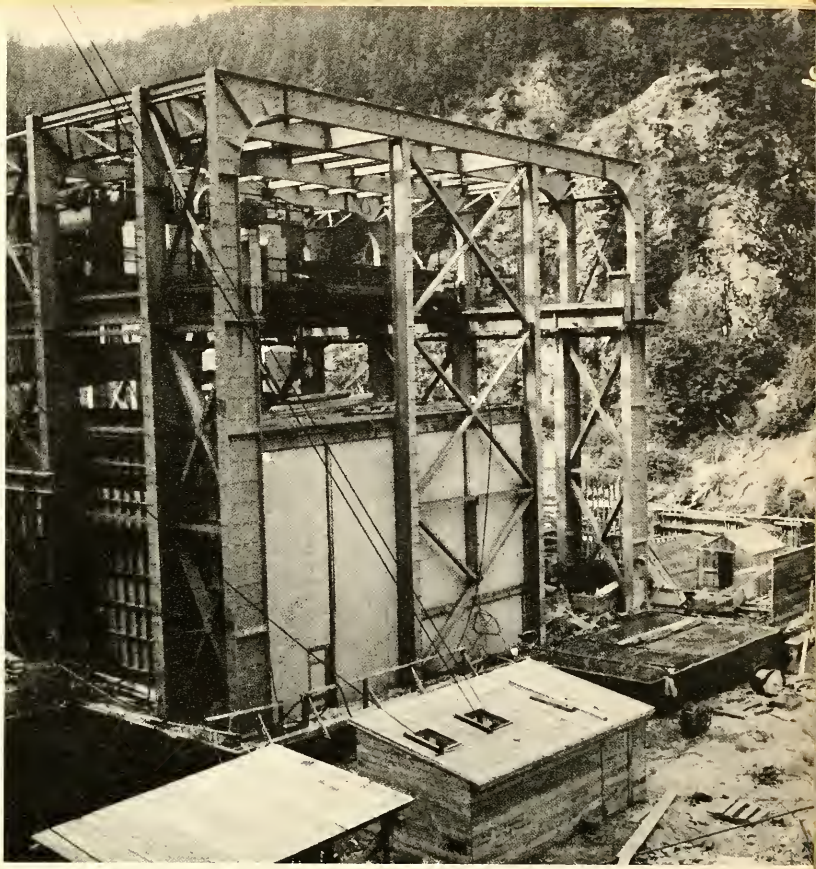
M = +2516'k.
V_F = 410k.
e = 2516 ÷ 410 = G 12'
$C = \frac{(G12 + 24G)410}{4.4G} = 790k.$

Stress in anchor bolts = $\frac{320}{2(7.549^{sq.in.})} = 25.2k/^{sq.in.}$

$\frac{1}{2} f_c (30)48 = 790k$ $\quad f_c = 1.10k./^{sq.in.}$

Allowable bending stress in base pl. = $18{,}000^{\#}/_{sq.in.}$
(See A.I.S.C.)





Code & Specifications

American Institute of Steel Construction.
(Exceptions noted below)

Allowable Stresses

For Bldg. Frame & Cols	20,000 #/□"	1.00f
For Crane Girders	18,000 #/□"	1.00f
Combined with Wind		1.32f
Combined with Earthquake		1.50f
Main anchor bolts		28,000 #/□" on net area.

Loads on Structure.

Dead Load on Roof (D.L.)	20 #/□'
Live Load on Roof (L.L.)	100 #/□'
Wind (Acts either direction all on one wall)	25 #/□'
Earthquake (E.Q.)	10% Gravity

Cranes (One 340 Ton Crane plus One 170 Ton Crane)
(Crane Impact 10% of max. & min. wheel loads)
(Lateral Crane 10% of hook load plus trolleys all on one rail)
(Longitudinal Crane 10% of max. wheel loads)
Misc. Loads (Lat. loads from takeoff structure & gantry crane)

Loading Combinations

(1) D.L. + L.L. Roof + Vert. Crane + Impact + Lat. Crane + Long. Crane	1.00f
(2) D.L. + ½ L.L. Roof + Vert. Crane + Impact + Lat. Crane + Long. Crane + Wind	1.33f
(3) D.L. + ½ L.L. Roof + D.L. of Crane + E.Q.	1.50f

Hook Load to 340 Ton Cr. = $\frac{2}{3}(465)$ = 310 Tons
Reaction of Bridge $102 \div 2$ = 51
Reaction of 2 Trolleys = $\frac{50.2}{65.2}(100)$ = 77

Total R = 438 Tons.

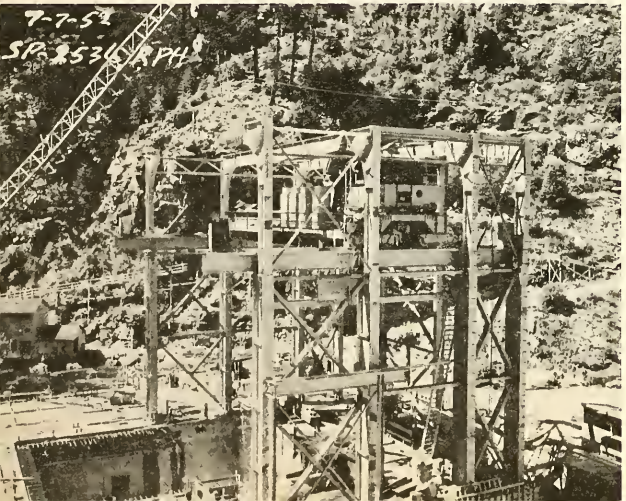
$\frac{R.}{\text{No. Wheels}} = \frac{438}{8} = 55$ Tons.

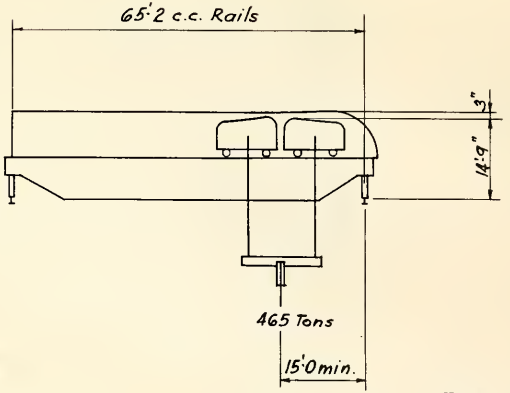
Hook Load to 170 Ton Cr. = $\frac{1}{3}(465)$ = 155 Tons
Reaction of Bridge = $50 \div 2$ = 25
Reaction of 1 Trolley = 38

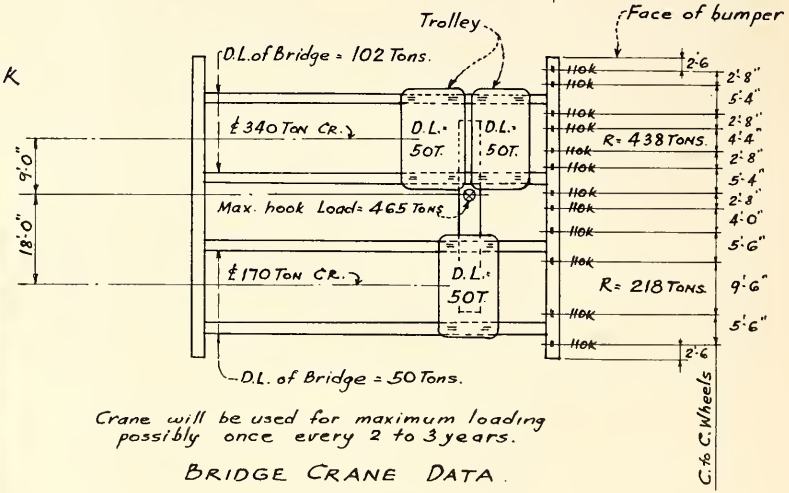
Total R = 218 Tons.

$\frac{R.}{\text{No. Wheels}} = \frac{218}{4} = 55$ Tons.

Max. Wheel Load without impact = 110 K





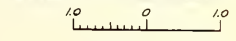


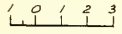
Crane will be used for maximum loading possibly once every 2 to 3 years.

BRIDGE CRANE DATA.

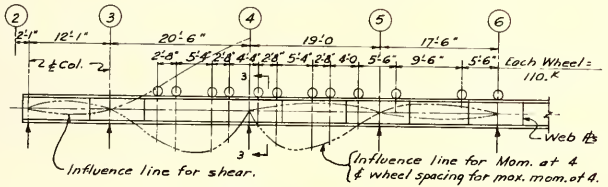
340 Ton Crane
170 Ton Crane
Max. Loading without impact.

CRANE GIRDER DESIGN AT 4



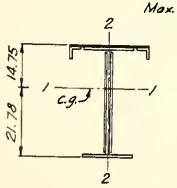


SCALES FOR INFLUENCE LINES



INFLUENCE LINES

From influence line max. neg. mom. at 4 without uplift = 1130'K
Max. neg. mom. at 4 with 1/16 uplift at any support = 1210'K
From influence line max. shear of 4_L without uplift = 421'K
Max. shear at 4_L with 1/16" uplift at any support = 432'K
Add 10% for Impact



SECTION 3-3

CRANE GIRDER AT 4

1-36WF 230	S.M. Top Flg.	= 1580 "³
2-L's 4x4x5/8	S.M. Bot. Flg.	= 1070 "³
1-Pl. 26 x 5/8	S.M. Axis 2-2	= 236 "³
2-Web Pl. 32 x 3/8	I_{1-1}	= 23,308 "⁴

Gross Area of Top Flg. = 46.17 0 "
Net Area of Top Flg. = 39.86 "
Gross Area of Bot. Flg. = 20.80 "
Net Area of Bot. Flg. = 17.90 "

Stat. Mom. of 2 L's 4x4x5/8 = 119.0 in.³
" " of 1 Pl. 26 x 5/8 = 234.0
" " of Top Flg of 36WF230 = 280.0
Total m_1 = 633.0 in³

Stat. Mom. of 2 web Pl's = 55.0 in.³
" " of Web of 36WF 230 = 63.0
Total m_2 = 751.0 in.³

Stat. Mom. of Bot. Flg. of 36WF230 = 445.0 in.³
" " of Web of 36WF230 = 159.0
" " of 2 Web Pl's. = 147.0
Total m_2 = 751.0 in.³

Gross Stress in top flg. = L.L. + D.L. + Impact + Lat. Crane
$= \frac{(1210+14+121)12}{1580} + \frac{121(12)}{236} = 10.2 + 6.15 = 16.35$ K/0"

Net Stress in Top Flg. $= \frac{(16.35)(46.17)}{39.86}$ = 18.9 K/0"

Gross Stress in Bot. Flg. $= \frac{(1210+14+121)12}{1070}$ = 15.1 K/0"

Net Stress in Bot. Flg. $= \frac{15.1 \times 20.8}{17.9}$ = 17.6 K/0"

Shear at top fillet $= \frac{V m_1}{Ib} = \frac{475(633)}{23,308\ (1.515)}$ = 8.5 K/0"

Fiber Stress at top fillet $= \frac{16.35\ (11.72)}{14.725}$ = 13.0 K/0"

Shear at c.g. $= \frac{V m_2}{Ib} = \frac{475\ (751)}{23,308\ (1.515)}$ = 10.1 K/0"

Shear at Bot. fillet $= \frac{V m_3}{Ib} = \frac{475\ (437)}{23,308\ (1.515)}$ = 5.9 K/0"

Fiber Stress at Bot. fillet $= \frac{15.1\ (13.41)}{21.78}$ = 13.5 K/0"

Principal Stress at Top fillet $= \frac{13.0}{2} + \sqrt{\frac{(13.0)^2}{4} + 8.5^2}$ = 17.2 K/0"

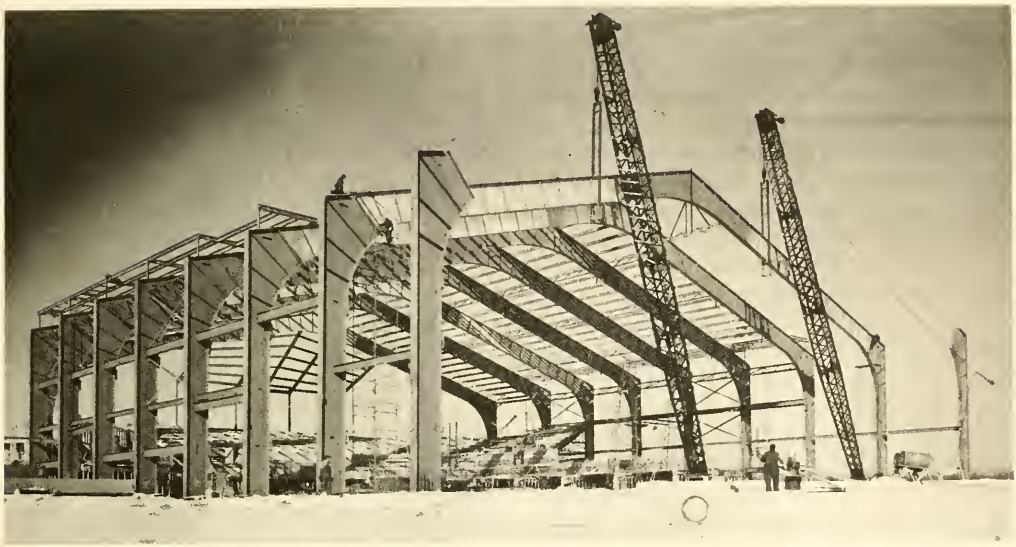
Shear taken by 2-3/8" Web P/s = 8.5 (0.75) = 6.4 K/Lin. In.
2-7/16" Fillet welds = 7.0 K/Lin. In.

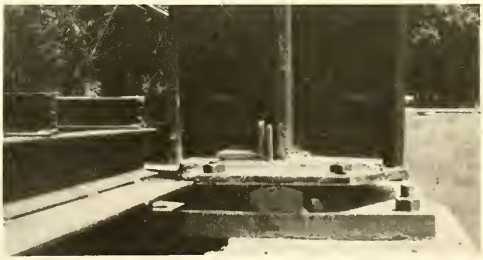
Mom. taken by 2-3/8" Web P/s $= \frac{fI}{c} = \frac{13(2408)}{12.16}$ = 2580 "K

Length of 2-7/16" Fillet welds to develop mom. of 2-3/8" web P/s $= \frac{2580}{32.(7)} = 11.5$". This figure added to the theoretical lengths required show that web p/s. should extend to the quarter points of span.

Allen County War Memorial Coliseum

FT. WAYNE, IND.
by
RALPH E. COBLENTZ*
Chief Engineer, A. M. Strauss, Inc.
Architects—Engineers
Fort Wayne 2, Ind.





*Author's Note: Endowed with originality, creativeness and a most receptive mind for new ideas in design and construction and for new materials, Ralph E. Coblentz also has the courage to try them. This unusual combination of aptitudes aided further by a most genial personality naturally attracts the biggest jobs in and around the Hoosier state. They would win anywhere. Of special interest in this structure are the column bases. After designing them as hinged, Coblentz lets his philosophy take command. He talks calmly and confidently to his bases, they go along with his philosophy, the hinges—mesmerized—change into everyday flat base plates and a courageous thought becomes a reality. The client saves considerably through his ingenuity. Engineers have a new precedent for study. Photograph shown displays the hermaphroditic base plate which remains in spirit and function a hinge; beside it for comparison is a scared column base with ties, trembling as in fear on expensive rockers—and its column span is less than one-quarter that of the Coblentz rigid frame! M.P.K.

The Allen County War Memorial Coliseum, located near Fort Wayne, Indiana, has a roof and bleacher construction supported on all welded rigid frames with a span of 224-feet center to center of cols. Rigid frames are spaced at 27′-0″ centers.

The first floor, or arena floor, is a 9″-thick flat slab construction supporting a 10″-thick ice floor slab containing brine pipes, insulation, etc. The entire basement is excavated and is to be used for an exhibition hall.

Bleacher seating is supported on reinforced concrete construction of which the outer ends of reinforced concrete riser beams are supported on the rigid frame bents, supporting the roof. This detail posed somewhat of a problem as the concrete bleacher construction is rigid and the steel rigid frames must be free to move with the expanding and contracting of the long roof rafter portions of the steel frame under loading and temperature changes. Many structures of this type have taken a simpler, but more expensive, way out by adding a column at the outer end of the bleacher beams making it entirely free of the construction supporting the roof. Another requirement which eliminated the possibility of using columns at this place was the fact that corridors, ramps, and promenade floors were adjacent to the rigid frame bents and additional space could not be spared. Out of these requirements came the decision to support the concrete riser beams on a link type hanger which allowed the rigid frame to move and still support its vertical load. The detail is so simple that it is a wonder that it had not been used before.

One of the outstanding features of this Coliseum is the long span all welded rigid frames supporting the roof construction. Each frame is built up entirely of plates with all shop fabrication done by welding and all field splices also welded. The frames were shop fabricated in five sections. Each section was set in a jig and flange plates welded to the web plate by a continuous weld. Shop welding was done by automatic machine using the new hidden arc-welding process.

The column sections were composed of a web plate 55″ × 5/8″ and flange plates 18″ × 2½″ thick. The rise from the base plate to the top of the knee was 54′-10″. Radius of haunch at the knee was 21′-0″. The five sections making up a complete frame consisted of two column sections, two rafter sections, and one center section. The rafter sections were composed of a web plate 72″ × 5/8″ and flange plates of 18″ × 1½″ thick flange plates. The radius of the haunch at the center of the span was 50′-0″. Total weight of one frame is 68 tons.

The rigid frames are tied at the base by two 2″ diameter tie rods with upset screw thread ends for turnbuckles at each end. The tie rods were field welded from 20′-0″ sections and accurately placed on the forms used for supporting the concrete flat slab arena floor. These rods were greased and wrapped with sisal paper to prevent any bond between the concrete floor slab and the tie rod. The rods were positioned to the exact center of the 9″ flat slab floor. In making the field welds the rods had to be raised slightly at the point of the weld to allow for the draw when the weld cooled, pulling the rods back to a straight line.

The rigid frames were set on concrete piers 36″ × 76″. A pocket 6″ deep was left at the top of the pier for shimming and grouting the frame base. No provision was made for a positive hinge at the base, although it is designed as such. Since the dead load was on the frame before any grouting of the base was done, it is assumed that the rotation of the base would be so slight as to preclude any expensive hinge details. Also, the long column lengths are sufficiently flexible to permit rotation of the knee.

Bracing was provided at each knee to stabil-

ize the haunch which was 12′-3″ deep at the deepest section. Also vertical bracing to stabilize the haunch at the center of the span which was 8′-3″ deep on the center line. Intermediate vertical bracing was spaced at 30′-0″ centers along the frame span. Horizontal bracing was placed in alternate bays.

Frames were designed to support a dead and live load of 46 lbs. per square foot of roof construction which consisted of 4″-thick precast perlite slabs spanning 6′-0″ and supported on 12″ purlins. Precast perlite slabs were chosen for the roof construction because of their combined uniform insulating value, strength, acoustical properties, and appearance since the underside is to be left exposed and unpainted. Ease and rapidity of erection is another point in favor of the precast slabs.

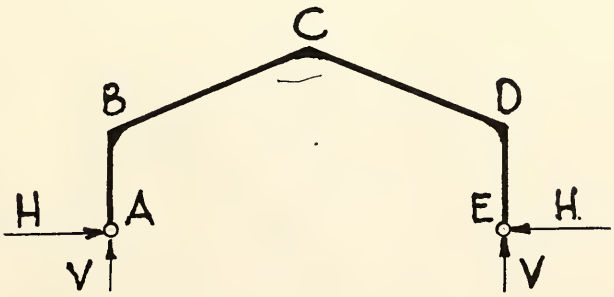
Design moments, shears and reactions were calculated separately for each of the following loads and then a logical combination made to produce the maximum moments, shears and reactions:

(a) Uniform dead and live load over the entire span
(b) Wind load on wall
(c) Wind load on roof
(d) Bleacher and mezzanine floor loads
(e) Temperature changes
(f) Concentrated load at center of span

The combination of the above loads which gave maximum knee moments, maximum center moments and maximum tie rod tension can be seen from the following table:

	MB	MC	MD	HA	VA	HE	VE
Uniform load	−5000	+2790	−5000	+94.3	+193..	+94.3	+193.
Wind on wall	+ 451	− 116	− 308	−22.8	− 3.4	+ 5.8	+ 3.4
Wind on roof	+ 497	− 90	− 435	− 9.6	− 5.4	+ 8.2	+ 5.4
Center load	− 37	+ 52	− 37	+ 0.8	+ 1.0	+ 0.8	+ 1.0
Bleacher load	+ 23	− 17	+ 23	+ 1.6	+ 36.	+ 1.6	+ 36.
Decrease in Temp.	+ 111	+ 180	+ 111	− 2.1		− 2.1	

(Above values are in kips and ft. kips)



(An increase in temperature would not occur with maximum snow load)

The knee which is the critical point of a rigid frame was designed for a maximum moment of 69,360,000 inch pounds and an unbraced length of 30'-0". Maximum tie rod tension was 110,700 pounds.

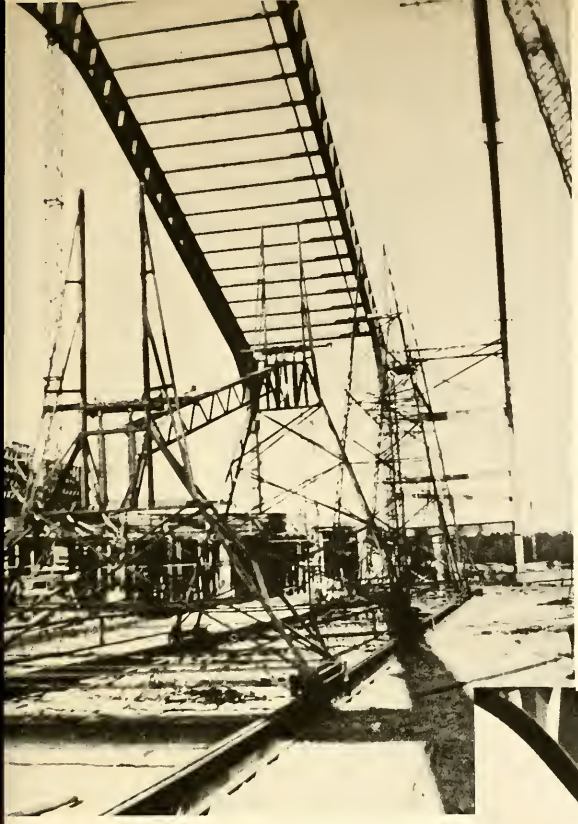
Erection of these long span rigid frames posed many problems. First, the frames had to be erected on a slab over a full basement which had a limiting live load of 330 pounds per square foot. Second, all field splices were 100% welded with dovetail splices. Third, location of hoisting equipment. Fourth, an assembly procedure to hold alignment of rafter sections for field welding and a welding procedure for the field splices to eliminate distortion.

Welding of the two rafter sections to the center rafter section was done by two welders working one to each side of the joint and duplicating their welds closely in order to keep distortion to the minimum. The welding time required four to five hours for the two men, totaling eight to ten man-hours for this connection.

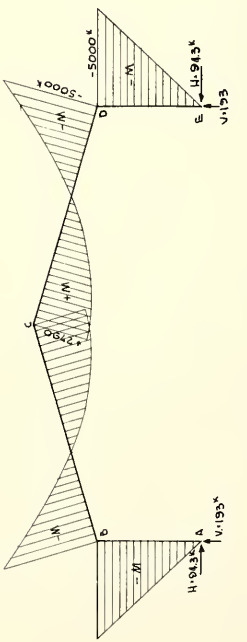
Welding of the rafter sections to the top column connections was done by two welders working in the same manner as for the above splice. The welding time required for this splice was nine to ten hours for two men, totaling 18 to 20 man-hours per splice.

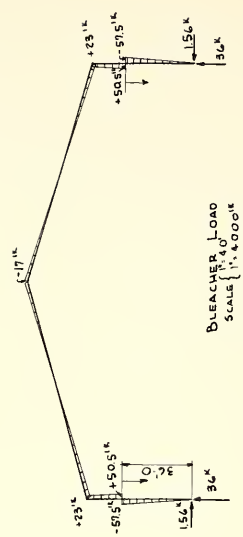
We discussed various theoretical methods of prestressing the tie rods to some determined figure, then arrived at a practical field method of handling the problem. Since the frame was set into position and all field splices welded, it was this final position of all members that we wished to maintain so that our stress pattern would be approximately as we had calculated. A turnbuckle was placed at the end of each rod at the base of each frame. These turnbuckles were tightened simultaneously by inserting crowbars in the turnbuckle and two men on each bar. Tightening was continued until the frame base started to give. This was the maximum prestress that could be placed in the rods and we felt would hold the base of our frame in position.

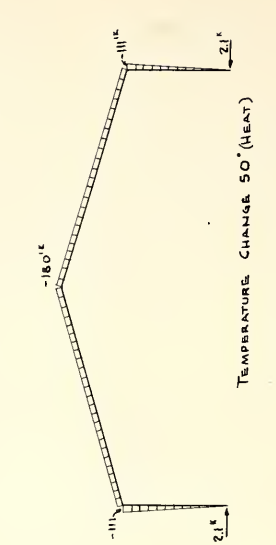
The method proposed by AISC in their publication, *Single Span Rigid Frames in Steel,* was used for preliminary design and a check made by "the exact method."

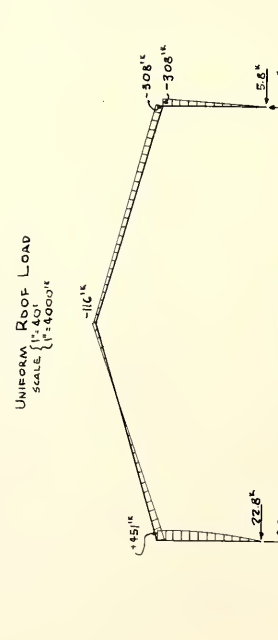


Views Showing Erection


-5000K
-5000K
D
E
H-943K
V-193
W-
W+
W+
W-
C
B
A
H-943K
V-193K


+23K
-57.5K
+50.5K
-17K
+50.5K
-57.5K
+23K
1.56K
36K
10'-2"
1.56K
36K
BLEACHER LOAD
SCALE 1"=40'
1"=4000K


-111K
-180K
-111K
2.1K
2.1K
TEMPERATURE CHANGE 50° (HEAT)


UNIFORM ROOF LOAD
SCALE 1"=40'
1"=4000K

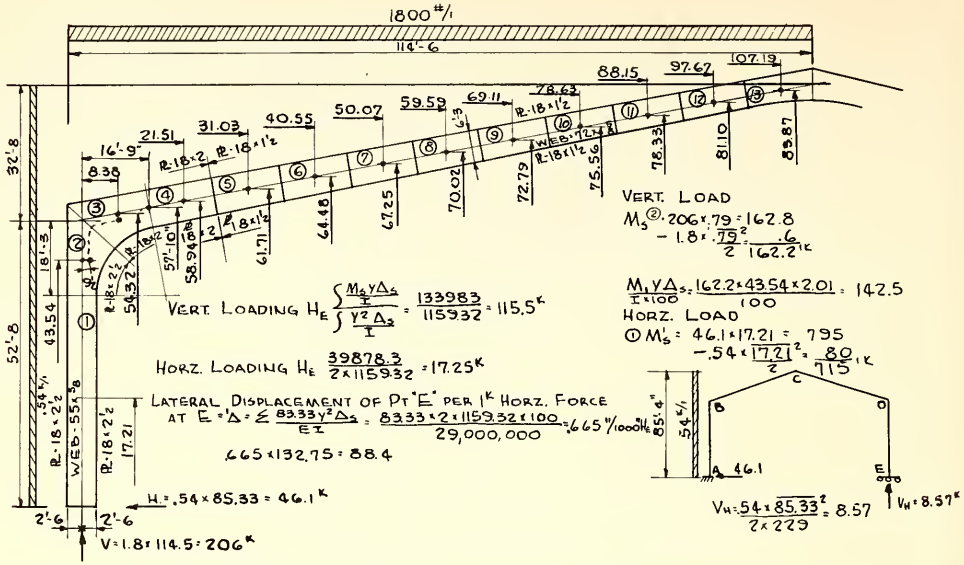
-308K
-308K
5.6K
339K
-116K
+451K
22.8K
339
WIND LOAD ON WALL
SCALE 1"=40'
1"=4000K

+111
+180
+111
21
17
TEMPERATURE CHANGE 50° (COLD)

-5780
-3850
-5780
-3990
48'0
48'0
SPLICE
30'0
20'0
40'0
10'0
+3150
3005
2550
1050
3000
-5780
-5780
MAXIMUM MOMENTS
SCALE 1"= 40'
1"= 4000'

-90
-435
8.15
54
+435
9.55
5.4
WIND LOAD ON ROOF
SCALE 1"= 40'
1"= 4000'

-371
.75
1
+52
-371
.75
1
CENTER LOAD
SCALE 1"= 40'
1"= 4000'



SECTION	MEMBER	Δ₂ FT.	x FT.	y FT.	d IN.	I IN⁴	I FT⁴	Δ₂/I	y²Δ₂/I ÷100	VERT. LOAD HALF OF FRAME M₃ FT. KIPS	VERT. LOAD HALF OF FRAME M₃yΔ₂/I ÷100	HORZ. LOAD LEFT OF ℄ M₃ FT KIPS	HORZ. LOAD LEFT OF ℄ M₃yΔ₂/I ÷100	HORZ LOAD RIGHT OF ℄ SECTION	HORZ LOAD RIGHT OF ℄ M₃ FT. KIPS	HORZ LOAD RIGHT OF ℄ M₃yΔ₂/I ÷100
1	PL 18x2½ PL 55x⅝	34.42	0	17.21	60	83165	4.02	8.58	25.42	0	0	715	1060	1'	0	0
2	"	17.0	.79	43.54	84	175431	8.48	2.01	38.20	162.2	143	1500	1315	2'	6.76	590
3	R-102 WEB-⅝	15.75	8.38	54.32	92	181493	8.77	1.79	52.80	1664.8	1620	1710	1660	3'	71.8	69.90
4	"	9.52	21.51	58.94	96	117940	5.69	1.68	58.30	4013.0	3970	1770	1750	4'	184.6	182.50
5	PL 18x⅝ R-72x⅜	9.52	31.03	61.71	96	92440	4.47	2.13	81.20	5530.0	7270	1810	2380	5'	266.0	350.0
6	"	9.52	40.55	64.48	96	92440	4.47	2.13	88.40	6870.0	9410	1850	2550	6'	347.5	476.0
7	"	9.52	50.07	67.25	96	92440	4.47	2.13	96.50	8060.0	11560	1880	2695	7'	429.0	615.0
8	"	9.52	59.59	70.02	96	92440	4.47	2.13	104.60	9080.0	13570	1900	2835	8'	511.0	763.0
9	"	9.52	69.11	72.79	96	92440	4.47	2.13	113.00	9930.0	15420	1920	2980	9'	592.0	916.0
10	"	9.52	78.63	75.56	96	92440	4.47	2.13	122.00	10640.0	17150	1950	3142	10'	675.0	1090.0
11	"	9.52	88.15	78.33	96	92440	4.47	2.13	131.00	11160.0	18600	1950	3255	11	755.0	1260.0
12	"	9.52	97.67	81.10	96	92440	4.47	2.13	140.50	11520.0	19920	1960	3390	12	837.0	14480
13	"	9.52	107.19	83.87	87	129476	6.25	1.53	107.40	11740.0	15050	1960	2510	13	919.0	1180.0
									1159.32		133983		31522			83563

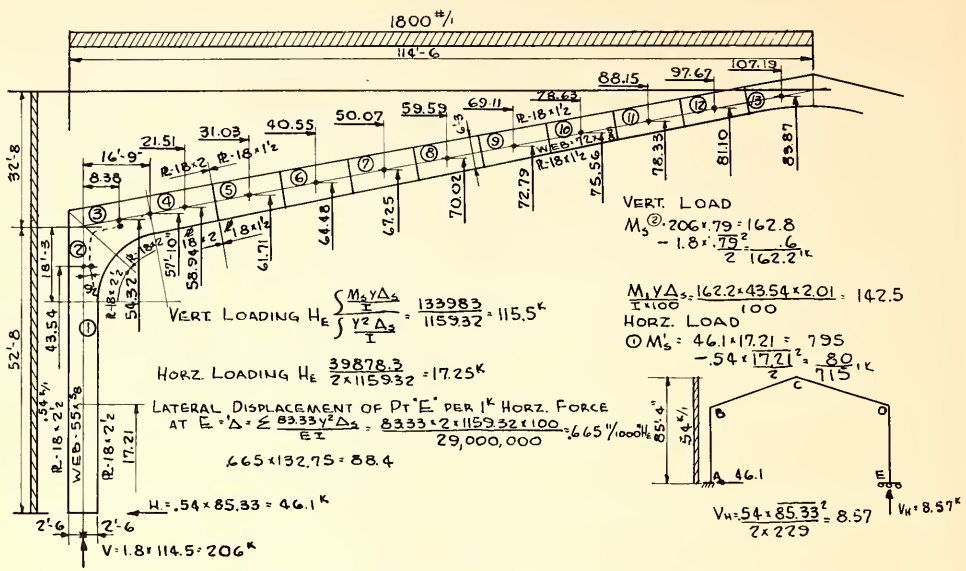
TOTAL HORZ. REACTION = H_E = 115.5 + 17.25 = 132.75ᵏ

H_A = 115.5 + 28.85 = 144.35

TOTAL VERT. REACTION = V_A = 206ᵏ

V_E = 206 + 8.57 = 214.57

8356.3
31522.0
39878.3



PROPERTIES OF FRAME										VERT. LOAD HALF OF FRAME		HORZ. LOAD LEFT OF ℄		HORZ LOAD RIGHT OF ℄		
SECTION	MEMBER	Δ₃ FT.	x FT	y FT.	d IN.	I IN⁴	I FT⁴	Δ₃/I	y²Δ₃/I ÷100	M₃ FT. KIPS	M₃yΔ₃/I ÷100	M₃ FT. KIPS	M₃yΔ₃/I ÷100	SECTION	M₃ FT. KIPS	M₃yΔ₃/I ÷100
1	18x2½ 55x5/16	34.92	0	17.21	60	83165	4.02	8.58	25.42	0	0	715	1060	1'	0	0
2	"	17.0	.79	43.54	84	175431	8.48	2.01	38.20	162.2	143	1500	1315	2'	676	590
3	18x2½ WEB-72x7/16	15.75	8.38	54.32	92	181493	8.77	1.79	52.80	1664.8	1620	1710	1660	3'	71.8	69.90
4	"	9.52	21.51	58.94	75	117940	5.69	1.68	58.30	4013.0	3970	1770	1750	4'	184.6	182.50
5	18x1½ 72x7/16	9.52	31.03	61.71	75	92440	4.47	2.13	81.20	5530.0	7270	1810	2380	5'	266.0	350.0
6	"	9.52	40.55	64.48	75	92440	4.47	2.13	88.40	6870.0	9410	1850	2550	6'	341.5	476.0
7	"	9.52	50.07	67.25	75	92440	4.47	2.13	96.50	8060.0	11560	1880	2695	7'	429.0	615.0
8	"	9.52	59.59	70.02	75	92440	4.47	2.13	104.60	9080.0	13570	1900	2835	8'	511.0	763.0
9	"	9.52	69.11	72.79	75	92440	4.47	2.13	113.00	9930.0	15420	1920	2980	9'	592.0	916.0
10	"	9.52	78.63	75.56	75	92440	4.47	2.13	122.00	10640.0	17150	1950	3142	10'	675.0	1090.0
11	"	9.52	88.15	78.33	75	92440	4.47	2.13	131.00	11160.0	18600	1950	3255	11	755.0	1260.0
12	"	9.52	97.67	81.10	75	92440	4.47	2.13	140.50	11520.0	19920	1960	3390	12	837.0	1448.0
13	"	9.52	107.19	83.87	87	129470	6.25	1.53	107.40	11740.0	15050	1960	2510	13	919.0	1180.0
									1159.32		133983		31522			83563

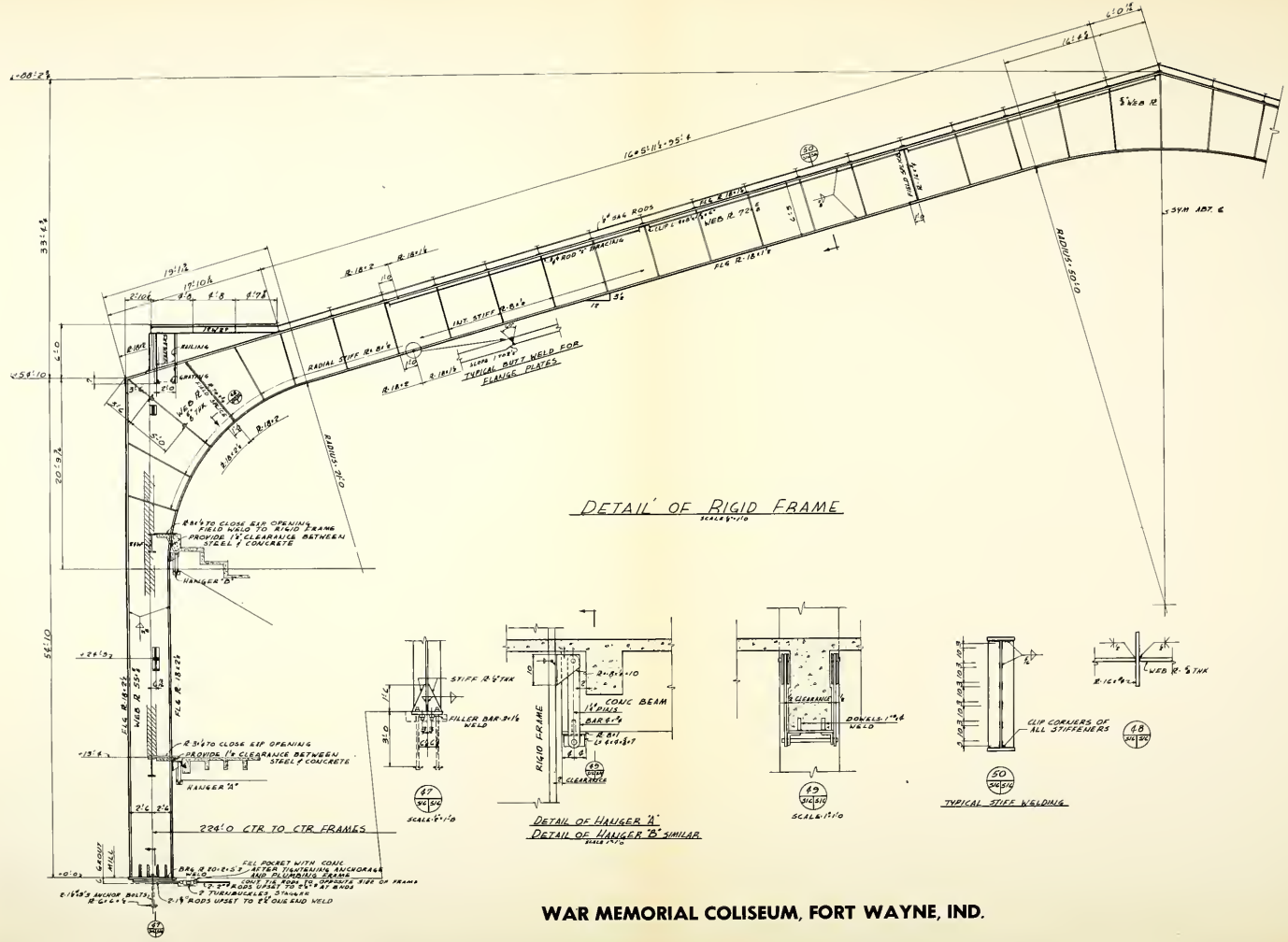
TOTAL HORZ. REACTION = H_E = 115.5 + 17.25 = 132.75 K

H_A = 115.5 + 2885 = 144.35

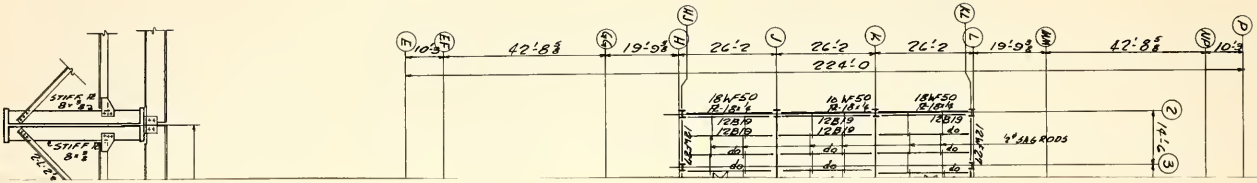
TOTAL VERT. REACTION = V_A = 206 K

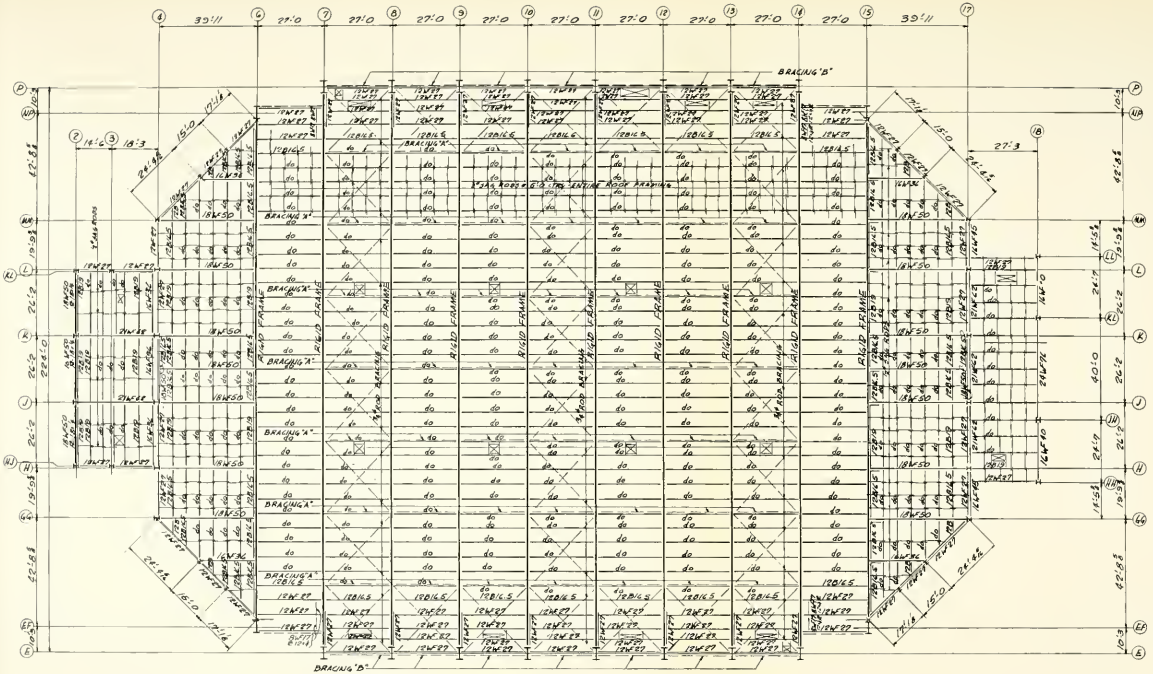
V_E = 206 + 8.57 = 214.57

8356.3
31522.0
39878.3

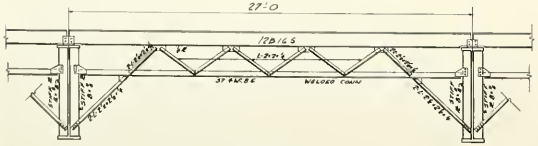


WAR MEMORIAL COLISEUM, FORT WAYNE, IND.

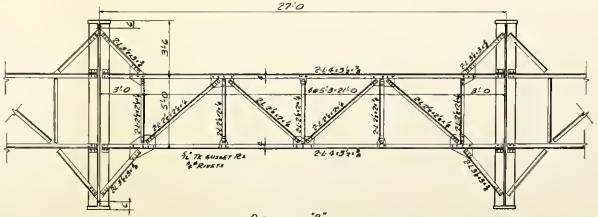

STIFF R
8"x⅞
STIFF R
8"x⅞
E
EF
42'-8⅜
G
19'-9⅜
H
HJ
2'-7½
J
2'-7½
K
2'-7½
KL
L
19'-9⅜
LM
42'-8⅜
M
10'-3
224'-0
18WF50
18WF50
18WF50
12B19
12B19
12B19
do
do
do
do
do
do
do
do
do
4" SAG RODS
2
3
14'-5"



ROOF FRAMING PLAN



BRACING "A"



BRACING "B"

WAR MEMORIAL COLISEUM, FORT WAYNE, IND.

Main Avenue Bridge Cleveland, Ohio

by
FRED L. PLUMMER*
(Formerly Chief Design Engineer,
Main Ave. Bridge Project)
Director of Engineering
Hammond Iron Works, Warren, Pa.

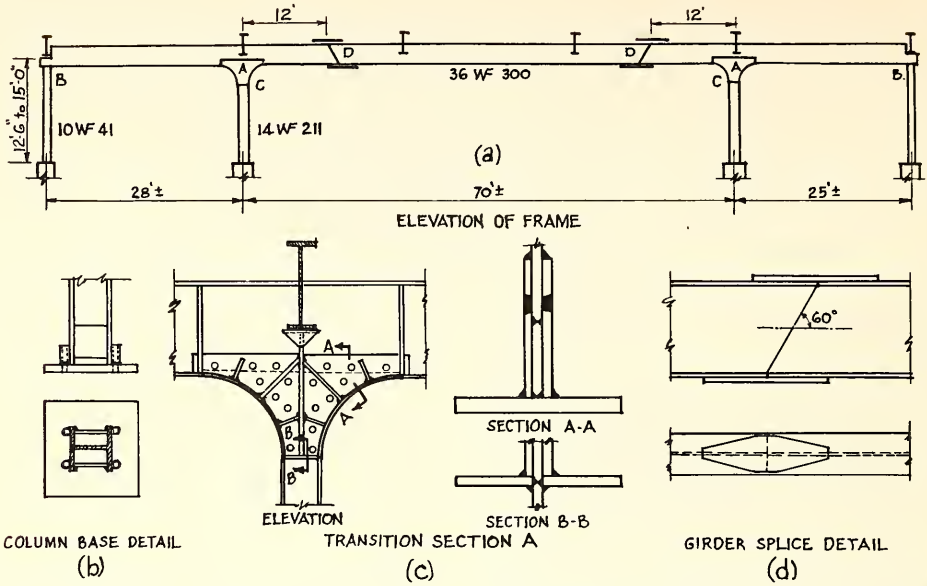
*Author's Note: Prime reason for presenting this article is the introduction of a design feature novel in this country, a transition section aiming to minimize brittleness in a highly stressed welded joint. It was projected by a novel type of man, engineer, scientist, research analyst, scholarly and accomplished, yet self-effacing and modest. Fred L. Plummer. While teaching engineering he was selected to head up this bridge job because of those special qualifications.

Rigid frame design was his first decision, welding followed, even though the year was 1939 when most other bridge engineers would not have tolerated such "new-fangled" ideas. Then—a question, what to do where the columns met head-on with the continuous girders, inducing crushing compression, triaxial stresses and consequent brittleness? The solution, automatic by progressive Plummer even though it had never before been attempted in the United States in a welded joint. He introduced a "transition section" of outside plates having flexible welded joints which divided the powerful concentration of compression into separate units of compression. And it was this separation which induced "breathing" and reduced to a minimum the possibility for brittle failure. A great idea which proved successful! M.P.K.

Two of the earliest all-welded rigid frame bridges constructed in this country form part of the approaches to Cleveland's Main Avenue Bridge constructed in 1939-40. Each bridge is a slightly skewed three-span structure carrying a 34 foot roadway and narrow escape walks, bringing the total width of each bridge to 40½ feet, on two continuous frames spaced about 26½ feet apart. The roadway on each bridge is built to a vertical curve, and on one of the structures a horizontal curve is also used. The center spans are about 70 feet flanked with short side spans of 25 to 50 feet.

The continuous welded frames are fabricated from standard wide-flange sections except for the transition sections A located at the tops of the interior columns as shown. Since service requirements to a large extent fixed the ratio of center span to side spans, it was found to be more economical not to provide a rigid connection at B between the girders and the end columns. At these points the girders rest on and are bolted to bearing plates welded to the tops of the columns.

The transition sections A provide for full continuity between the girders and the interior columns. These sections were formed by flame cutting out a section of the flange and adjacent web of the girder to a depth of 3 inches from the outside of the flange. The transition section was then built up and shop-welded to the girder as shown in detail. The web consists of three plates while the flange is a single plate of the same thickness as that of the flange of the



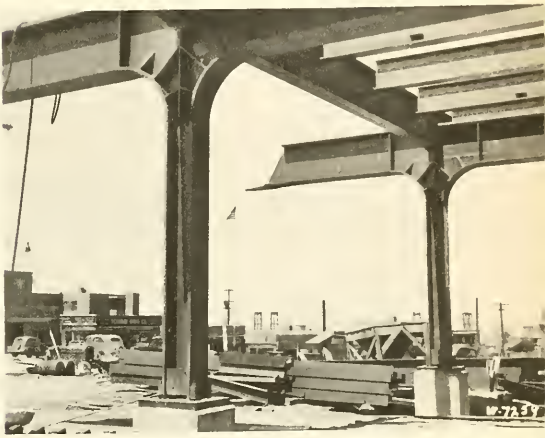
girder and the column. The center vertical stiffener was provided to receive and distribute the floor beam reaction while the purpose of the other stiffeners is to support the curved compression flange and to stiffen the highly stressed web adjacent to this flange. In determining the spacing and location of these stiffeners consideration was also given to the appearance of the resulting structure.

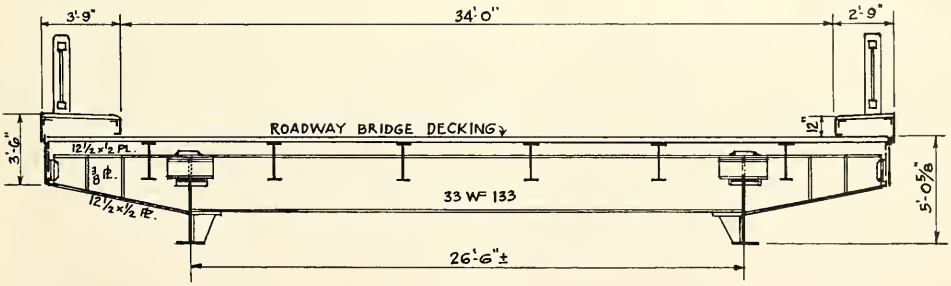
The outside web plates were used for two reasons. First, both tests and analytical studies indicate that in such sharply curved sections very high compression and shear stresses are created in the inner flange and adjacent web, whereas relatively low tension and shear stresses are built up in the outer flange and adjacent web. Second, experience in Europe had indicated that the intersection and overlapping of welds in corners or at other points where the surrounding metal is held rigidly might result in the creation of biaxial or triaxial stresses, which would cause the weld metal to fail as a very brittle material rather than as a ductile metal. The use of these outside plates provided a very small but it is believed sufficient amount of lateral flexibility to eliminate the possibility of such a "brittle material" failure. European engineers, who frequently use "cutouts" in all stiffeners and lateral plates to avoid such weld intersections, have commented very favorably on this feature of the design.

The only field welds in the frames were made at C and D. The four columns were erected first and bolted to the base plates and concrete piers. The columns were finished at the lower end for bearing and were not welded to the base plates. An end section of the girder was then swung in position, bolted to the end column at B and tack-welded to the inner column at C. The other end section of the girder was then placed and finally the center section of girder was dropped into position resting on the lower flange splice plates which had been shop-welded to the end girder sections. The erection of the two frames was carried on simultaneously, and floorbeams were placed in

position to provide lateral stability. This procedure made possible quick erection without the use of any falsework.

An H-20 loading was assumed in the design. Stresses created by lateral wind forces, longitudinal and centrifugal effects of live loads, and temperature changes were computed and combined with those created by the direct dead and live loads. The floor stringers were designed and detailed for full continuity for the entire length of the structures. Similarly, each floorbeam and its corresponding cantilever brackets were designed for fully continuous joints. For the welded frames hinged supports were assumed at the tops of the end columns, 80 per cent fixed supports at the bottom of the interior columns, and fully continuous joints at the tops of the interior columns. The end columns were designed to carry the superimposed load, giving due consideration to bending from longitudinal, lateral and temperature forces, and to the bending imposed upon the top of the column by the deflection of the adjacent girders of the frame.

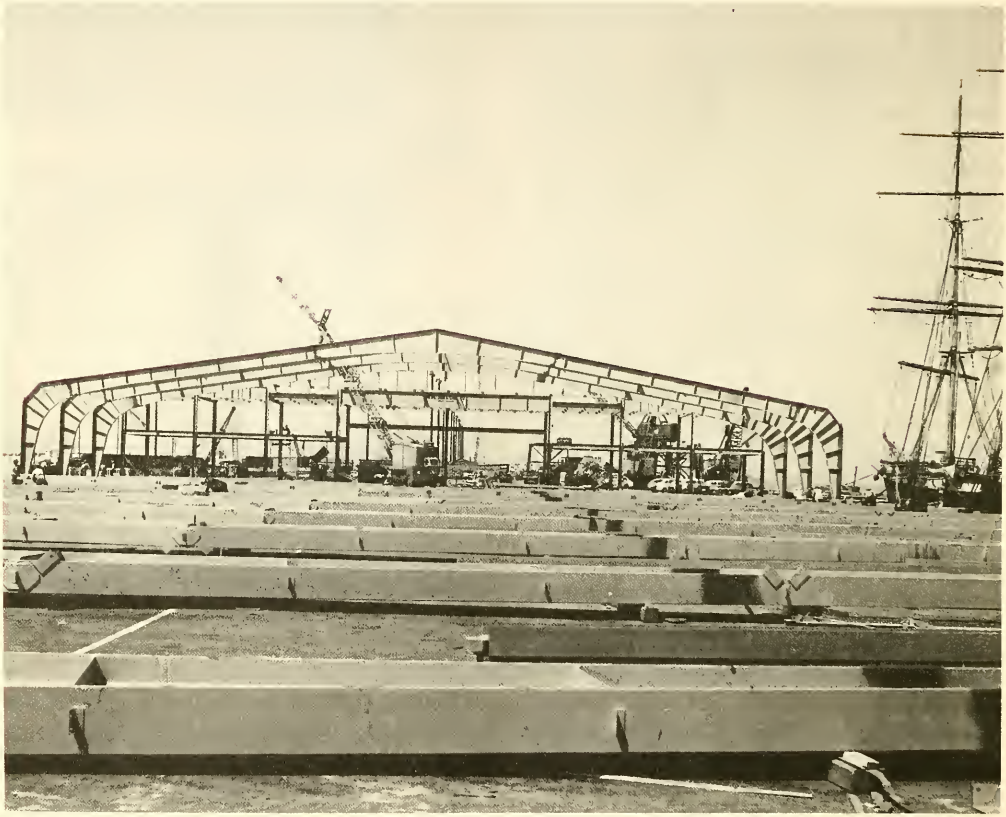




TYPICAL CROSS SECTION

Transit Shed, Long Beach, Cal.
200 Ft. Span*

by
R. R. SHOEMAKER
Chief Harbor Engineer
Long Beach, Cal.



*Author's Note: This structure embodies many unique features. Its size, 200 ft. x 1152 ft. makes it one of the largest clear-span transit sheds in the world. The welded steel rigid frames were nicely proportioned, the knees sloped around outside corners, the stiffeners were very well spaced. The need of design for horizontal seismic loadings, the ties placed far in advance of the shed construction—their embedment deep in the earth fill, the problem of loadings from the deep-seated subsidence, all are highly interesting.

A study of this entire job from superstructure down into the foundations left me with high admiration for the proven ability of Chief Engineer Shoemaker and for R. D. Sandham, Assistant Harbor Engineer, who was responsible for all the detailed design on the project. Slides and drawings of the structure attracted attention at several engineering colleges where they were studied following my lectures.

Several photographs are shown here of the shop fabrication done by Pacific Iron & Steel Co. of Los Angeles. Each frame was shipped in five sections, welded together 70% on the ground, then raised into place by three cranes working together. Henry Roberts Co. the erectors did the work in record time by this procedure.

From its conception in design, through fabrication and erection, this job was a masterly performance. M.P.K.

(The following data has been condensed from information furnished)

Live load per sq. ft. of horizontal projection on the roof, per the Long Beach Building Code ..... 20 lb. per sq. ft.

Dead load of roof structure, excepting the dead load of the rigid frames themselves .... 14 lb. per sq. ft.

Our building code, as well as the state law, stipulated a minimum wind load of 15 pounds per square foot, treated as acting horizontally, but, in our own instance, we had at the time of the design of this shed provision in the Long Beach Building Code requiring design for horizontal seismic loads equal to 10% of gravity, and, inasmuch as this building had poured concrete side walls averaging approximately ten inches in thickness, the seismic factor was the controlling element of design. The seismic load is applied at the points on the structure where it logically is tributary. The building code provides for the same unit stresses as are stipulated in the specifications of the American Institute of Steel Construction, with provision for increasing these allowable unit stresses in the amount of 33⅓% where seismic stresses are being combined with dead load and live load stresses.

The steel rod ties used on this project were placed far in advance of the detail design and construction of the superstructure, and the area occupied by the building was utilized for port purposes for some months in advance of commencing the building. The design was unique in that on the pierhead or waterfront side of the water side line of building foundations, there was located a vertical sheet pile bulkhead and a reinforced concrete wharf deck. This reinforced concrete wharf deck functions as a horizontal beam or wall to support the steel sheet pile bulkhead and transmit the thrust from the earth fill back of the bulkhead to the horizontal steel ties. This thrust is

Erecting 1st Frame

carried in tension by the steel ties and carried through the concrete of the front line of building foundations and across the 200-foot width of the building to timber pile foundations beneath the rear columns of the rigid frames. The footings at the rear side of the building incorporated batter piles designed to take the bulkhead thrust, and the total area of steel in the ties between the two footings was calculated on the basis of 20,000 pounds per square inch for the combined bulkhead reaction, and for the stress developed by reason of the loading on the frame. The ties were encased in 14-inch by 38-inch reinforced concrete struts which, being embedded 70 inches in the earth fill, gave us some degree of resistance against seismic forces tending to decrease the span.

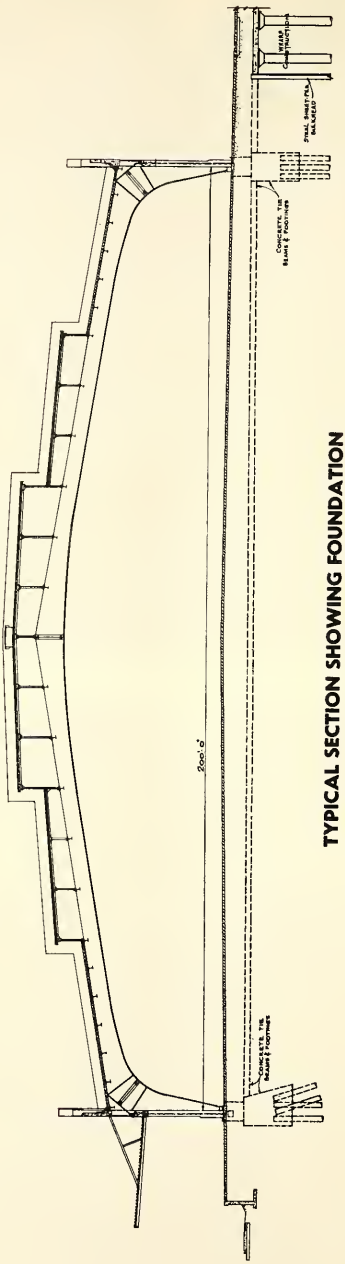
We have one or two more unusual loadings to which these frames are subjected which were not fully anticipated at the time of design. These stresses result from the deep seated subsidence which occurs in our immediate area, presumably as a result in a drawdown of fluid pressures in the oil field beneath, and which subsidence is not uniform. The differential subsidence within the 200-foot span is not serious in view of the fact that the supports are hinged, but, since the subsidence is very deep seated and is differential in quantity, it is accompanied by a horizontal movement which from year to year changes in direction. Thus 200-foot frames might tend to be slightly elongated at the elevation of the tie, depending upon the characteristics of the pile footings and the location of the structure. On the other hand, the same 200-foot spans may in later years be subjected to a tendency for shortening due to the same underground causes. However, due to the short width of the structure and the fact that the ties are 62 inches below the pin connections, it is entirely probable that cantilever action of the buried concrete footings will compensate for any small amount of movement.

The rigid frames were designed by the column analogy.

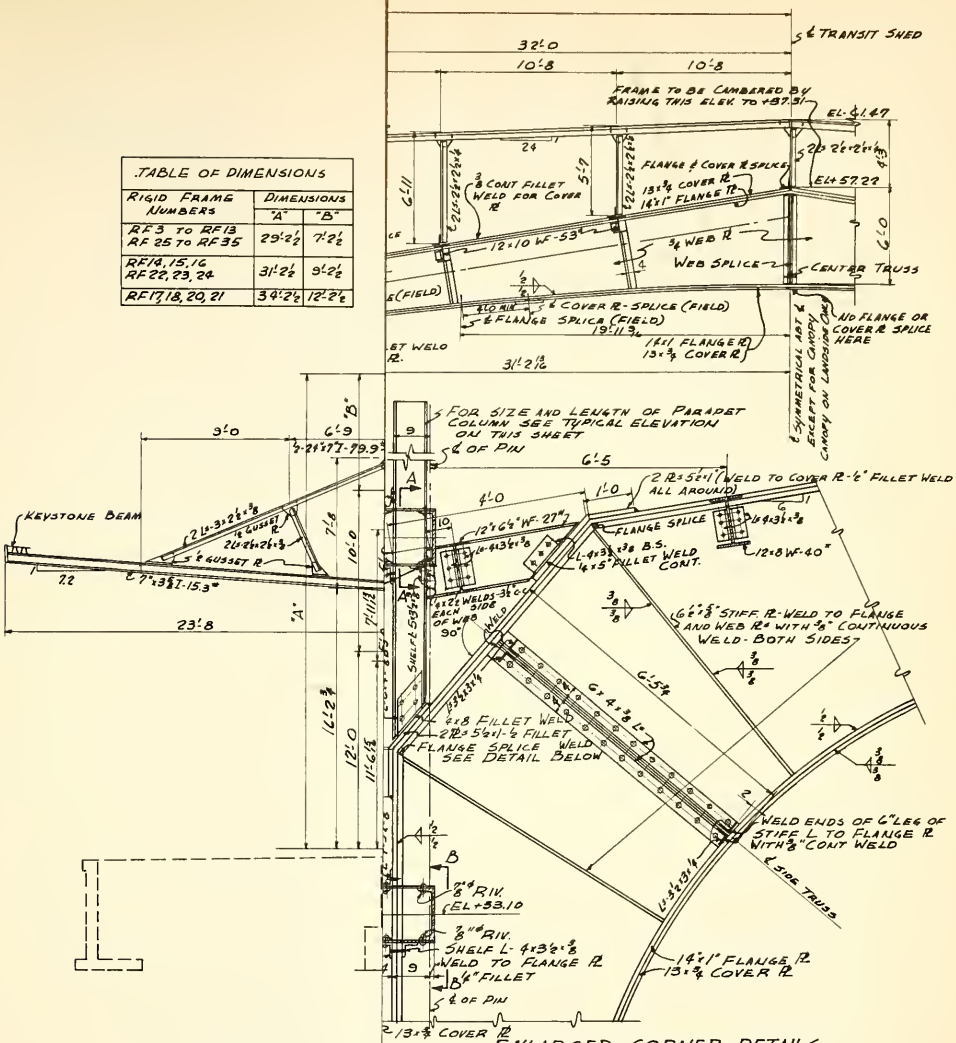
The following data refers only to the forces developed in the frame due to gravity loads, but includes the Long Beach Building Code requirement of 20-lbs. per square foot live load; making the total average gravity load equal to 43-lbs. per square foot on the horizontal projection.

The horizontal force, due to the above loading, developed at the hinges is closely 164 kips. The bending moment at the haunch 2789.7 ft. kips, and at the crown 561.3 ft. kips.

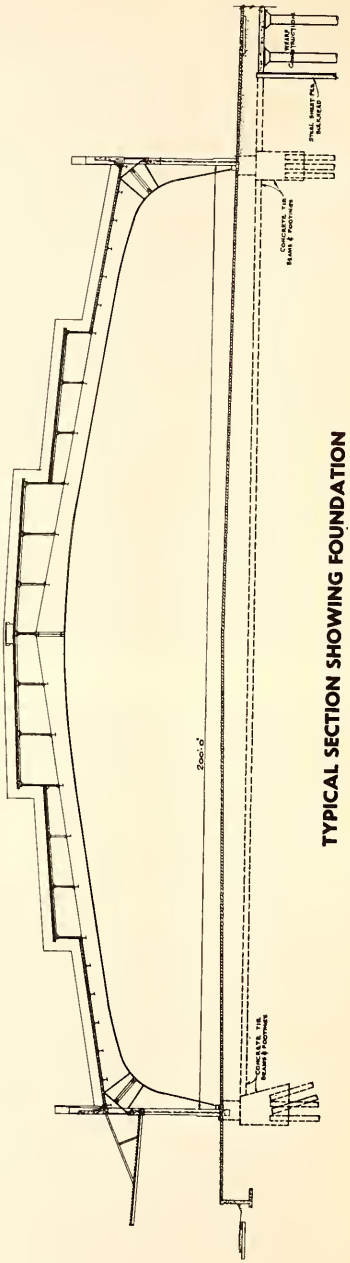
← Shop Fabrication →



TYPICAL SECTION SHOWING FOUNDATION



LONG BEACH, CAL.



TYPICAL SECTION SHOWING FOUNDATION

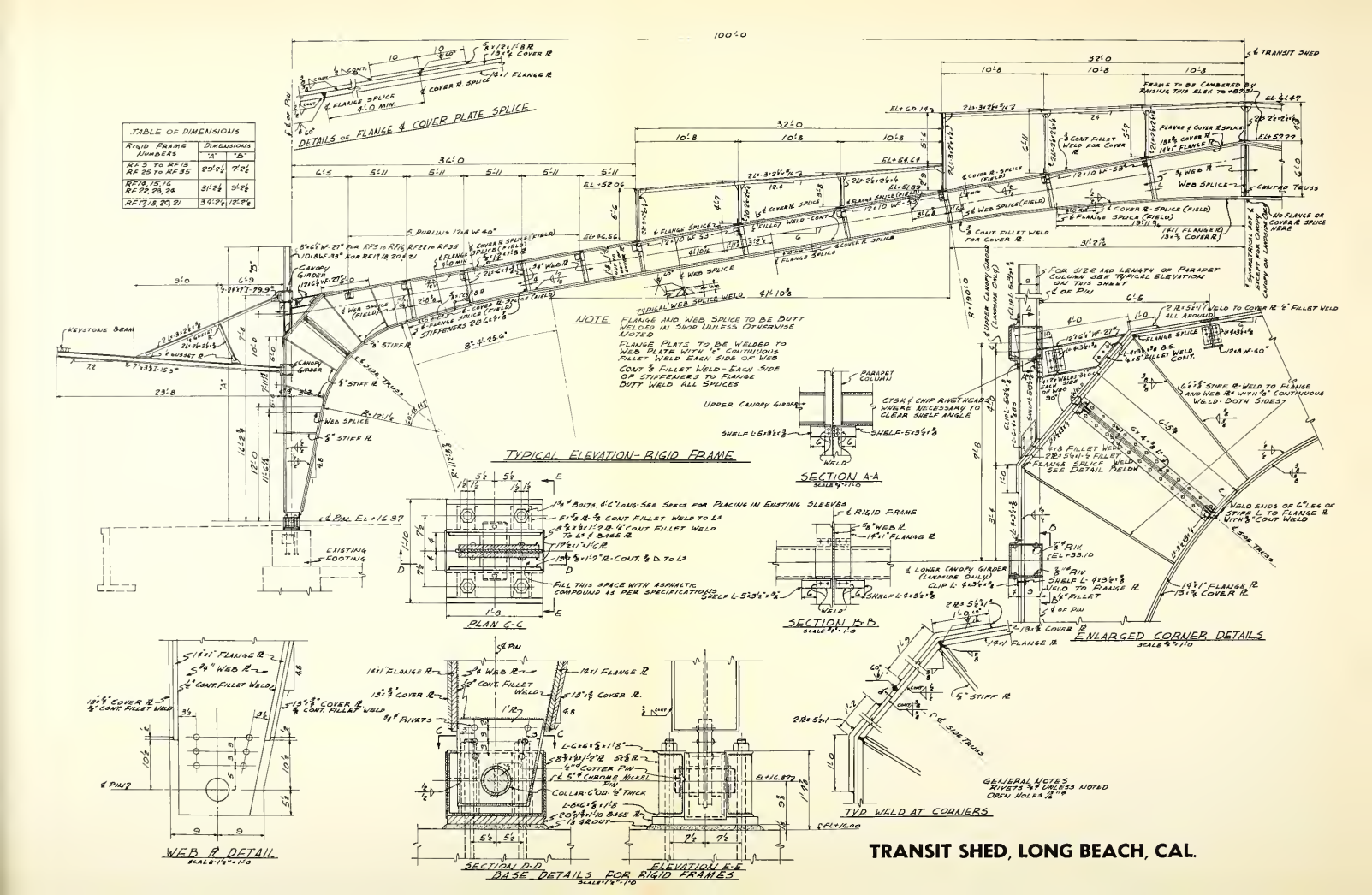


TABLE OF DIMENSIONS
RIGID FRAME NUMBERS
DIMENSIONS
"A"
"B"
RF 3 TO RF 18
RF 25 TO RF 35
29'-0½"
7'-2½"
RF 19, 15, 16
RF 22, 23, 24
31'-2½"
9'-2½"
RF 17, 18, 20, 21
34'-2½"
12'-2½"
DETAILS OF FLANGE & COVER PLATE SPLICE
TYPICAL ELEVATION - RIGID FRAME
NOTE
FLANGE AND WEB SPLICE TO BE BUTT WELDED IN SHOP UNLESS OTHERWISE NOTED
FLANGE PLATES TO BE WELDED TO WEB PLATE WITH ⅜" CONTINUOUS FILLET WELD EACH SIDE OF WEB
CONT ⅜ FILLET WELD - EACH SIDE OF STIFFENERS TO FLANGE
BUTT WELD ALL SPLICES
SECTION A-A
SECTION B-B
PLAN C-C
WEB R DETAIL
SECTION D-D
ELEVATION E-E
BASE DETAILS FOR RIGID FRAMES
ENLARGED CORNER DETAILS
TYP. WELD AT CORNERS
GENERAL NOTES
RIVETS ⅞" Ø UNLESS NOTED
OPEN HOLES 1⅛" Ø
TRANSIT SHED, LONG BEACH, CAL.

Three-Hinged Frames 132 Ft. Span*

Philips Electrical Works, Eindhoven, Holland

by

J. R. BOUTEN

Director and Architect

These rigid frames were erected in 1951 for a recreation building 132 ft. wide, 200 ft. long. Bents were spaced 40 ft. on centers. Following is excerpt from letter received from Director Bouton, reproduced without change—the word "truss" meaning rigid frame:

"This hall had to house an exhibition at the commemoration of the 60th anniversary of Philips Electrical Industries in May 1951 and afterwards to be used as a sports hall for the Philips employees, especially for three tennis courts.

The idea arose in September 1950, it had to be (and was) ready 1st of May, 1951.

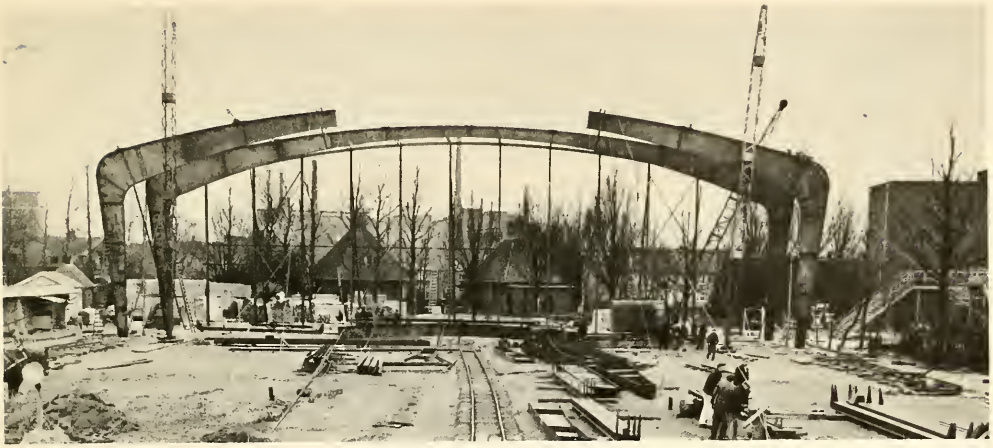
Originally I had in mind to make glued timber trusses, as being used many times for similar halls in Europe, especially in Sweden. The difficulty was that each truss had to be divided in at least six parts to enable transport. The joints between these parts, if constructed in wood, would have been rather complicated and ponderous and therefore I decided to use welded steel.

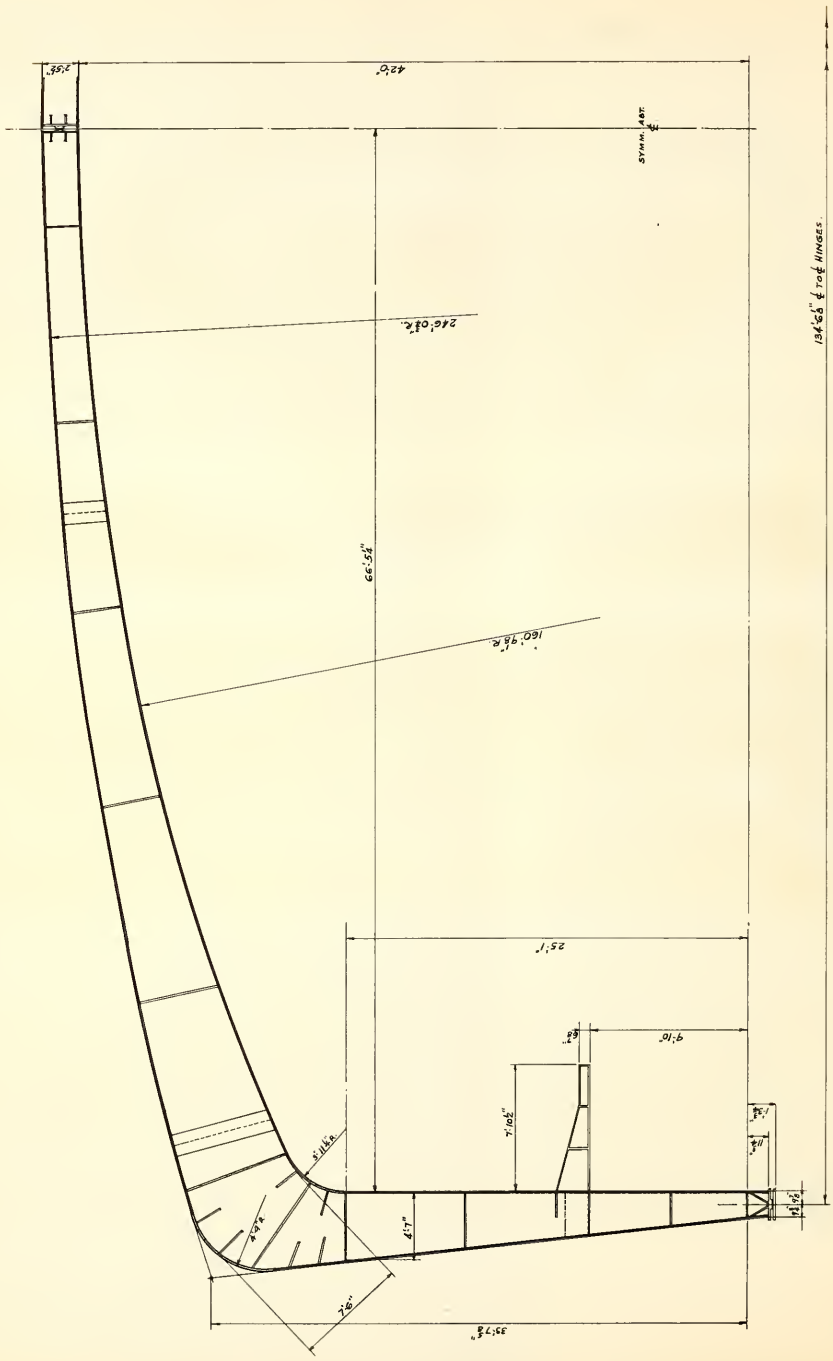
To obtain not only a better outside architecture but mainly to improve the inside impression as well, I made the innerside of the stanchions vertical and found the necessary increase of the sections by including the outside flange. I think this solution, apart from some other secondary advantages, has been a great success, and I am of the opinion that due to this solution the inside of the hall gives the impression of being much wider than it really is.

The trusses are three-hinge joint type, transported to the side in six sections. As you will see on the photographs, the stanchions were connected with the corner by screwbolts on the ground. We preferred this type of connection because by welding these joints these rather heavy parts had to be turned over for the welding on both sides. The middle joint was temporarily fixed and hoisted up by a caterpillar crane."

*Author's Note: My effort to include a good-looking three-hinged frame in this Manual led to search afar. Selection of this structure built in Holland was made on the basis of beauty in architecture and excellence in structural design. The photographs here tell a much better story about the frames than anything I could write. Also given above is a reproduction without change of part of a very interesting letter from Director Bouton, architect of this impressive building. It should prove of special interest to architects as well as to engineers. His fresh style of expression, undiluted by me, may give the reader a pleasant change.

Accompanying the drawings and photographs which Architect Bouton sent me was a complete set of design computations comprising 28 typed sheets in the Flemish language, neatly bound, and including stress diagrams. They will be loaned out to colleges for study. M.P.K.





PART CROSS SECTION, EINDHOVEN, HOLLAND

Boston Navy Yard, Extension to Structural Shop, 102 Ft. Span*

J. F. JELLEY, Rear Admiral, CEC, USN
Chief, Bureau of Yards & Docks, Navy Dept.

A. AMIRIKIAN, Head Designing Engineer

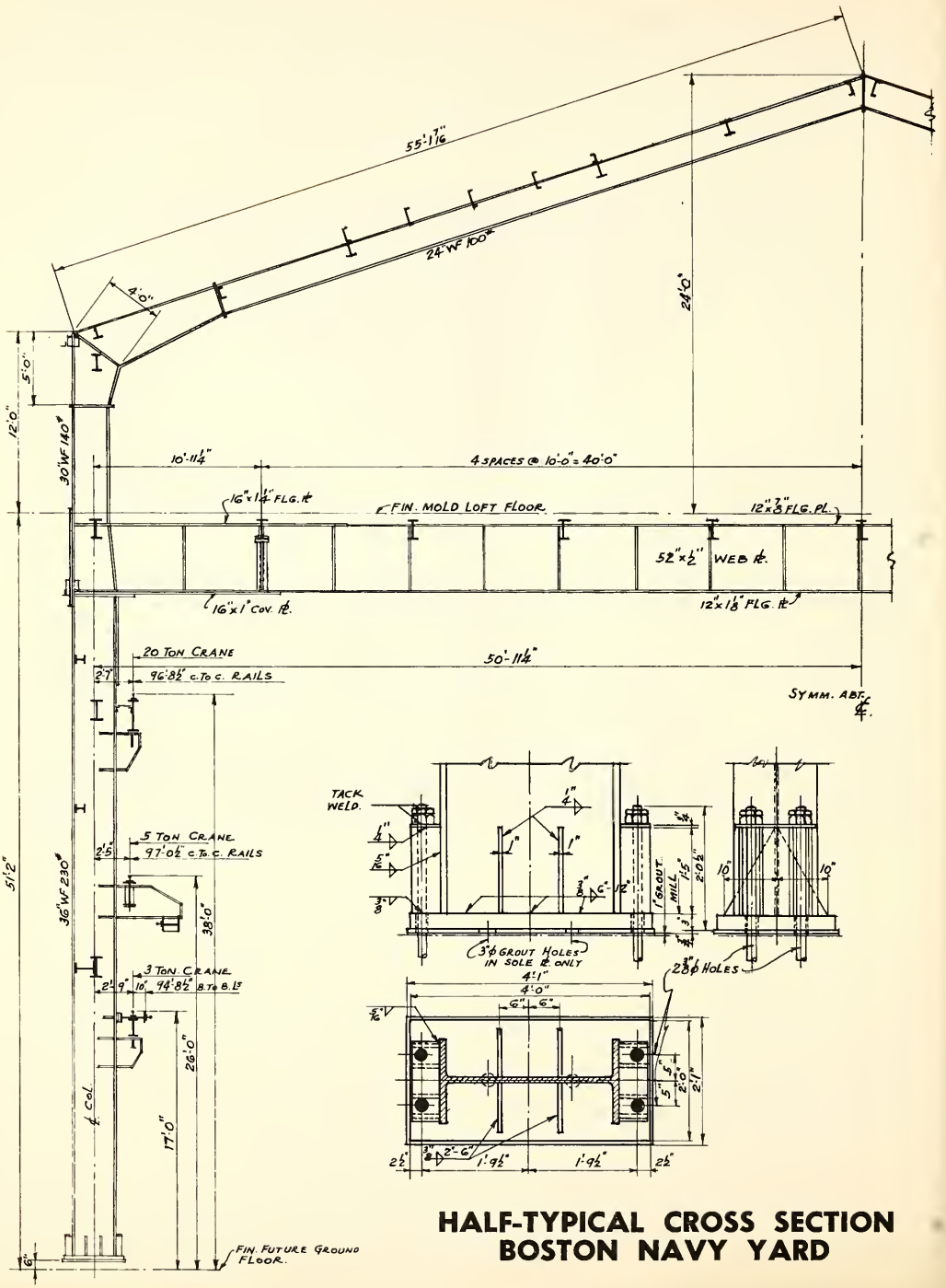
*Author's Note: In 1938 the Bureau of Yards & Docks invited me, among others, to review and criticise their proposed new Standards of Design and new Specifications for Structural Steel Welding. Nothing embodied more than their proposed Standards, the highest available technical knowledge of that day. Those in private practice who were inclined to think of service departments as "moth-eaten" saw here a most progressive organization from Big Chief Ben Moreel down through the ranks. An esprit de corps had been developed which in the field of engineering accomplishments spelled "CAN DO" for everything, everywhere, every time.

That spirit got an added shot forward with the coming of their present Chief, Admiral J. F. Jelley, Naval Academy, Cornell and Rensselaer, striking symbol of the "SPIRIT OF CAN DO."

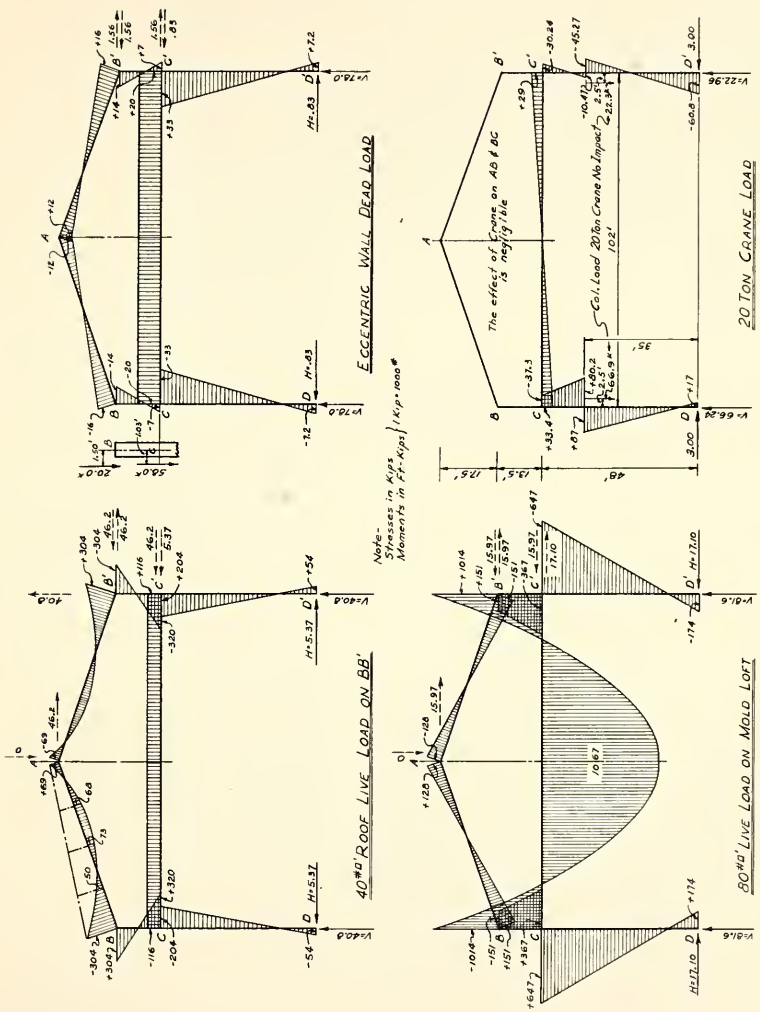
As to their "whiz-bang" Engineer of Design, A. Amirikian, who here hasn't heard of him? (I see no hands raised.) Always one jump ahead of the times, he introduced welded drydocks for the old riveted type (thus saving the government millions) and welded rigid frames for buildings, produced a book "Analysis of Rigid Frames" telling you what you did not know, made innovations in concrete design, etc., etc. And probably at this moment he has next year's supply of original designs all sewed up for future delivery.

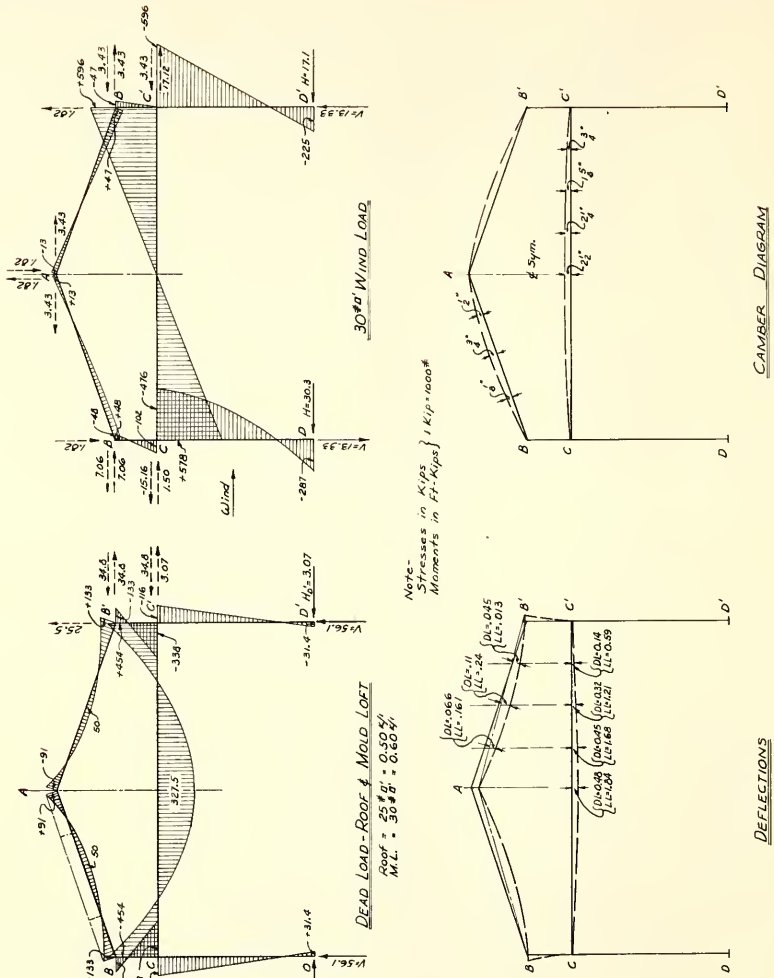
The Boston Navy Yard job (1939) shown here is typical of his early work and his thoroughness in analysis (slope deflection) from which you can learn considerable today. This design was presented for alternate bids with a conventional one of riveted trusses and proved much lower in costs.

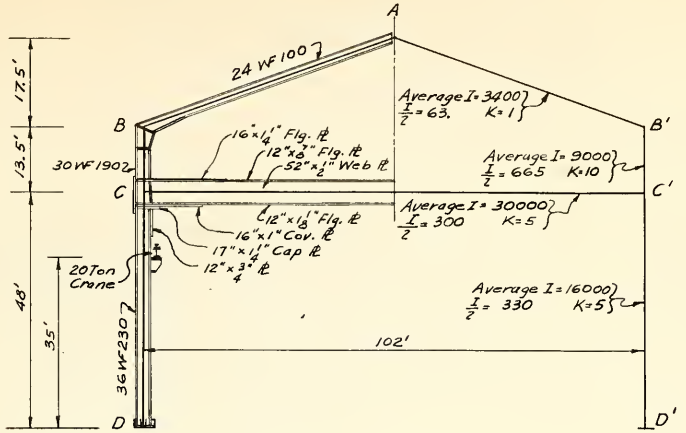
As an old Army man, with CAN DO ideas, I have but one regret for my country—that the Bureau of Yards & Docks belongs to the Navy Department. And the hope persists that President Ike, as another old Army man, will agree with me—say CAN DO to my suggestion and transfer them, lock, stock and Chief Jelley to the ARMY! I'm sure that my old Tank battle-buddy General George G. Patton wouldn't have thought them too bad to soldier with us. M.P.K.



HALF-TYPICAL CROSS SECTION BOSTON NAVY YARD

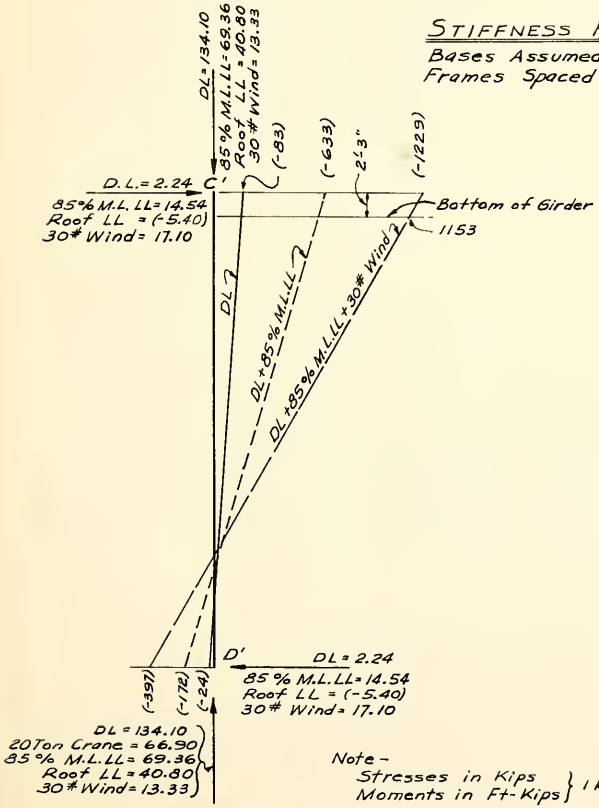

ECCENTRIC WALL DEAD LOAD
40#/0' ROOF LIVE LOAD ON BB'
80#/0' LIVE LOAD ON MOLD LOFT
20 TON CRANE LOAD
Note-
Stresses in Kips
Moments in Ft-Kips
Kip = 1000#
The effect of Crane on AB & BC
is negligible
Col. Load 20 Ton Crane No Impact
V=78.0
V=40.8
V=81.6
V=22.96
V=66.24
H=.83
H=5.37
H=12.10
H=11.10
D'
3.00
48'
35'
13.5'
12.5'

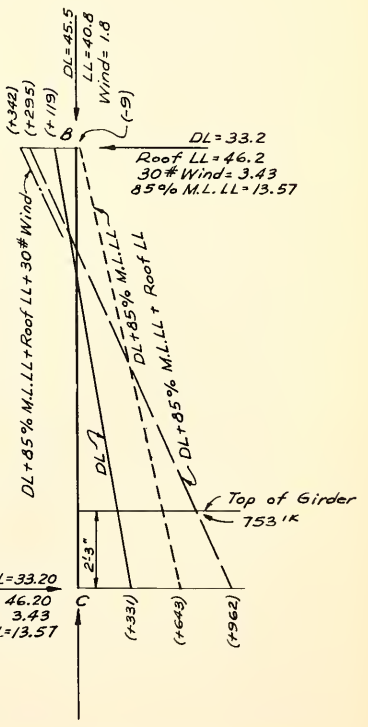

DEAD LOAD - ROOF & MOULD LOFT
Roof = 25#/ft² = 0.50%
M.L. = 30#/ft² = 0.60%
30#/ft WIND LOAD
Wind
Note-
Stresses in K/ps
Moments in Ft-Kips
1 K/p = 1000#
DEFLECTIONS
CAMBER DIAGRAM



STIFFNESS FACTORS

Bases Assumed 50% Fixed
Frames Spaced 20'-0 c.c.

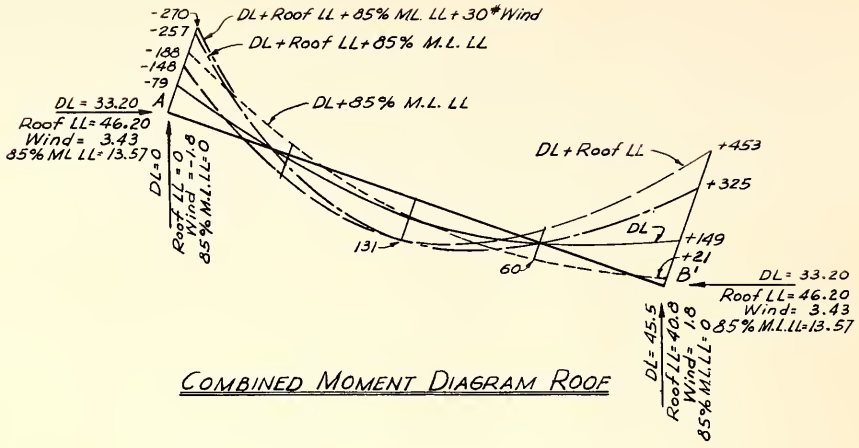




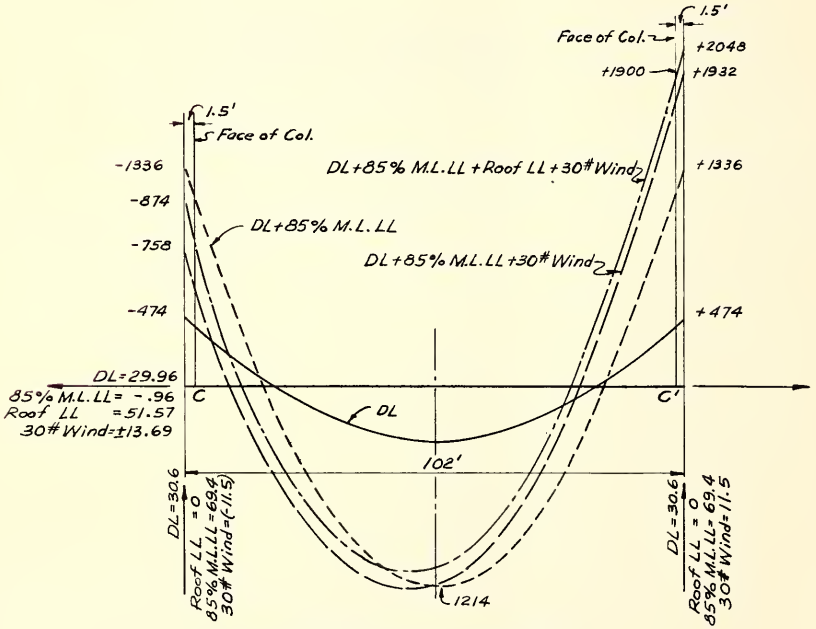
Note –
Stresses in Kips
Moments in Ft-Kips
} 1 Kip = 1000#

COMBINED MOMENT DIAGRAM
LEEWARD COL. BELOW MOLD LOFT FL.

COMBINED MOMENT DIAGRAM
LEEWARD COL. ABOVE MOLD LOFT FL.



COMBINED MOMENT DIAGRAM ROOF



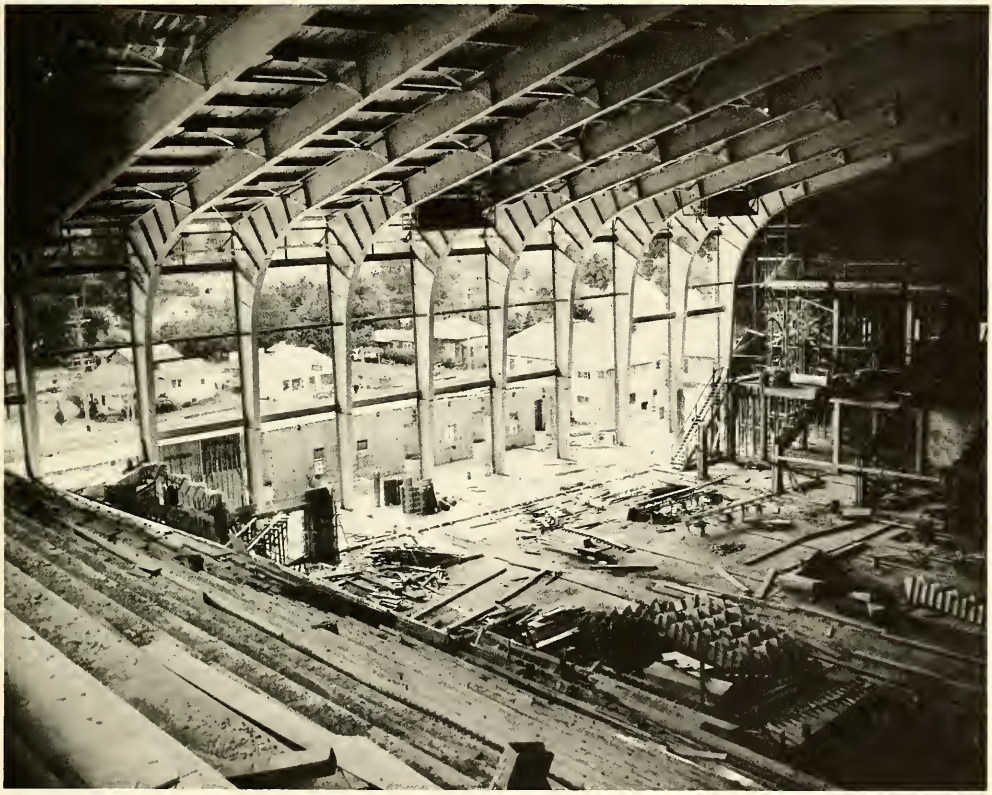
COMBINED MOMENT DIAGRAM
MOLD LOFT FLOOR

Note-
Stresses in Kips
Moments in Ft-Kips
} 1 Kip = 1000#

Oregon State College Pavilion Corvallis, Ore., 222 Ft. Span*

PROF. GLENN W. HOLCOMB
Oregon State College

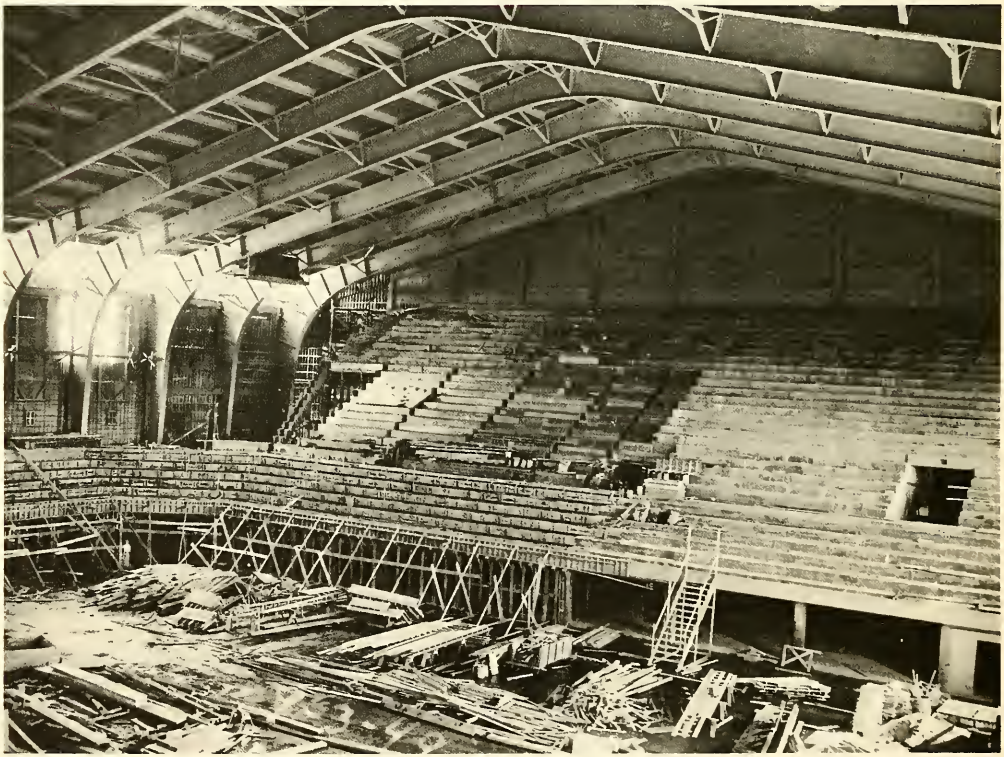
MILES K. COOPER & ROWLAND S. ROSE
Structural Engineers, Portland 4, Ore.



Seating accommodations for approximately 10,000 basketball fans were provided in the new Pavilion erected for Oregon State College at Corvallis, Oregon. It is a complete separate unit and marks the first step of a series of improvements to the present athletic facilities as well as being a place for miscellaneous student body gatherings.

*Author's Note: This structure, when completed in 1948, was of near record size. An excellent design, well proportioned throughout, its stiffeners nicely spaced, it is presented here as an interesting structure for study. Note the splice through the haunch, most uneconomical, but dictated by shipping conditions. The above paper by Mr. Rosé, contributed in cooperation with Prof. Holcomb, gives the general description. A brass model, made to a small scale at Oregon State College, checked closely the computed deflections. M.P.K.

Outstanding structural feature of the reinforced concrete and structural steel building is the use of twelve steel rigid frame arches of 222′-0″ span and 84′-0″ rise, making them among the largest in the country. Legs of the arches taper from 2′-6″ overall depth at the bottom to 13′-0″ diagonal at the haunch. The remainder has a uniform depth of 4′-6″ except for the gracefully curved inside flange at the haunches and center ridge. The arches are supported at the first floor on 4½″ diameter cold rolled steel pins which in turn are supported on steel base plates anchored to the concrete foundation. The outward thrust of each arch leg is taken by a single 2¼″ tie rod attached to each base plate and provided with turnbuckles for take up. These rods rest in recesses in the first floor, which were filled with concrete after the tie rods had been finally adjusted. Use of these rigid frame arches spaced on 20′-0″ centers permitted the elimination of all columns and incorporation of a steep balcony line for good vision while still retaining a comparatively low roof line. Another unusual feature of the building is the balcony layout. A major portion of the balcony seats are placed parallel to the long way of the 106′×170′ playing floor for better vision and because of the greater depth in this direction the steel roof arches are set lengthwise of the main playing floor.

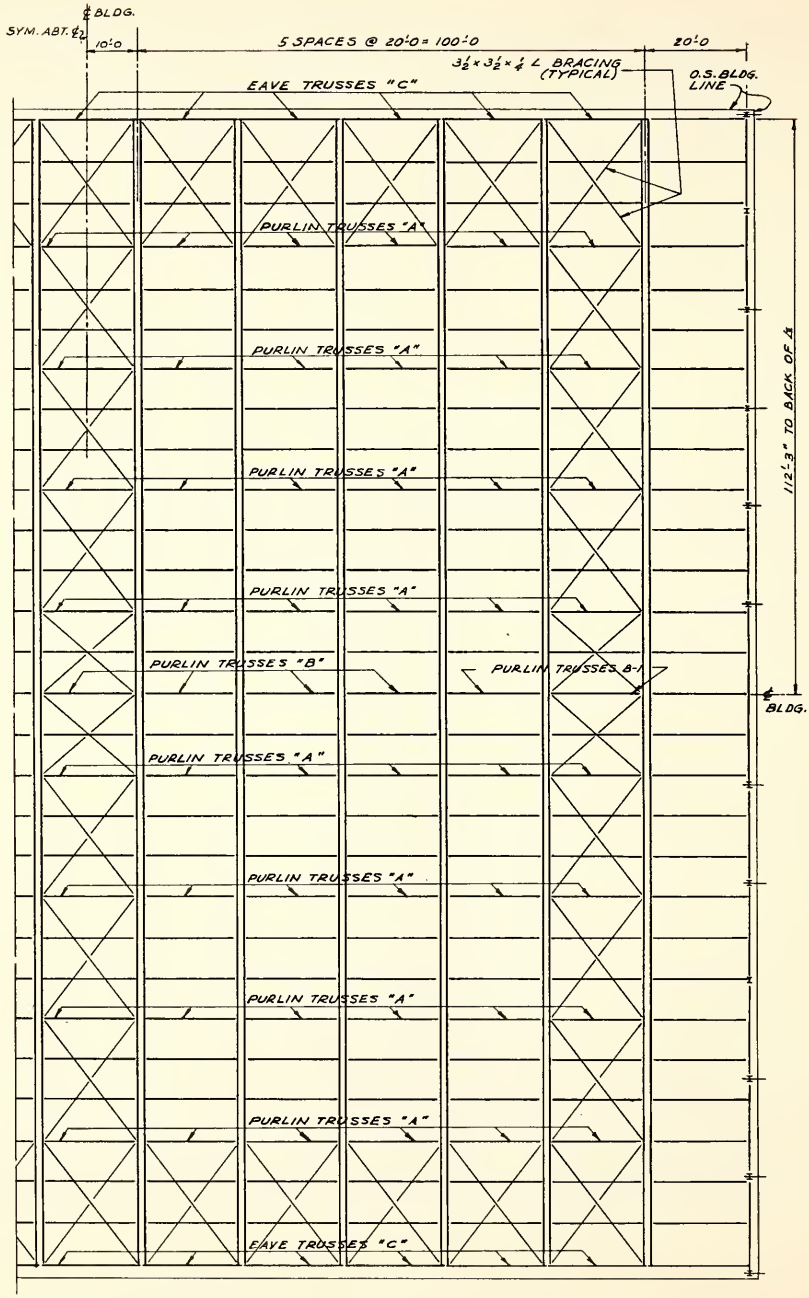


The arches themselves, though designed as a two hinged arch, are each composed of seven separate pieces with a total weight of sixty tons. Transportation facilities necessarily limited the size of the pieces which could be shipped; therefore, considerable field erection was required. The two legs of each frame were set first and held in position by guy cables and a wood post. The haunch splice plates were then attached to the legs, after which the first rafter section was swung into place and attached. The outer end of the first rafter section was then jacked to correct elevation by means of a thirty ton jack mounted on a wooden tower. The second rafter section was then cantilevered from this point. After both sides of a single frame had progressed to this point, the center ridge section was lowered into position and the arch completed. Erection of all arches was accomplished by means of two fixed derricks which were positioned three times for the erection of the twelve frames, which were set one at a time and braced to the previously erected arch. Preliminary erection was 90% bolted, after which riveting crews accomplished all final splicing.

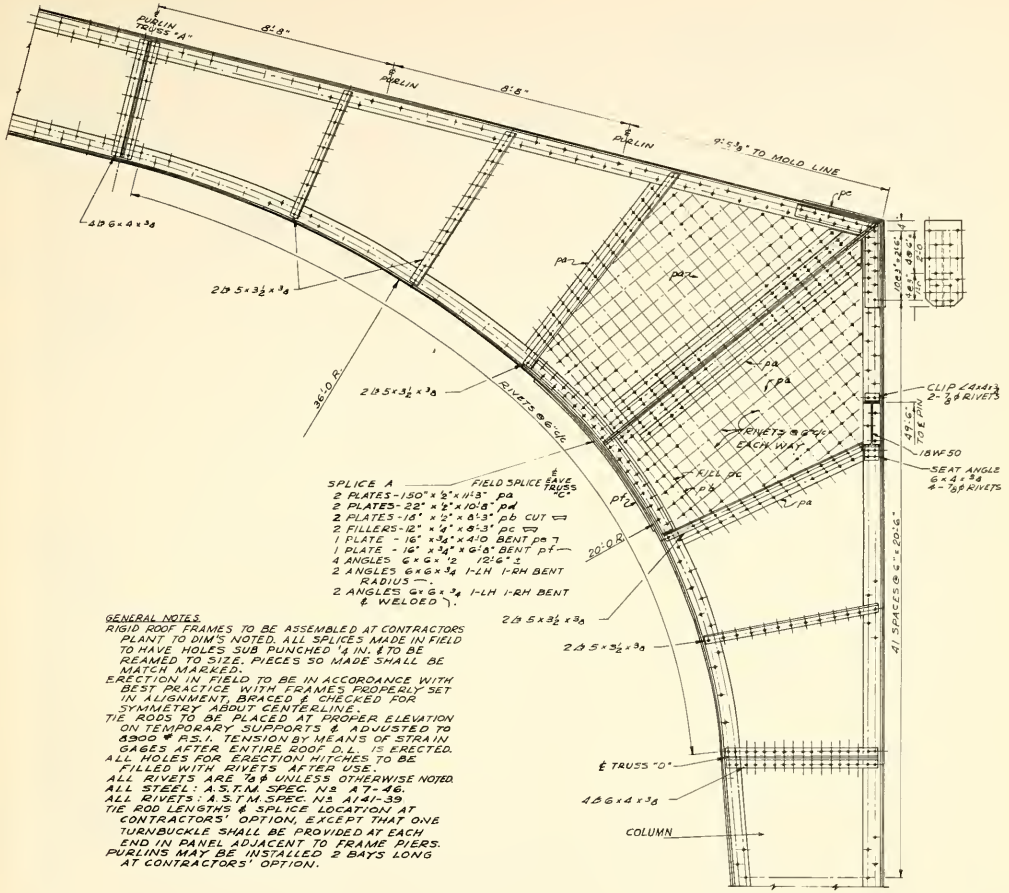
Main lateral bracing for the frames consists of thirteen separate lines of truss type bridging between which are placed 10″ steel roof beams to support the two inch wood roof deck. A conventional built-up roof was laid over the roof deck and the underside was acoustically treated.

The remainder of the building is composed of reinforced concrete construction. The first floor has an enclosed area of 263′-2″ × 226′-10″, or approximately 1.35 acres, and is devoted to various concessions, student offices, athletic facilities and corridors in addition to the main playing floor. The basement occupies the entire area under the first floor and is devoted to athletic facilities and storage. The second floor area is utilized for balcony access corridors, rest rooms and concession rooms. The balcony, constructed entirely of reinforced concrete, has a main access aisle at the second floor level entirely around the playing floor. Six rows of seats are provided below the aisle entirely around the four sides of the main floor. Above the aisle, twenty-two rows of seats on the two lengthwise sides of the playing floor are provided.

Support for the playing floor is provided by the structural concrete first floor which is recessed six inches to provide room for 4×4 sleepers 2′-0″ on centers, over which are laid 2×5 planks at right angles. After these were laid, a select quality 13/16″ maple floor was laid to provide as near perfect a playing floor as possible.



HALF ROOF FRAMING PLAN



SPLICE A — FIELD SPLICE AT ℄ EAVE TRUSS "C"

2 PLATES - 150" x ½" x 11'-3"	pa
2 PLATES - 22" x ½" x 10'-8"	pd
2 PLATES - 18" x ½" x 8'-3"	pb CUT ⇨
2 FILLERS - 12" x ¼" x 8'-3"	pe ⇨
1 PLATE - 16" x ⅜" x 4'-10" BENT	pa ⅂
1 PLATE - 16" x ⅜" x 6'-8" BENT	pf — RADIUS —
4 ANGLES 6 x 6 x ½	12'-6" ±
2 ANGLES 6 x 6 x ⅜	1-LH 1-RH BENT
2 ANGLES 6 x 6 x ⅜ ℄ WELDED ⅂	1-LH 1-RH BENT

GENERAL NOTES.

RIGID ROOF FRAMES TO BE ASSEMBLED AT CONTRACTORS PLANT TO DIM'S NOTED. ALL SPLICES MADE IN FIELD TO HAVE HOLES SUB PUNCHED 1/4 IN. ℄ TO BE REAMED TO SIZE. PIECES SO MADE SHALL BE MATCH MARKED.

ERECTION IN FIELD TO BE IN ACCORDANCE WITH BEST PRACTICE WITH FRAMES PROPERLY SET IN ALIGNMENT, BRACED ℄ CHECKED FOR SYMMETRY ABOUT CENTERLINE.

TIE RODS TO BE PLACED AT PROPER ELEVATION ON TEMPORARY SUPPORTS ℄ ADJUSTED TO 6300 # P.S.I. TENSION BY MEANS OF STRAIN GAGES AFTER ENTIRE ROOF D.L. IS ERECTED.

ALL HOLES FOR ERECTION HITCHES TO BE FILLED WITH RIVETS AFTER USE.

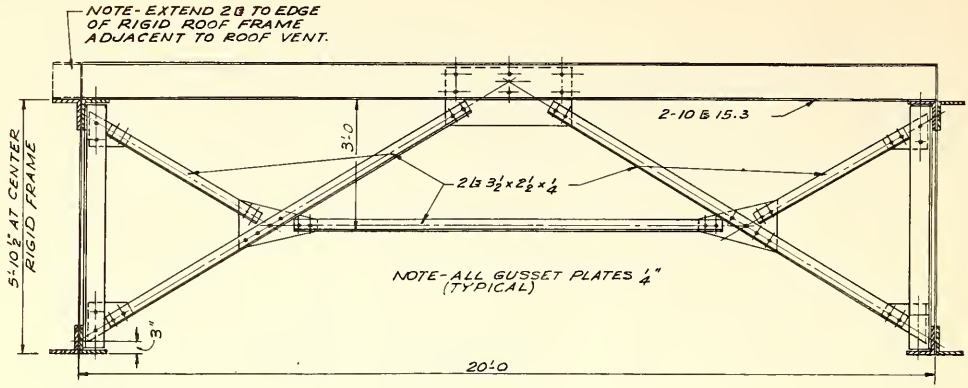
ALL RIVETS ARE ⅞ ø UNLESS OTHERWISE NOTED.

ALL STEEL: A.S.T.M. SPEC. NO. A7-46.

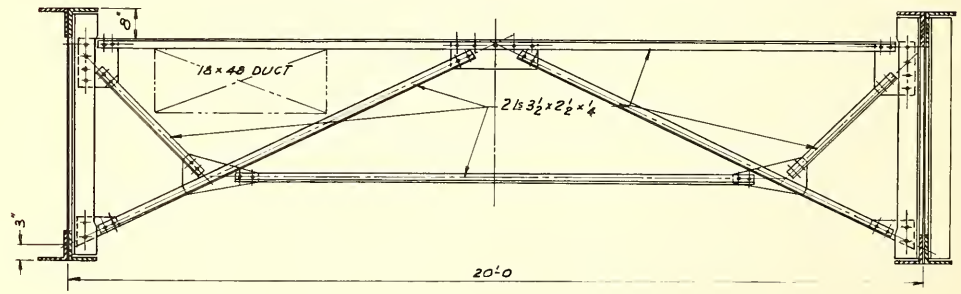
ALL RIVETS: A.S.T.M. SPEC. NO. A141-39

TIE ROD LENGTHS ℄ SPLICE LOCATION AT CONTRACTORS' OPTION, EXCEPT THAT ONE TURNBUCKLE SHALL BE PROVIDED AT EACH END IN PANEL ADJACENT TO FRAME PIERS.

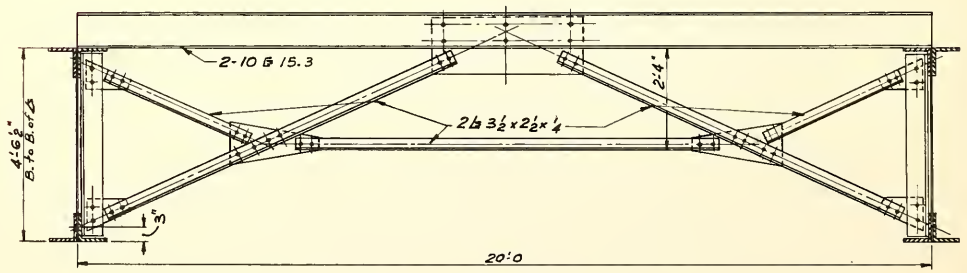
PURLINS MAY BE INSTALLED 2 BAYS LONG AT CONTRACTORS' OPTION.


NOTE- EXTEND 2 @ TO EDGE OF RIGID ROOF FRAME ADJACENT TO ROOF VENT.
5'-10½" AT CENTER RIGID FRAME
3'-0
2-10 B 15.3
2 ⌊s 3½ x 2½ x ¼
NOTE- ALL GUSSET PLATES ¼" (TYPICAL)
3"
20'-0

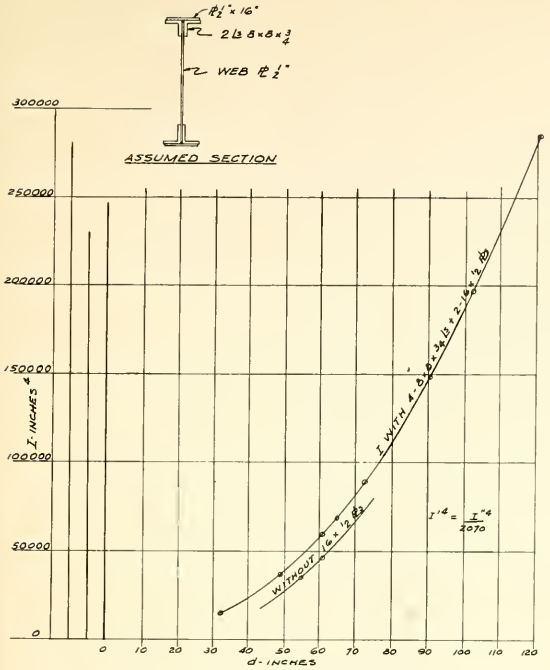
PURLIN TRUSS "B1"



TRUSS "D"

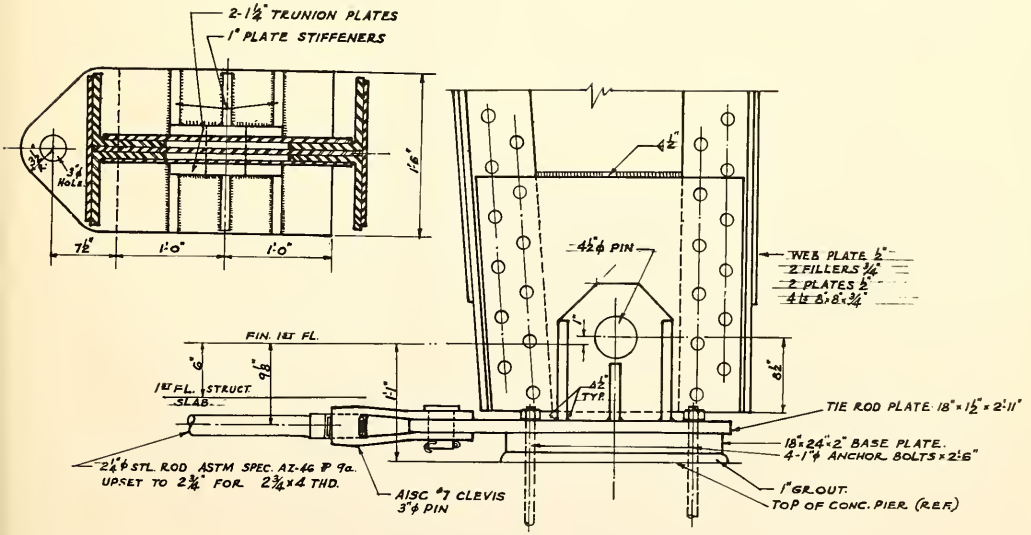


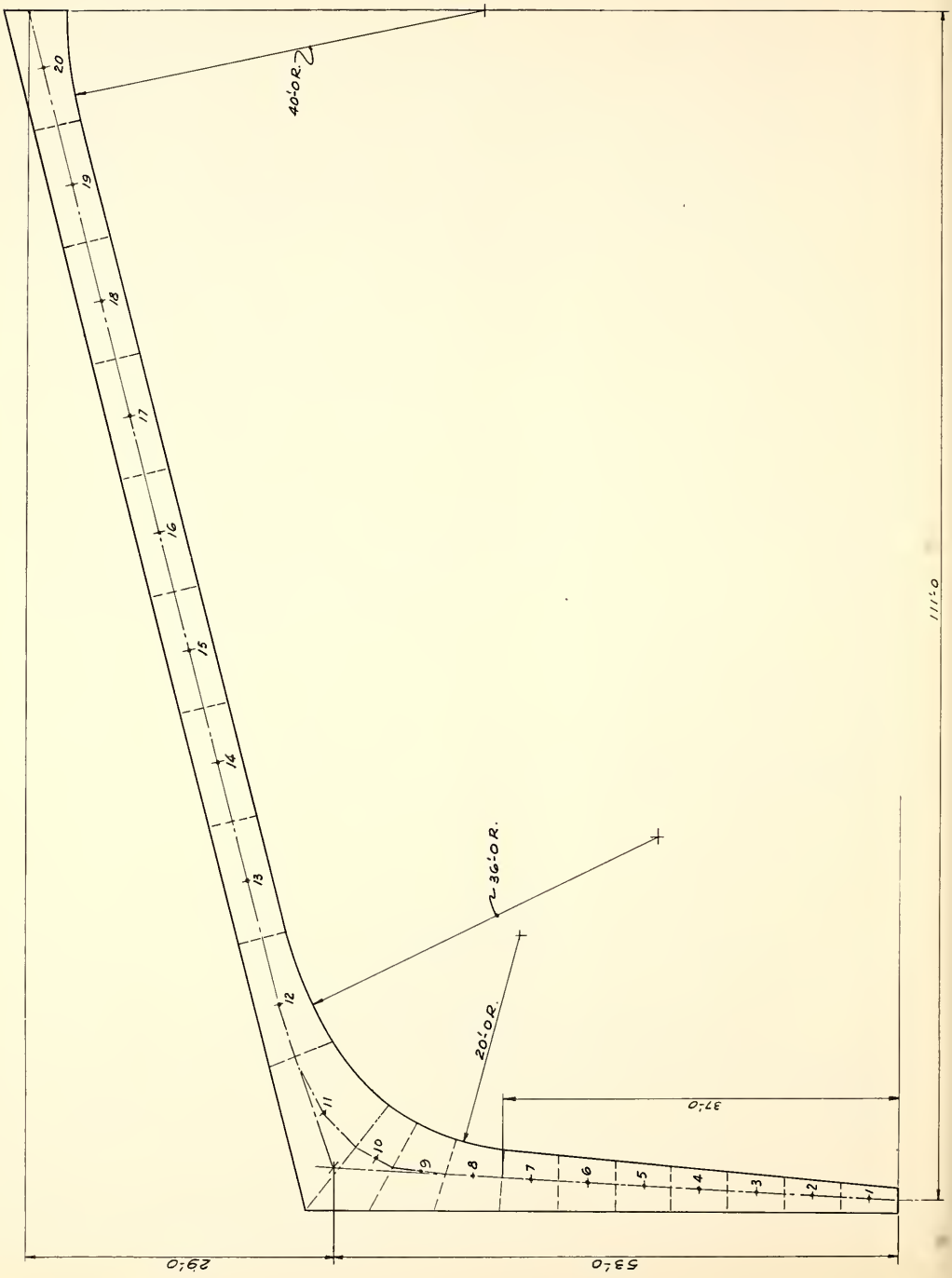
PURLIN TRUSS "A"



MOM. VERT. LOADS
1050# / LIN. FT.

Pt	x	y	ℓ-x	M
1	0	2.7	222.	0
2	.2	8.0	221.8	23.3
3	.4	13.3	221.6	46.5
4	.6	18.6	221.4	69.8
5	.8	23.9	221.2	93.
6	1.0	29.2	221.0	116.
7	1.3	34.5	220.7	150.
8	1.6	39.8	220.4	185.
9	2.2	44.0	219.8	254.
10	3.6	48.8	218.4	413.
11	8.0	53.8	214.0	898.
12	18.3	58.2	203.7	1950.
13	29.4	61.4	192.6	2970.
14	40.2	64.2	181.8	3830.
15	51.2	66.9	170.8	4580.
16	62.0	69.7	160.0	5210.
17	73.0	72.6	149.0	5710.
18	83.9	75.5	138.1	6080.
19	94.8	78.3	127.2	6340.
20	105.6	81.0	116.6	6500.




40'-0 R.
36'-0 R.
20'-0 R.
111'-0
53'-0
29'-0
37'-0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20

CHECK BY EXACT METHOD

PROPERTIES OF FRAME							VERT. LOAD		HOR. LD. TO LEFT		HOR. LD. TO RIGHT		
Pt	$\frac{\Delta s}{2}$ FT.	$\frac{I}{FT.^4}$	$\frac{\Delta s}{I} = A$	x FT.	y FT.	$y^2 \frac{\Delta s}{I}$	M_s 'K	$M_s y \frac{\Delta s}{I}$	M_s' 'K	$M_s' y \frac{\Delta s}{I}$	Pt	M_s' 'K	$M_s' y \frac{\Delta s}{I}$
1	5.3	5.25	1.0	0	2.7	7	0	0	88	238	1'	0	0
2	5.3	7.56	.7	.2	8.0	45	23.3	130	250	1400	2'	1	6
3	5.3	10.15	.52	.4	13.3	92	46.5	322	402	2785	3'	2	14
4	5.3	13.1	.4	.6	18.6	138	69.8	520	542	4030	4'	4	30
5	5.3	16.7	.32	.8	23.9	183	93.	710	671	5130	5'	5	38
6	5.3	21.2	.2	1.0	29.2	171	116.	680	787	4590	6'	6	35
7	5.3	33.3	.16	1.3	34.5	190	150.	828	894	4930	7'	7	39
8	5.3	43.4	.12	1.6	39.8	190	185.	883	990	4730	8'	8	39
9	5.3	71.6	.07	2.2	44.0	135	254.	783	1050	3230	9'	10	31
10	5.3	137.4	.04	3.6	48.8	95	413.	806	1112	2170	10'	22	43
11	11.25	95.0	.12	8.0	53.8	348	898.	5800	1148	7400	11'	49	316
12	11.25	29.0	.39	18.3	58.2	1320	1950.	44300	1131	25700	12'	112	2550
13	11.25	17.0	.66	29.4	61.4	2485	2970.	120500	1083	43900	13'	180	7300
14	11.25	17.0	.66	40.2	64.2	2715	3830	162000	1053	44600	14'	245	10350
15	11.25	17.0	.66	51.2	66.9	2950	4580	202000	1003	44300	15'	312	13750
16	11.25	17.0	.66	62.0	69.7	3200	5210	239500	950	43700	16'	378	17400
17	11.25	17.0	.66	73.0	72.6	3480	5710	273000	895	42800	17'	443	21200
18	11.25	17.0	.66	83.9	75.5	3760	6080	303000	838	41700	18'	512	25500
19	11.25	17.0	.66	94.8	78.3	4050	6340	328000	782	40400	19'	579	29900
20	11.25	21.8	.52	105.6	81.0	3410	6500	273500	715	30100	20'	642	27000
TOTALS						29064		1958462		397833			155541

$H = \frac{1958462 \times 2}{29064 \times 2} = 66000\#$ HOR. LD. $M_s = \frac{W \times (2-x)}{2}$ $R_y = 6100\#$ $R_N = 33000\#$

HOR. LD. $\frac{397833 + 155541}{2 \times 29064} = \frac{9500}{75500}$

3.5% HIGH

LD. TO RT. CTR. $M_s = 6100\,(111 - x)$

LD. TO LEFT CTR. $M_s = 33000\,(y) - [6100 + \frac{400\,y^2}{2}]$

RIGID FRAME

SUMMARY

H = 73000 # MAX.

PT.	d PLT."	I	MAX. MOM. FT·KIPS	A FLG.	V KIPS	FLG. STRESS BASED ON V	7/8 ϕ RIVETS @17500	FLANGE STRESS BASED ON M. f_s	WEB AREA GROSS	WEB SHEAR #/D"	$\frac{h}{t}$	$\frac{64000000}{(\frac{h}{t})^2}$
1	32	15100	-197	30.88	183.5	8650	2½"	2500	16 0"	12000	32	
2	37	21187	-585	30.88	DO	6900	3¼"	6150	18.5	10300	42	
3	43	28350	-970	DO	DO	6000	3¾"	8900	21.5	8300	54	
4	48	35716	-1360	DO	DO	5200	4½"	11000	24.	7900	64	
5	53	45750	-1745	DO	DO	4500	5"	12000	27.5	6900	74	11700
6	58	56200	-2140	DO	DO	3000	5¾"	13300	29.	6600	84	9000
7	64	69120	-2520	DO	DO	2700	6"	13900	32	5950	96	7000
8	72	85500	-2910	DO	DO	2300	6"	14100	36	5300	112	STIFF. 5100
9	90	148345	-3300	DO	DO	1770	6"	12000	45	4250	148	STIFF. 2900
10	120	284700	-3680	DO	DO	1230	DO	9300	60	3200	208	STIFF. 1475
11	102	196700	-3879	DO	183.5	1470	DO	12100	51	3600	172	STIFF. 2150
12	60	60000	-2845	DO	168.9	2620	DO	17200	30	5650	88	8300
13	54	35371	-1900	22.88	155.8	2720	DO	13000	27	5750	76	11350
14	54	35371	-1056	DO	142.3	2500	DO	9700	27	5250	76	DO
15	54	35371	345	DO	127.9	2250	DO	3250	27	4750	76	DO
16	54	35371	826	DO	114.3	2000	DO	7550	27	4250	76	DO
17	54	35371	1167	DO	101.2	1600	DO	10700	27	3750	76	DO
18	54	35371	1359	DO	87.9	1530	DO	12500	27	3250	76	DO
19	54	35371	1420	DO	73.4	1280	DO	13100	27	2700	76	DO
20	60	45100	1392	30.88	60.1	1280	DO	11200	30	2000	88	8300

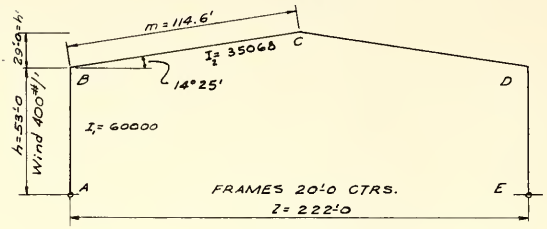
MAX. MOM. CONDITION

RIGID FRAME

BY SINGLETON METHOD

$U^L = 1050\#/'$ $K_1 = \frac{I_2}{I_1} \times \frac{h}{m} = .27$ $K_2 = \frac{h'}{h} = .547$

$K_3 = 3 + K_2 = 3.547$ $K_4 = \frac{3}{h} = 4.2$ $Q = .0645$ $Q = .0481$



GIVEN:

GRAVITY LD. 1050#/L.F.

WIND 20#/0' 400#/L.F.

LL = 25#/0'

DL = 16.6#/0'

UNIFORM GRAVITY LD.

$R_A = R_E = \frac{1}{2} w\ell = \frac{1050 \times 222}{2} = 116,500\#$

$H = Q_1 K_4 w\ell = .0645 \times 4.2 \times 1050 \times 222 = 63100\#$

$M_B = M_D = -Hh = 63100 \times 53 = -3,370,000'\#$

$M_C = [.125 - Q_1(1 + K_2)]\, w\ell^2 = -1,305,000'\#$

WIND ON COL.

$R_A = \frac{1}{2}\frac{h}{\ell} ph = \frac{1}{2}\frac{53}{222} \times 400 \times 53 = -2530\#$

$R_E = 2530\#$

$H_E = \frac{1}{4}[5 + .27 + 6 \times .547 + 12]\, Q_2 ph = 4240\#$

$H_A = 400 \times 53 - 4240 = 16900\#$

$M_B = 16900 - \frac{1}{2} \times 400 \times 53)\, 53 = +334,000'\#$

$M_D = -4240 \times 53 = -224,500'\#$

$M_C = [\frac{400 \times 53}{4} - 4240(1 + .547)]\, 53 = -66300'\#$

TOTAL H_E

63100
4240
5370
72710#
USE 73000#

TOTAL M_B & M_D

3,370,000
224,500
284,500
3,879,000'#

WIND ON RAFTER

$R_A = \frac{1}{222}(53 + \frac{29}{2}) 400 \times 29 = -3510\#$

$R_E = 3510\#$

$H_E = [2 \times .27 + 1.25 \times .547(4 + .547) + 6] = 5370\#$

$H_A = 400 \times 29 - 5370 = 6230\#$

$M_B = 6230 \times 53 = 330,000'\#$

$M_d = -5370 \times 53 = -284,500'\#$

$M_C = 3510 \times \frac{222}{2} - (1 + .547) 5370 \times 53 = 330,000 - 440,000 = -50000$

TOTAL AT C

1,305,000
66,300
50,000
1,421,300

BY A.I.S.C. METHOD

$F = \frac{I_2}{I_1}\frac{h}{m} = 27$ $\phi = \frac{h'}{h} = .547$

$K = .065$ $K_1 = .45$

$H_E = \frac{.065 \times 1050 \times 222^2}{53} = 63500\#$

$R = 1050 \times 111 = 116,500$

TOTAL H = 73000 # USE.

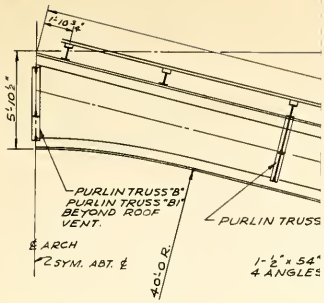
WIND = $20\#/0' \times 20 = 400\#/'$

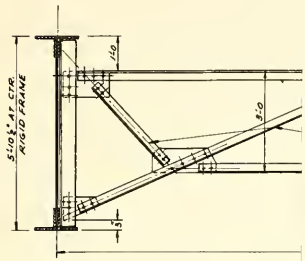
$H_E = K_1 ph = .45 \times 400 \times 53 = 9500\#$

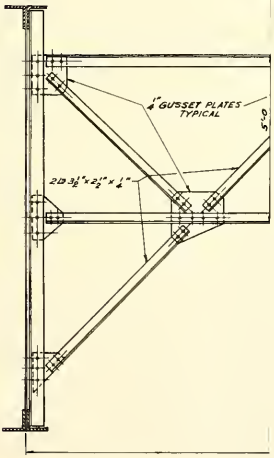
$M_B = 63500 \times 53 = 3,365,000'\#$ (CHECKS WITH M_B SINGLETON)

$M_C = \frac{116500 \times 222}{4} - 63500(B_2)$

6,470,000 - 5,210,000 = 1,260,000 '# (CHECKS CLOSE WITH M_C SINGLETON)


1'-10 3/4"
5'-10 1/2"
PURLIN TRUSS "B"
PURLIN TRUSS "B1"
BEYOND ROOF
VENT.
PURLIN TRUSS
℄ ARCH
2 SYM. ABT. ℄
40'-0 R.
1-1/2" x 54"
4 ANGLES


5'-10 1/2" AT CTR
RIGID FRAME
1'-0
8'-0

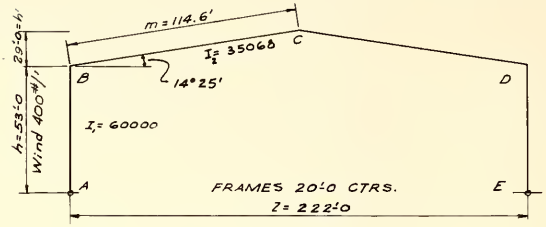

1" GUSSET PLATES
TYPICAL
5'-0
2 @ 3 1/2" x 2 1/2" x 1/4"

RIGID FRAME

BY SINGLETON METHOD

$U^L = 1050\#/' \quad K_1 = \frac{I_2}{I_1} \times \frac{h}{m} = .27 \quad K_2 = \frac{h'}{h} = .547$

$K_3 = 3 + K_2 = 3.547 \quad K_4 = \frac{3}{h} = 4.2 \quad Q = .0645 \quad Q = .0481$



GIVEN:

GRAVITY LD. 1050#/L.F.

WIND 20#/□' 400#/L.F.

LL = 25#/□'

DL = 16.6#/□'

UNIFORM GRAVITY LD.

$R_A = R_E = \frac{1}{2} wl = \frac{1050 \times 222}{2} = 116,500\#$

$H = Q_1 K_4 wl = .0645 \times 4.2 \times 1050 \times 222 = 63100\#$

$M_B = M_D = -Hh = 63100 \times 53 = -3,370,000'\#$

$M_C = [.125 - Q_1(1+K_2)]\, wl^2 = -1,305,000'\#$

WIND ON COL.

$R_A = \frac{1}{2}\frac{h}{l} ph = \frac{1}{2}\frac{53}{222} \times 400 \times 53 = -2530\#$

$R_E = 2530\#$

$H_E = \frac{1}{4}[5 + .27 + 6 \times .547 + 12]\, Q_2 ph = 4240\#$

$H_A = 400 \times 53 - 4240 = 16900\#$

$M_B = 16900 - \frac{1}{2} \times 400 \times 53)53 = +334,000'\#$

$M_D = -4240 \times 53 = -224,500'\#$

$M_C = [\frac{400 \times 53}{4} - 4240(1 + .547)]53 = -66300'\#$

TOTAL H_E

63100
4240
5370
72710#
USE 73000#

TOTAL M_B & M_D

3,370,000
224,500
284,500
3,879,000'#

WIND ON RAFTER

$R_A = \frac{1}{222}(53 + \frac{29}{2}) 400 \times 29 = -3510\#$

$R_E = 3510\#$

$H_E = [2 \times .27 + 1.25 \times .547(4 + .547) + 6] = 5370\#$

$H_A = 400 \times 29 - 5370 = 6230\#$

$M_B = 6230 \times 53 = 330,000'\#$

$M_d = -5370 \times 53 = -284,500'\#$

$M_c = 3510 \times \frac{222}{2} - (1 + .547)5370 \times 53 = 350,000 - 440,000 = -50000$

TOTAL AT C

1,305,000
66,300
50,000
1,421,300

BY A.I.S.C. METHOD

$F = \frac{I_2}{I_1}\frac{h}{m} = 27 \quad \phi = \frac{h'}{h} = .547$

$K = .065 \quad K_1 = .45$

$H_E = \frac{.065 \times 1050 \times 222.^2}{53} = 63500\#$

$R = 1050 \times 111 = 116,500$

TOTAL H = 73000 # USE.

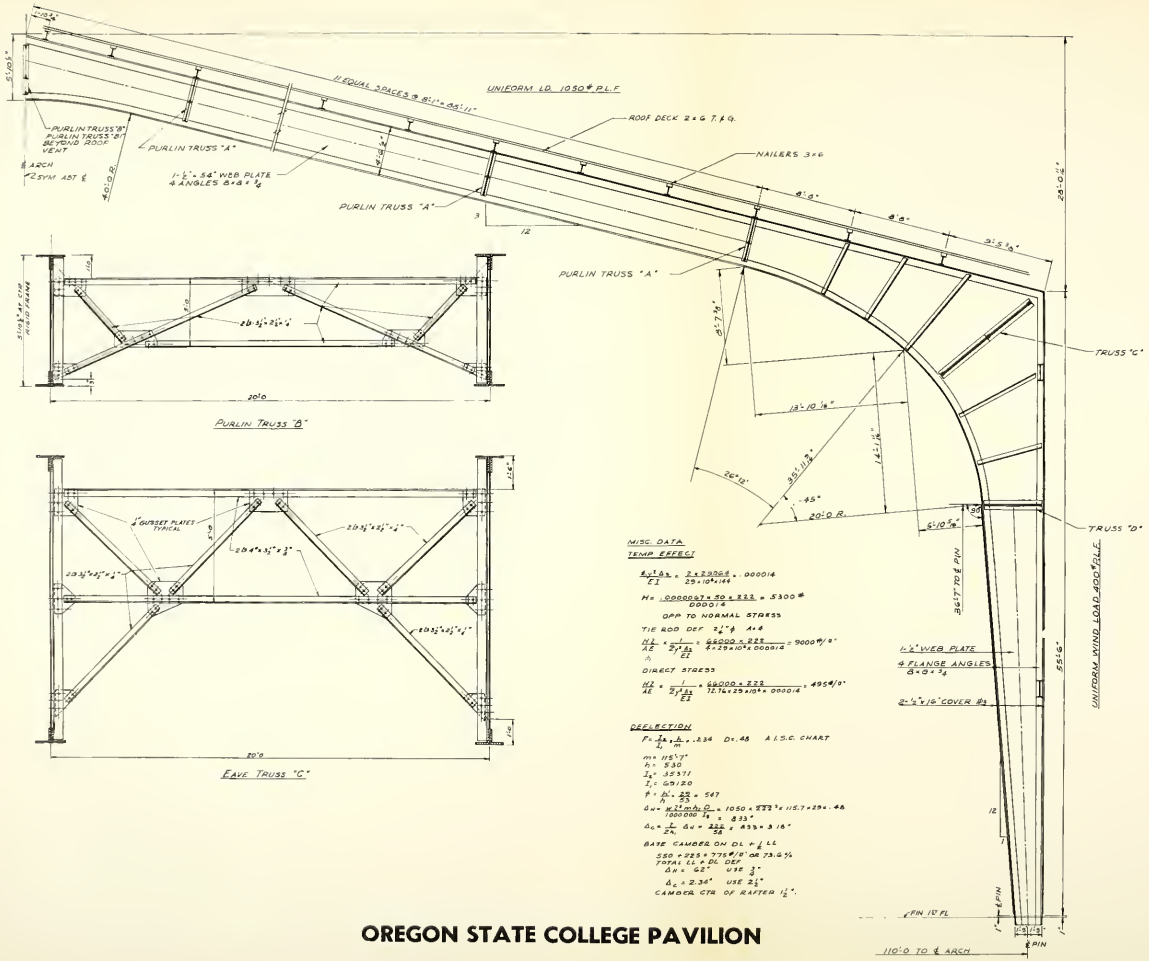
WIND = 20#/□' × 20 = 400 #/'

$H_E = K_1 ph = .45 \times 400 \times 53 = 9500\#$

$M_B = 63500 \times 53 = 3,365,000'\#$ (CHECKS WITH M_B SINGLETON)

$M_C = \frac{116500 \times 222}{4} - 63500\,(B_2)$

6,470,000 - 5,210,000 = 1,260,000'# (CHECKS CLOSE WITH M_C SINGLETON)



MISC. DATA

TEMP EFFECT

$$\frac{\alpha t L_H}{\frac{f}{E}} = \frac{.0000067 \times 50 \times 222}{\frac{29 \times 10^6}{144}} = .000014$$

$$H_t = \frac{.0000067 \times 50 \times 222}{.000014} = 5300\#$$

OPP TO NORMAL STRESS

TIE ROD DEF 2¼"ϕ A=4

$$\frac{HL}{AE} = \frac{1}{2\left(\frac{A_1}{E_1}\right)} = \frac{66000 \times 222}{4 \times 29 \times 10^6 \times .000014} = 9000\#/\square"$$

∴

DIRECT STRESS

$$\frac{HL}{AE} = \frac{1}{2\left(\frac{A_1}{E_1}\right)} = \frac{66000 \times 222}{12.76 \times 29 \times 10^6 \times .000014} = 4954\#/\square"$$

DEFLECTION

$F = \frac{I_x}{I_c} \cdot \frac{L}{m} = .234$ D = .48 A.I.S.C. CHART

$m = 115'-3"$

$I_x = 35371$

$I_c = 6791.20$

$\beta = \frac{M}{H} = \frac{55}{55} = 547$

$\Delta_H = \frac{W L^3 m h_1 Q}{1000000 \, I_x} = \frac{1050 \times 222^3 \times 115.7 \times 294 \times .48}{1000000 \times 35371} = 8.33"$

$\Delta_{LL} = \frac{L}{2n} \cdot \Delta_H = \frac{222}{58} \times 8.33 = 3.18"$

RATE CAMBER ON DL + ½ LL

550 + 275 = 775#/ft OR 73.6 %

TOTAL LL + DL DEF

$\Delta_H = 6.2"$ USE 6"

$\Delta_L = 2.38"$ USE 2½"

CAMBER CTR OF RAFTER 1½".

OREGON STATE COLLEGE PAVILION

An Architect Speaks for Welded Rigid Frames

by
GEORGE M. McCONKEY*
Professor of Architecture
College of Architecture and Design
University of Michigan

This is a viewpoint of an Architect who is very much interested in the actual integration of the construction design of a building with its aesthetic design.

Where is the architect or engineer who is not completely captivated by the beautiful curves and camber possibilities inherent in the rigid frame? More and more frequently we are encountering the reality of the steel ribbed vault with its smooth arch ribs sweeping across the ceiling of a great modern room of huge span, breaking with a smooth easy curve at the ridge and down into the columns at the walls to their bearing on the ground.

Again, when one sees a roof abruptly changing its angles from skylight to slope and from column to column, the supporting ribs following every change in the direction and slope, he begins to realize that there seems to be no limit to the ability of the rigid frame to enclose space in any form or type of structure whatsoever.

The old laboring post and lintel or truss on the column seems so much the work of the "old school Engineering" when compared to these new modern types where the imagination of the designer is given such free rein.

Just as the majestic thirteenth century Gothic turned the prosaic stone lintel into the great pointed vaults of that day, so the steel rib is turning our heavy apparently sagging old trusses into the graceful camber of a modern steel rib thrust down into its columns like staunch buttresses, again, today in many modern buildings, with the graceful curves reminding us of this still beautiful gothic.

The almost ideally perfect but still modern material "steel" coupled with the welders art can transform the design of large box shaped rooms into unlimited possibilities of new form when the roof is the principle factor in the design.

In almost any conceivable structure or space enclosure the steel rib will supply almost any shape demanded in supporting the outer envelope of a structure. While the beautiful shape of the parabolic arch almost exactly follows a typical line of thrust and is in itself the very essence of economy as pure compression; it nevertheless fails to take the full advantage like the rigid frame of steel's capacity to bend as well as shorten under the pressure so that the variations in the direction of a rib can be changed almost without restriction from the parabolic arch to much greater freedom in the design.

Nature does not seem to lean altogether towards economy but rather it tends towards the bountiful and profuse. Take for instance the shape of the trees on either side of an avenue of elms meeting high over the middle of a roadway, their general effect is quite similar to the curve of some of our recently designed rigid frames.

The first dwelling made with human hands no doubt got its form and shape from lashing

Author's Note: In the chapter "A Private Chat With Architects," the need for integration of architecture and engineering was emphasized and its application illustrated in actual practice. But students of architecture at the U of M, Ann Arbor, have long been taught to plan and think as architect-engineer entities by Prof. George M. McConkey. Qualified by education and experience to teach both architecture and engineering, he presents advanced structural design with such gay insouciance as to enthrall his students—they lap it up. And because so many of them have sung his praises, I thought how appropriate it would be to save him as our last speaker, if only to say a few words.

So pull up a chair, every one, let the whole family gather around. And instead of introducing my friend George—it would be with a hoopla—I shall turn the microphone over to his legion of former students scattered around the country, in Europe, in the far East, during more than forty years. So come in wide world. And here they come:

"He is a student's teacher" — "one, you could never forget" — "a man you are proud to study under" — "a humorist" — "a fair and unbiased judge of others" — "a credit to the profession."

Is that all? No, but it does give some idea of the almost universal esteem for him. So I'm sure, we shall enjoy the next few minutes.

M.P.K.

growing young saplings together at the top and with thatch as a covering again gives us a good example of the same shape found in some modern steel ribs. The steel rigid frame is so far and away above the restricted action of a brick or stone arch and even the ponderous compromise of reinforced concrete that it obviously just has to be used.

We are no longer restricted to the use of the framed triangle, since the welded steel curved forms with their rigid angles can as well enclose any shape that fancy may dictate. The only requirement is the proper strength and continuity of the angular connections of each such frame.

Although its very so-called rigidity gains a resiliency in these angular connections and thus the ability to safely yield a little further in a sudden gust, the rigid frame behaves somewhat like a tree trunk swaying a small amount to the wind. Steel, however, minimizes these yielding movements to a very small fraction of those in wood construction and at the same time retains the desirable resiliency.

The framework of triangles so common in the truss and the bracing of the average structure have the very small yielding of tension and compression; the yield of bending is a magnified form of these small tensile and compressive movements; and the rigid frame carries these same bending stresses right around the angles of the structure undiminished, instead of letting them die out and begin again at every interesection as in the ordinary triangulated frame so common in bridges and buildings seen everywhere.

Any arch slightly modified into a rigid frame and carrying a roof over a large vaulted interior encloses not only the space desired for the unobstructed view of the interior but seems to enhance the lines of perspective and creates the feeling of larger space and greater lightness and loftiness. It introduces the vertical leg into the arch and gives unobstructed free space for the high seats in the balcony of a great hall. Of course when a rib wanders considerably from a given line of thrust and the cost of the steel mounts accordingly, it becomes apparent that the dimensions must be increased to hold down this added cost. The very increase in the dimensions, however, satisfies the eye just as a muscular athlete shows his ability to resist the strains of a heavy load.

But, the real point to emphasize is the ability of the rigid steel frame with the aid of the welder to take a direction different from the line of thrust when necessary to carry the load of a roof around an angle. Thus a designer may conceive new curves and new forms for any future great interior span. This makes it possible with judicious use of this new freedom in design to give added interest and usefulness even greater beauty of form and proportion with but slight increase or decrease of dimension to satisfy good proportion or clear a sight line.

In other words, why not use steel as a material up to the safe limit of its capacity for not only its structural possibilities, but for its aesthetic possibilities as well?

We must not be too hesitant to use a little more of a material to gain some of these ends, just as we never hesitate to omit columns from the middle of a room although it takes more material to do this. Just as a clumsy truss may give way to a soaring steel rib, even when first cost increases are sometimes encountered, this may merge into savings when maintenance, painting and decorating are considered.

The computations necessary in the design of a fine looking steel rib will remain exacting and scientific. But if reviewed in the light of experience and knowledge expounded in the preceding chapters the factors of cost and safety will be no problem.

Its possibilities are exciting to any architect

honesty of expression on both exterior and in- or engineer well enough versed in the theoretical aspects to undertake the design of these new structural forms. When he lets his imagination have a field day he may come up with a smashing new design that tops all the old shapes and one possible only in welded rigid frame construction.

The modern trend in Architecture is just passing through a stage of emphasis on the flat roof but this emphasis is swinging back again to favor the lofty nave of the church, the vaulted ceiling of the gymnasium, the auditorium, the concert hall, the fieldhouse, the memorial and recreation buildings of all types and sizes. The design of all these types of buildings almost demands some form of the rigid frame.

Most architects and designers during the present period of high and rising prices are stressing with special emphasis bare necessity and simplicity in construction with a minimum in maintenance. They are just beginning to realize that the rigid frame fits perfectly into such a program and at the same time while replacing the old overhead truss abolishes a costly ceiling and turns an unused attic space into a lofty new clean-cut modern interior with lower maintenance cost and simpler construction.

Simplicity is a keynote of modern architecture, to do the job in as direct a manner as possible and with a minimum expenditure of labor in the shop or in the field; good proportion with a minimum of decorative ornament and a pleasing distribution of the masses with an honesty in expressing on the exterior what the interior is and what purpose its use may be.

Thus, a satisfactory design must take advantage of the modern rigid frame with its absolute terior of the building, its desirable simplicity, its rapidity of erection, and its clean cut appearance—a single rib instead of a multiplicity of parts.

There can never be any regret — only the pride of accomplishment on the part of designer, fabricator and builder when a new great room is unveiled to the public with its smooth lofty ribs in beautiful swinging curves.

This is modern architecture at its best and the steel rigid frame will contribute a large part in its success.

New York Subways—4 Span Welded Rigid Frames

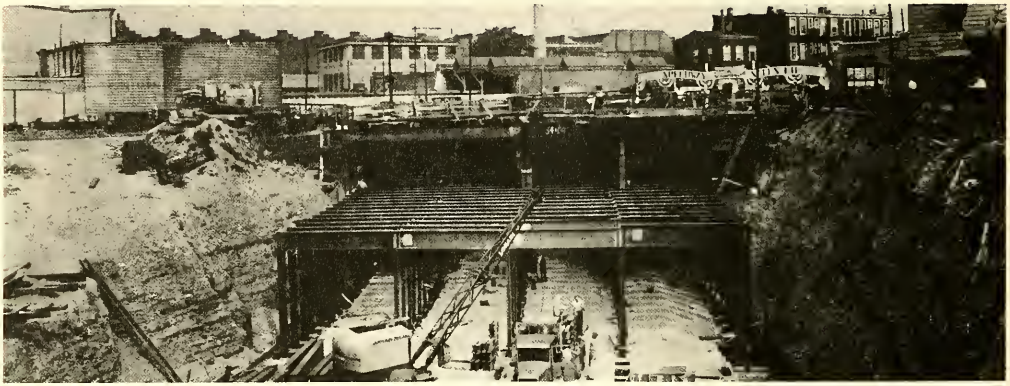
LINCOLN ARC WELDING FOUNDATION

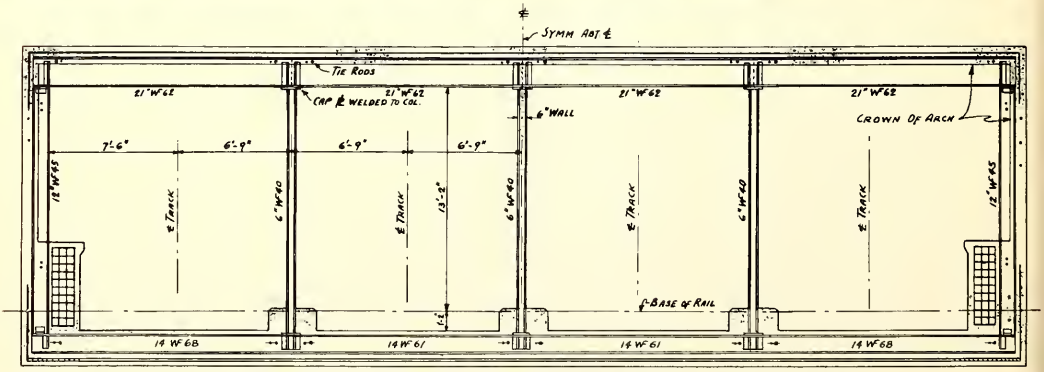
Author's Note:

Drawings of prize-winning design in the Lincoln Arc Welding Foundation Prize Contest submitted by Mr. N. D. Brodkin, outstanding subway design engineer, from basic design by Martin P. Korn.

Present type of construction consists of articulated riveted bents.
Savings in weight by welded rigid frame design 22% above water
Savings in weight by welded rigid frame design 28% below water
An official committee appointed by the Board of Transportation, City of New York, has reported favorably on the feasibility, praticality and economy in rigid frame designs for New York Subways.

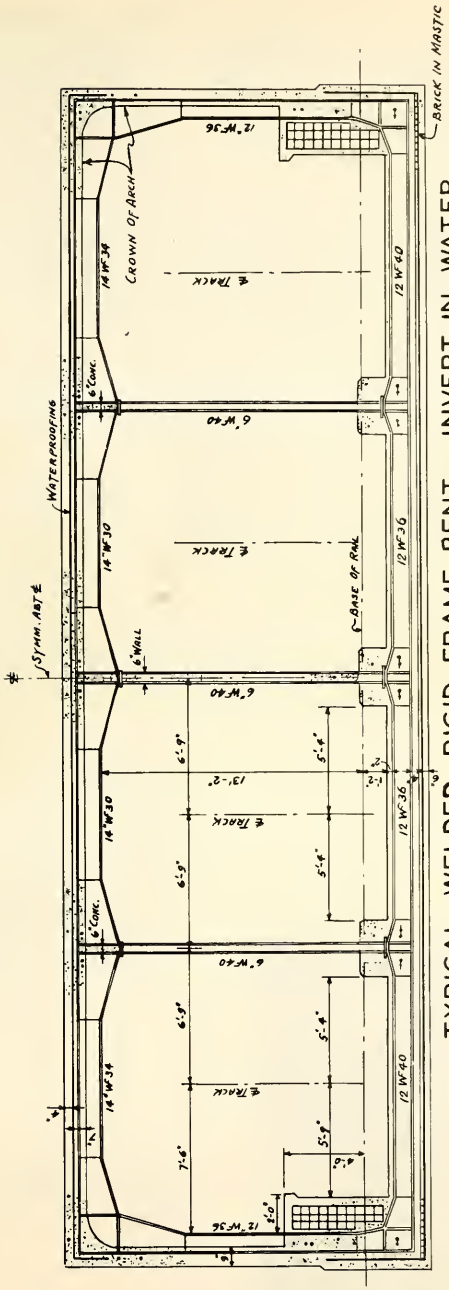
M.P.K.





TYPICAL RIVETED BENT — INVERT IN WATER

BENTS : 5'-0" C. C. ESTIMATED WEIGHT = 11040 LBS.

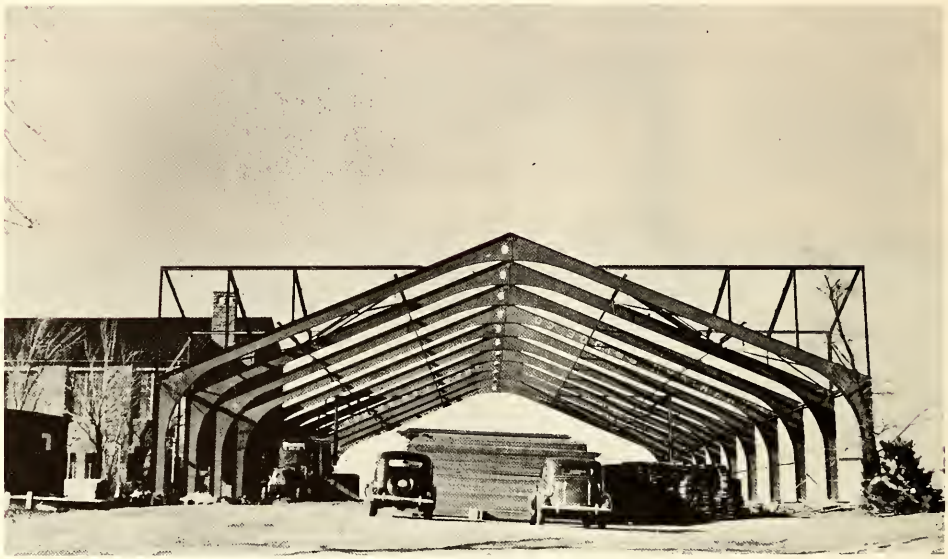


TYPICAL WELDED RIGID FRAME BENT — INVERT IN WATER

BENTS : 5'-0" C.C. ESTIMATED WEIGHT = 7900 LBS.

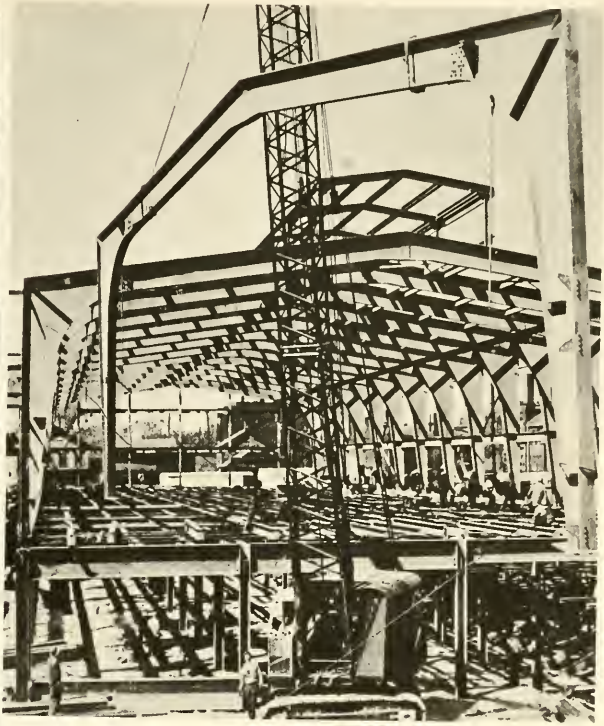
NOTE : DESIGNS ARE FOR AVERAGE COVER = 14'-0"

American Bridge
Division
U. S. Steel Co.



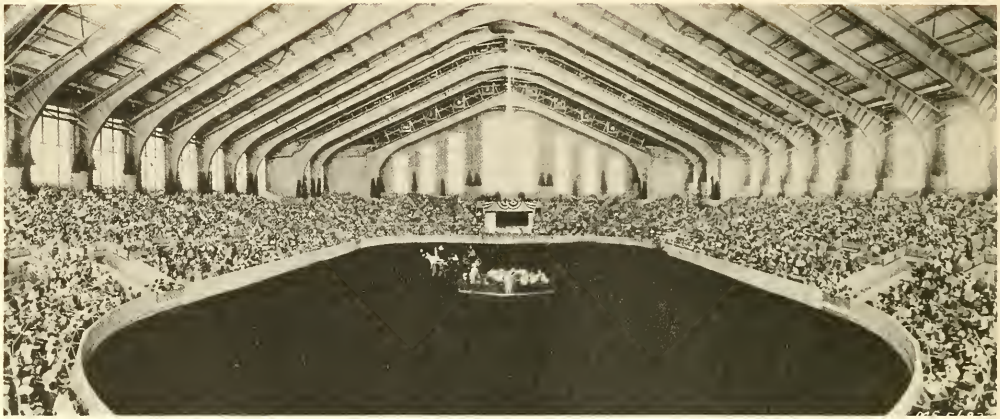
Field House for University of Colorado, Boulder, Colo.
Riveted Beam Arches, span 127'-6". Built 1936.

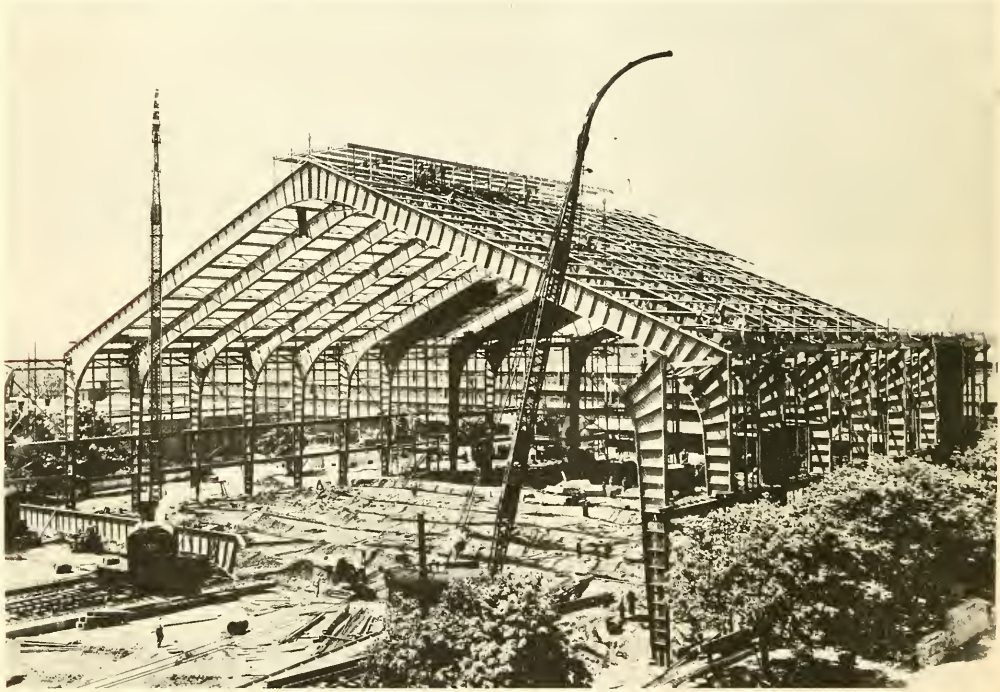
Field House for University of Chicago, Chicago, Ill.
14 riveted split Beam Arches, span 168'-2". Built 1931
This was the first installation of Beam Arch Construction as designed by American Bridge.



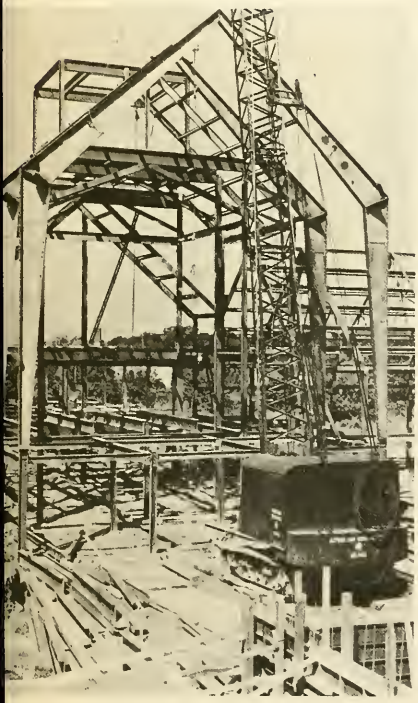
(Above) Bottling Plant, Stroh Brewery Co., Detroit, Mich. Span 87'-4", 0 to 0.

One of Four All-Welded Rigid Frames for Broad St. Bridge, Seymour, Conn. Each frame 68'-0" span, out-to-out, on vertical curve, for deck bridge, 1'-9½" deep at crown 4'-0" deep at knees. Built from split beams with web plate inserts. (1939)

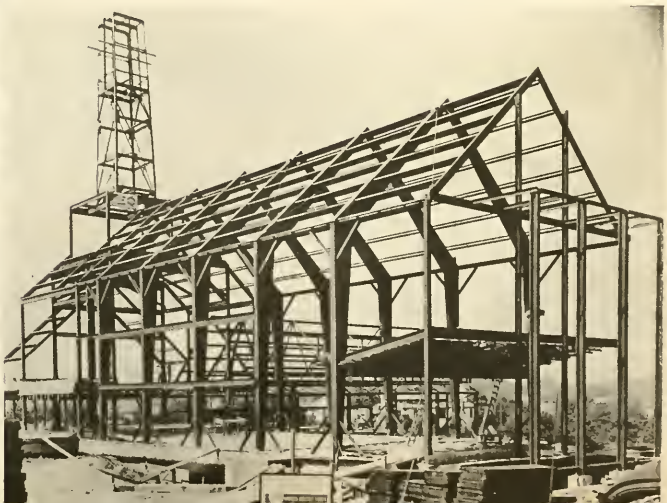


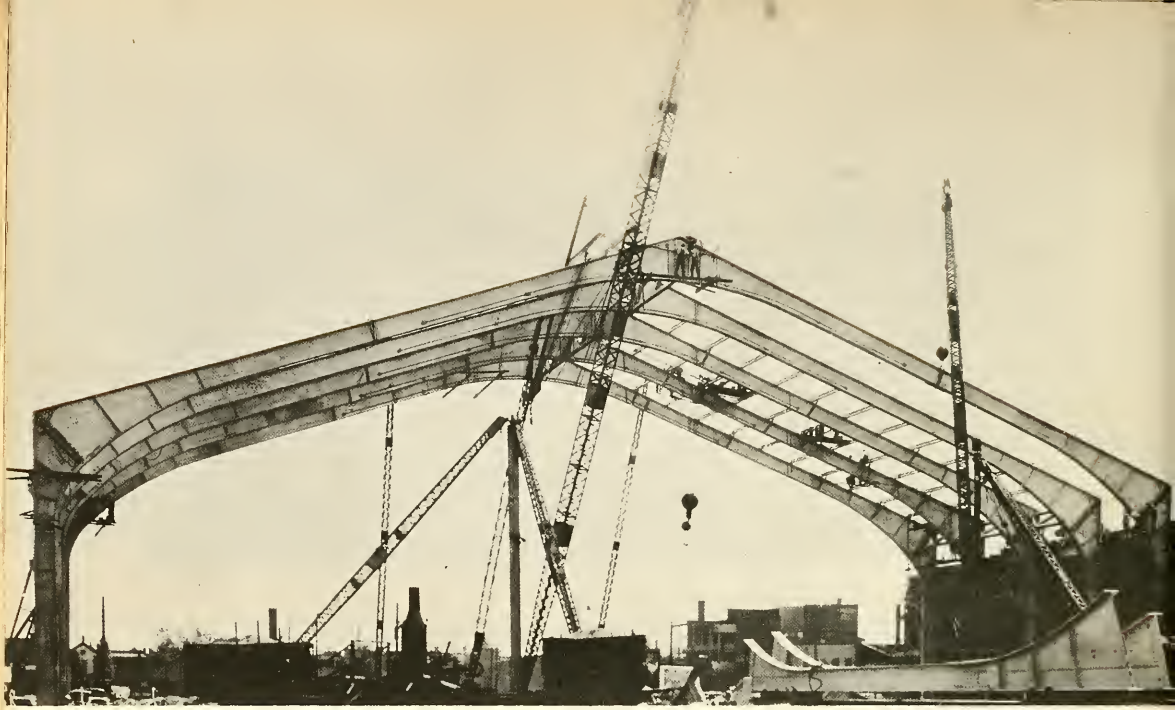


Riding Ring for National Guard Armory, West Orange, N. J.
Riveted Beam Arches, span 142'-0". Built 1937
Balcony on one side 18'-8" wide.

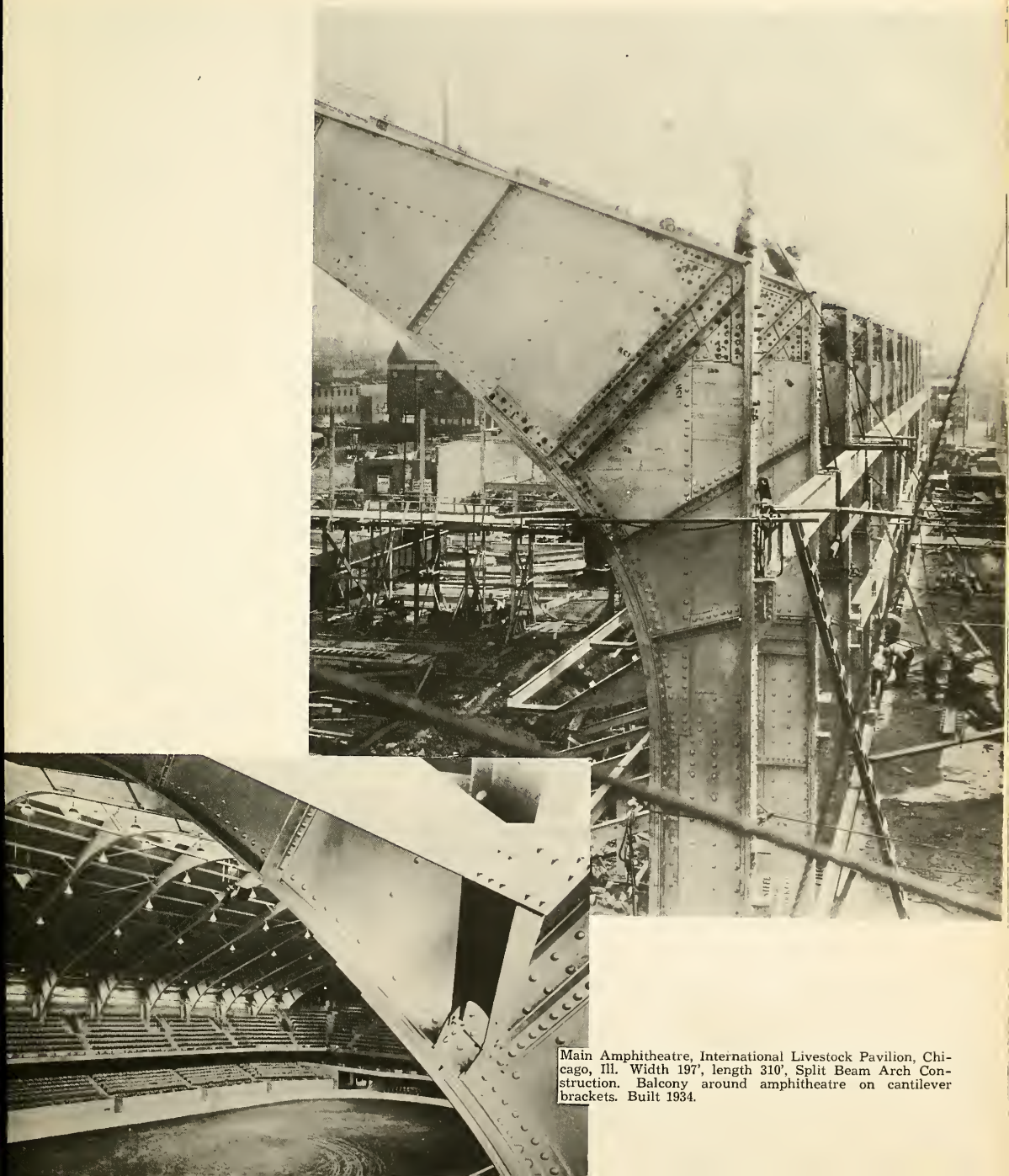


Congregational Church, Manhasset, L. I., N. Y.
Span 45'. Height to peak of roof about 50'.



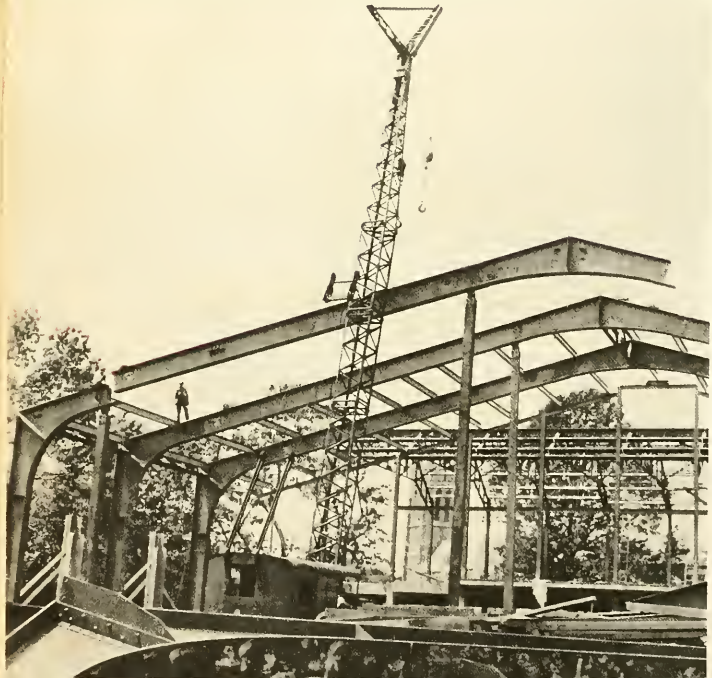


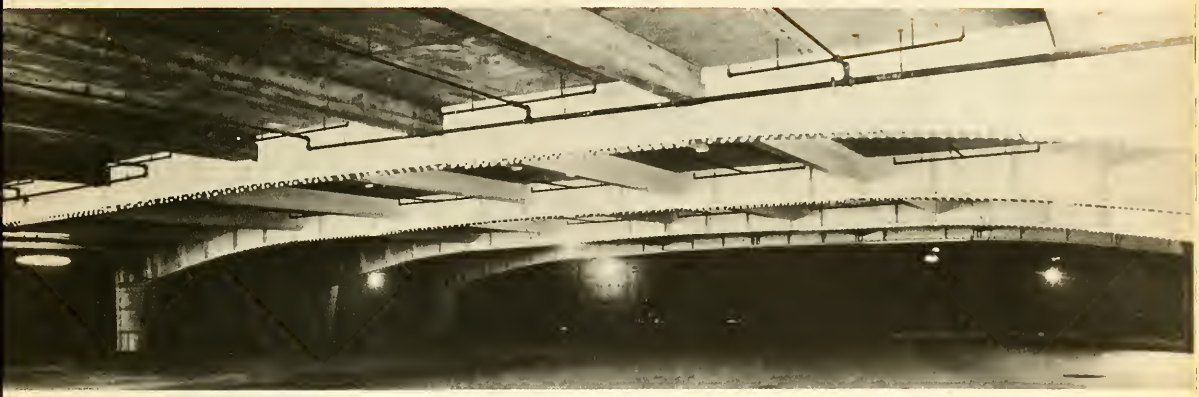
Main Amphitheatre, International Livestock Pavilion, Chicago, Ill.

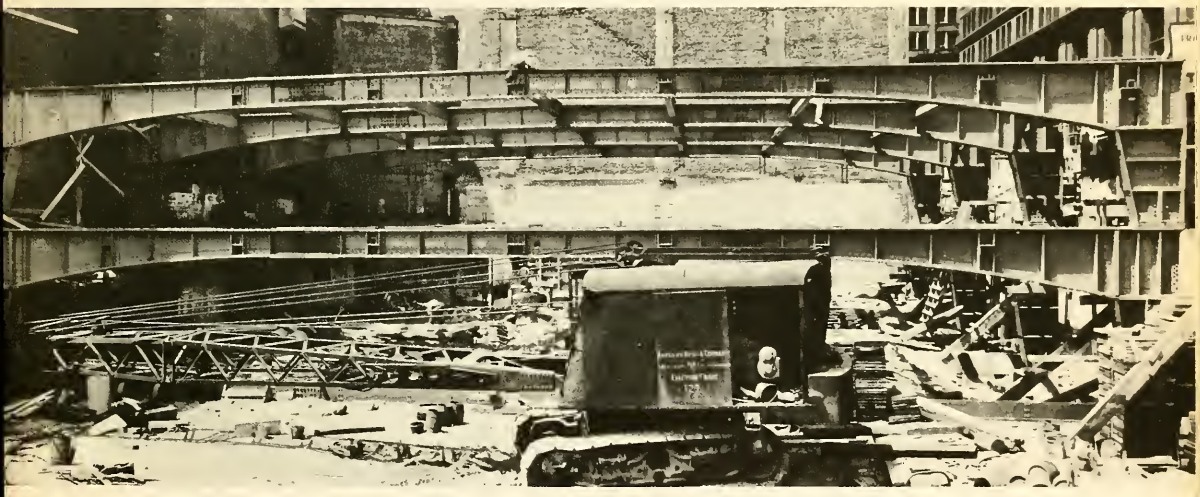


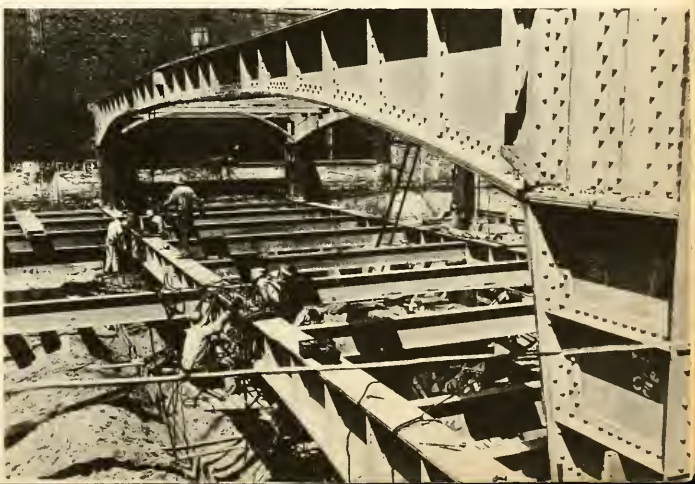
Main Amphitheatre, International Livestock Pavilion, Chicago, Ill. Width 197', length 310', Split Beam Arch Construction. Balcony around amphitheatre on cantilever brackets. Built 1934.

Field House for St. Joseph's College, Philadelphia

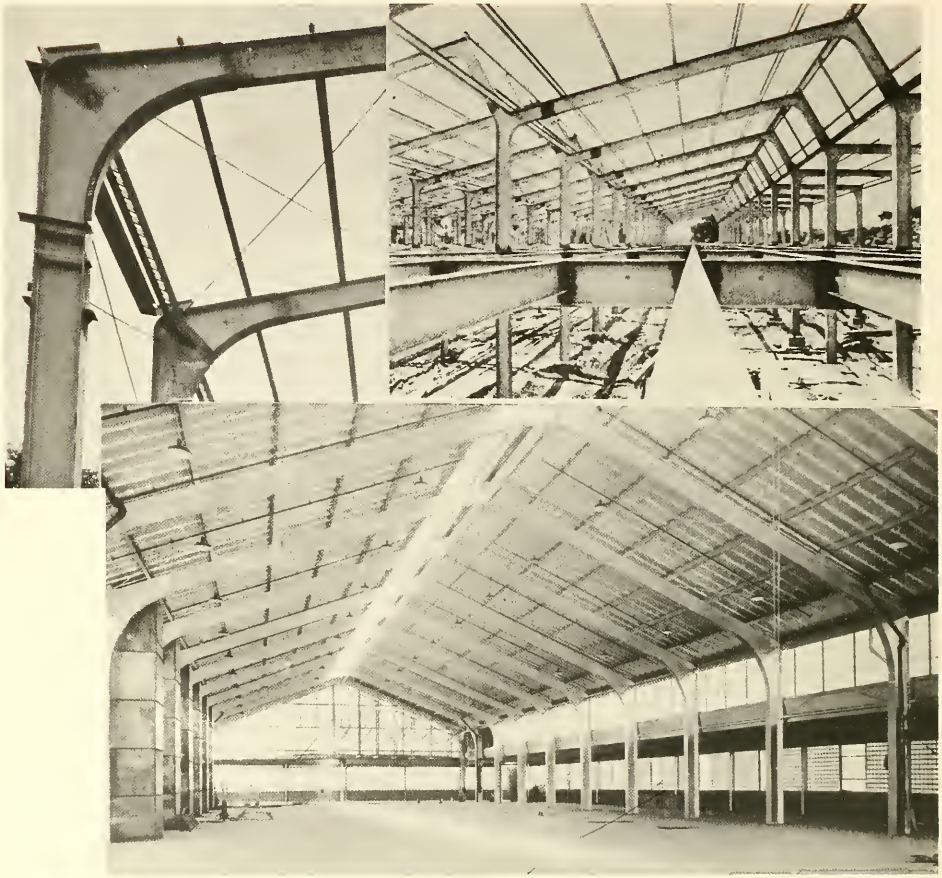








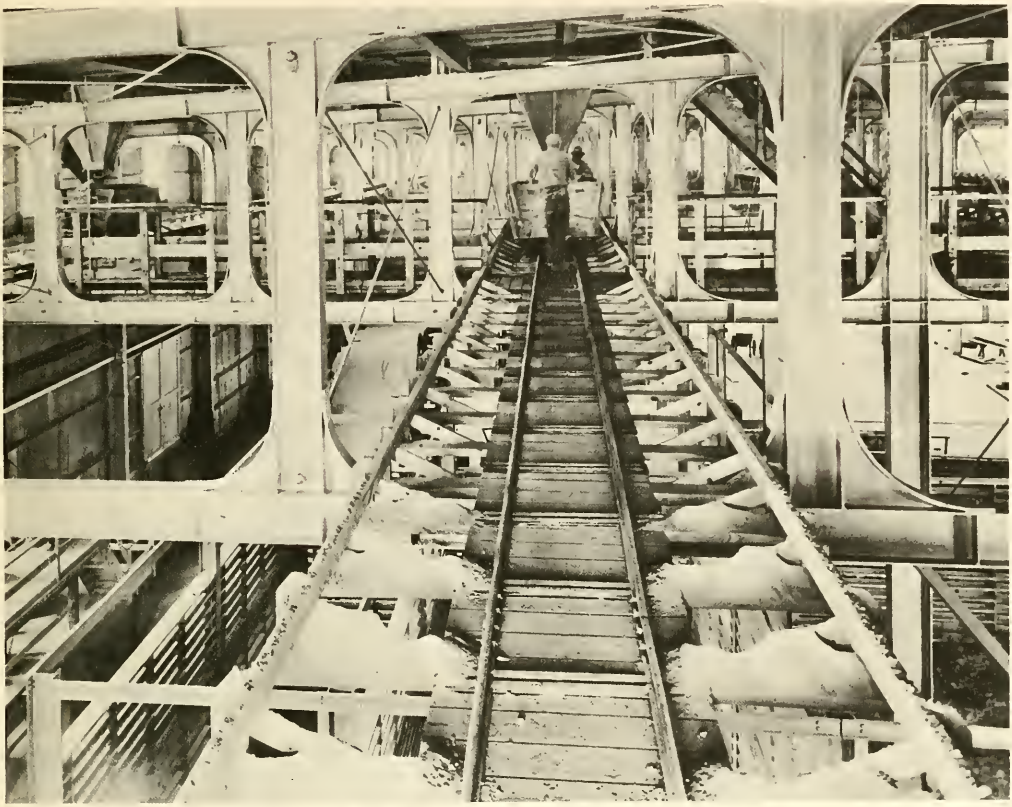
Miscellaneous Rigid Frame Structures

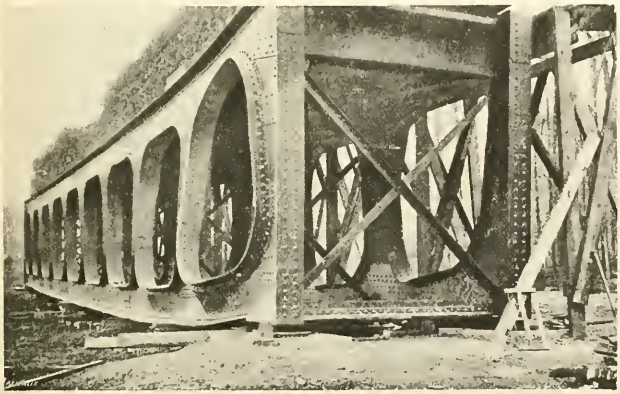


(Above) Three Views of Lincoln Electric Co. Plant, Cleveland, Ohio. Built 1937

(Left) Portland Mfg. Co. Plant
Portland, Mich.

(Designed and built by Austin Co.)



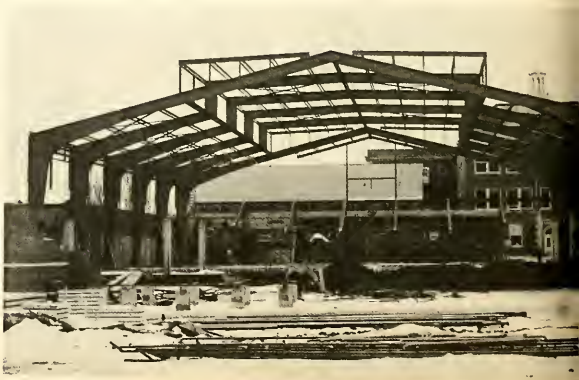


(Top) Welded Vierendeel Truss at International Agricultural Plant, Chicago.
(Designed and built by Austin Co.)

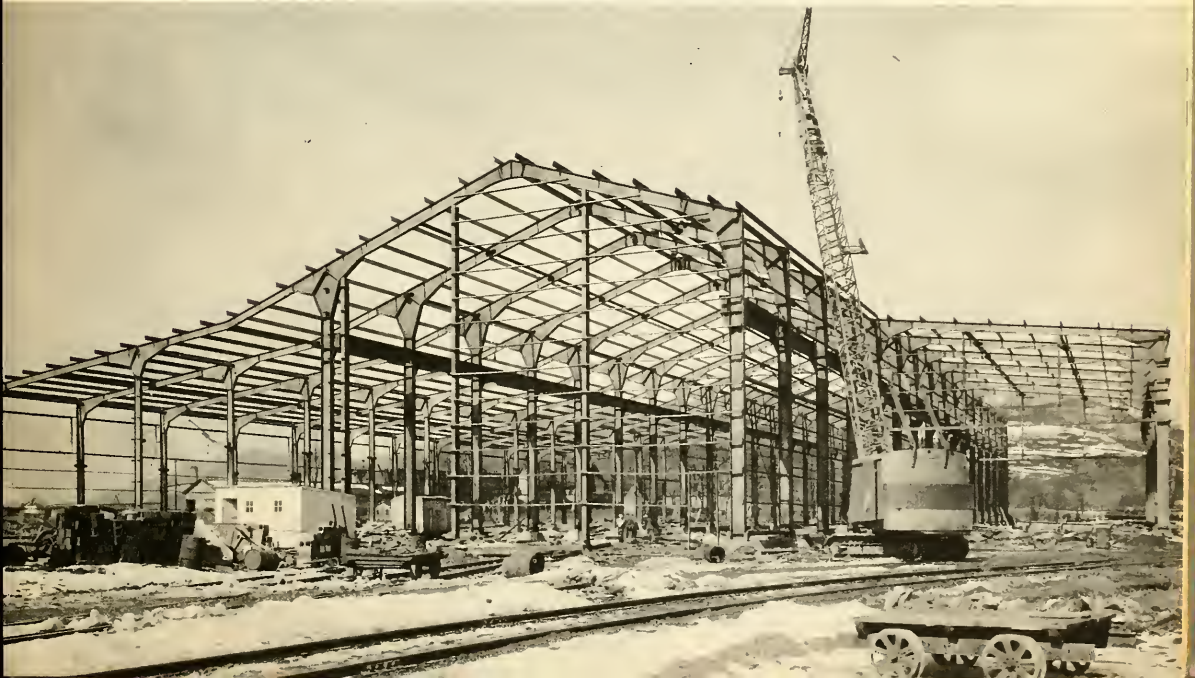
(Left) Original Vierendeel truss of riveted construction, tested by Prof. Vierendeel, 1897.
Note: Riveted Vierendeel trusses do not look much different today. Compare with welded truss above.

Big rigid frame fieldhouse (1950) transforms little village of Laingsburg, Mich., pop. 800, into the bustling City of Laingsburg with emphasis on schools and churches—building America's future. (Designed by Giffels & Vallet, Inc., L. Rosetti, Detroit, Mich.)

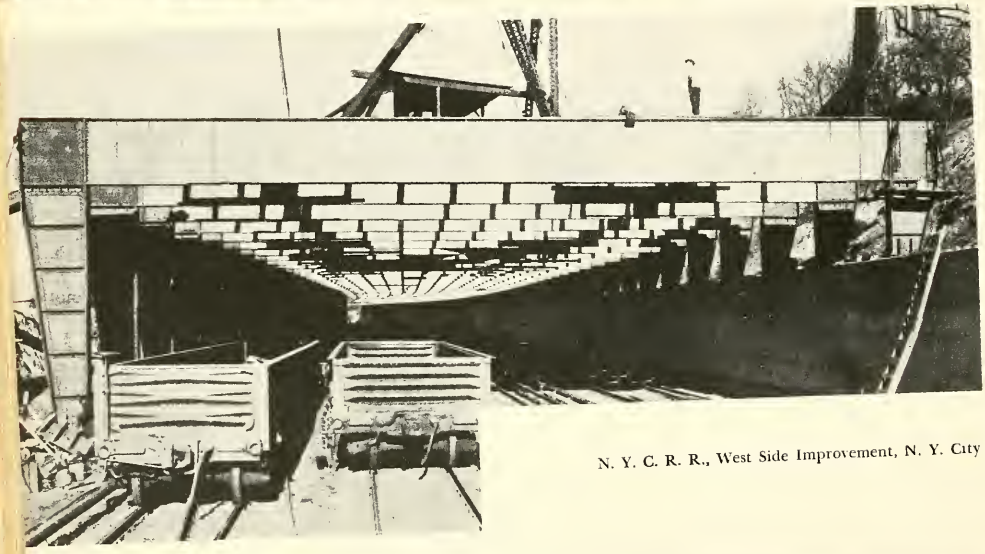
Clerestory Rigid Frames for High School, Ft. Collins, Colo.
Designed by Milo S. Ketchum, Consulting Engineer, Denver, Colo.
(Photo, courtesy of Midwest Iron & Steel Works Co., Denver, Fabri



All-welded Industrial Plant of Functional Design
(Photo, courtesy of Pittsburgh-Des Moines Steel Co.)

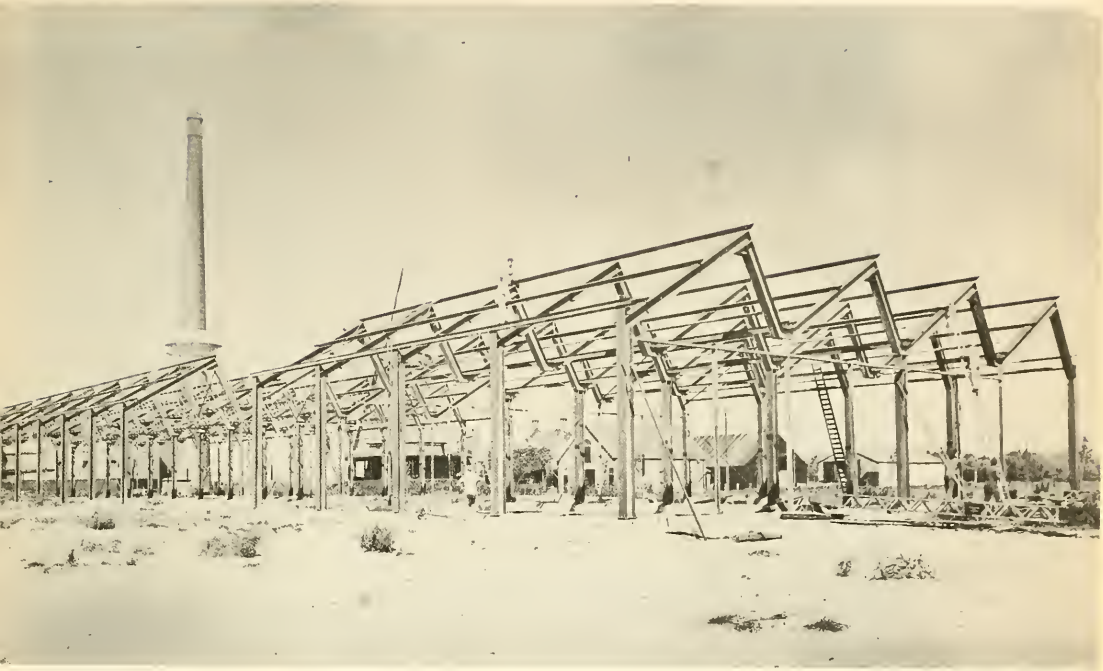


(Left)
Palmer Ave. Bridge, Yonkers, N. Y. Built 1931. (See page 94) (Courtesy of Arthur G. Hayden)

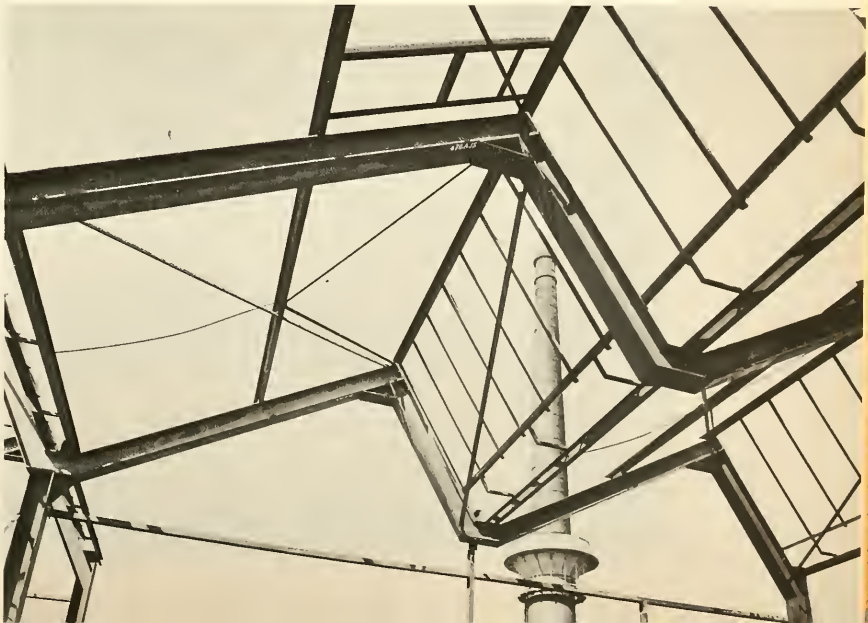


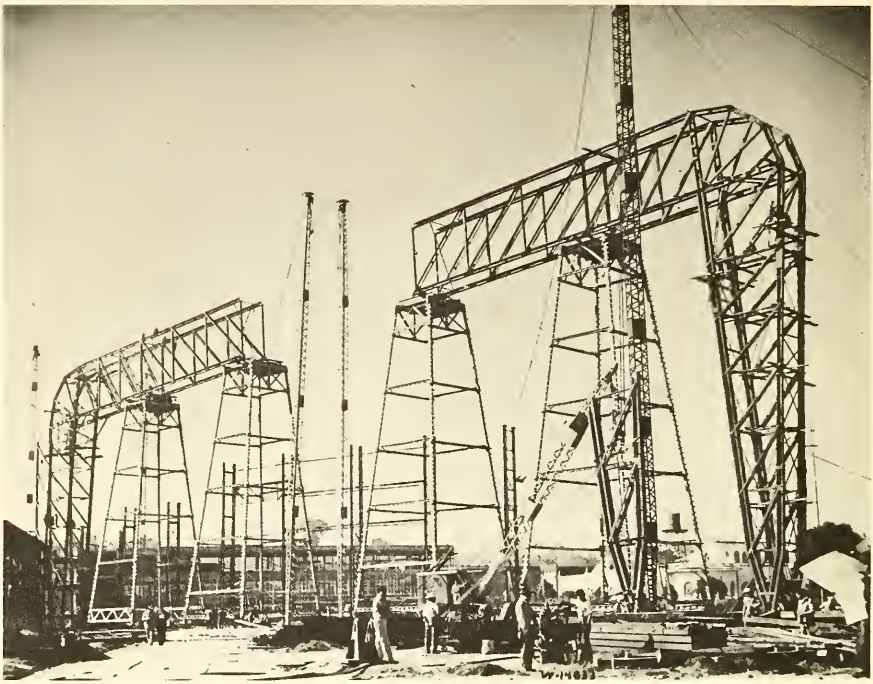
N. Y. C. R. R., West Side Improvement, N. Y. City

All-Welded Rigid Frame Bridge, Lave Ave., Greenwich, Conn. (Simple, strong and streamlined, designed by John F. Willis, now Engineer of Bridges & Structures. Gilded, ornamental iron

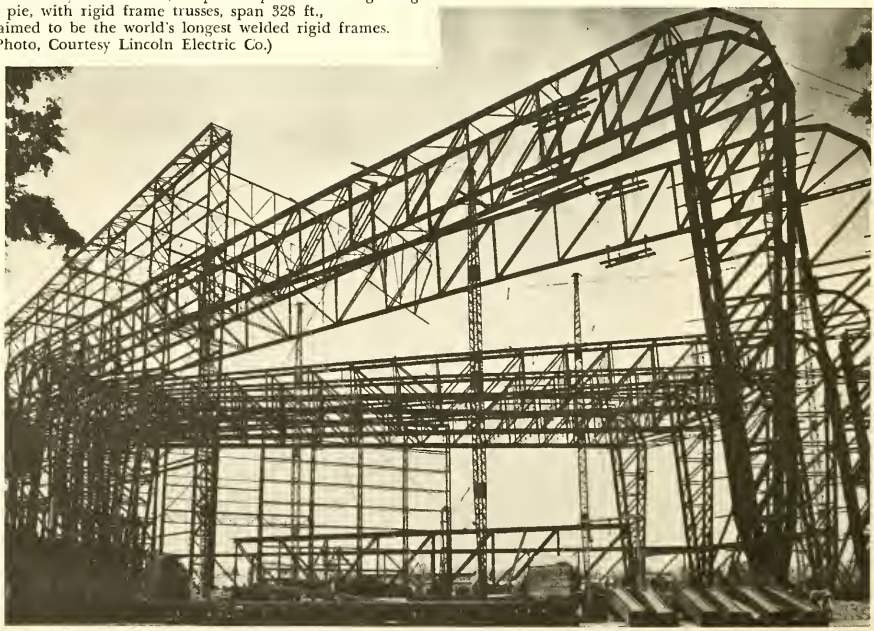


Chemical Plant, Holland.
(Courtesy of DeVries Robbe & Co., Holland)





Mexico City Auditorium, shaped in plan like a big wedge of pie, with rigid frame trusses, span 328 ft., claimed to be the world's longest welded rigid frames. (Photo, Courtesy Lincoln Electric Co.)



Date Due

MAY 6 '54			
OCT 14 '57			
DEC [illegible] '58			
MAY [illegible] '60			
MAY 16 '62			
APR 1967 LP			
SEP 11 '71	SEP [illegible] '71		
MAY 28 '75	MAY 28 '75		
[illegible] 12 '80	JUN 24 '80		
DEC 15 '83			
Ap27,87	APR 26 1984		
OCT 27 1985	OCT 22 1988		
MAY 10	MAY 16 1990		

85

Steel rigid frames manual: des sci
693.7K84s
3 1262 03232 4674

693.7
K84s